

FUNDAMENTALS OF CONTROL SYSTEM

① What is Control System?

It is an interconnection of components that gives desired outputs / response. The main ~~max~~ objective of any control system is to maintain the output at desired output.

Control System requires several elements / components like - Sensors, final control element to maintain the control variable at a reference input.

Transducer -

It is a device that transforms one type of physical quantity to another type (Electrical quantity)

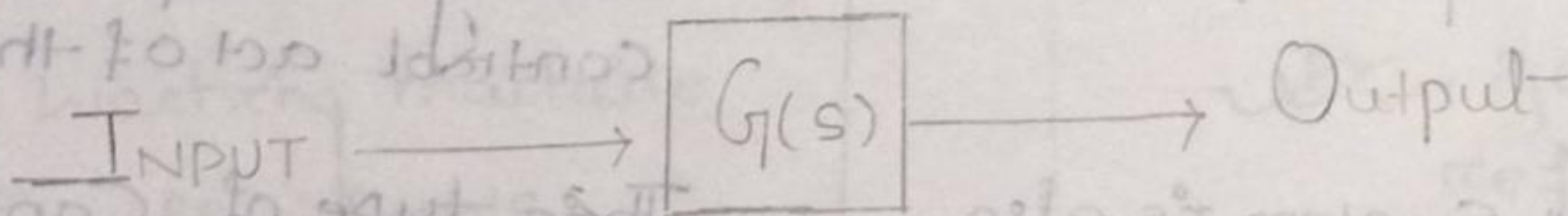
Classification Of Control System:-

They are classified into 2 types.

- (i) Open loop type
- (ii) Close loop type.

Open Loop Control System -

In this configuration the feedback of measurement is not connected to forward path or controller. It is observed that the output has no effect on control system.



I Advantages -

Simple in Construction.

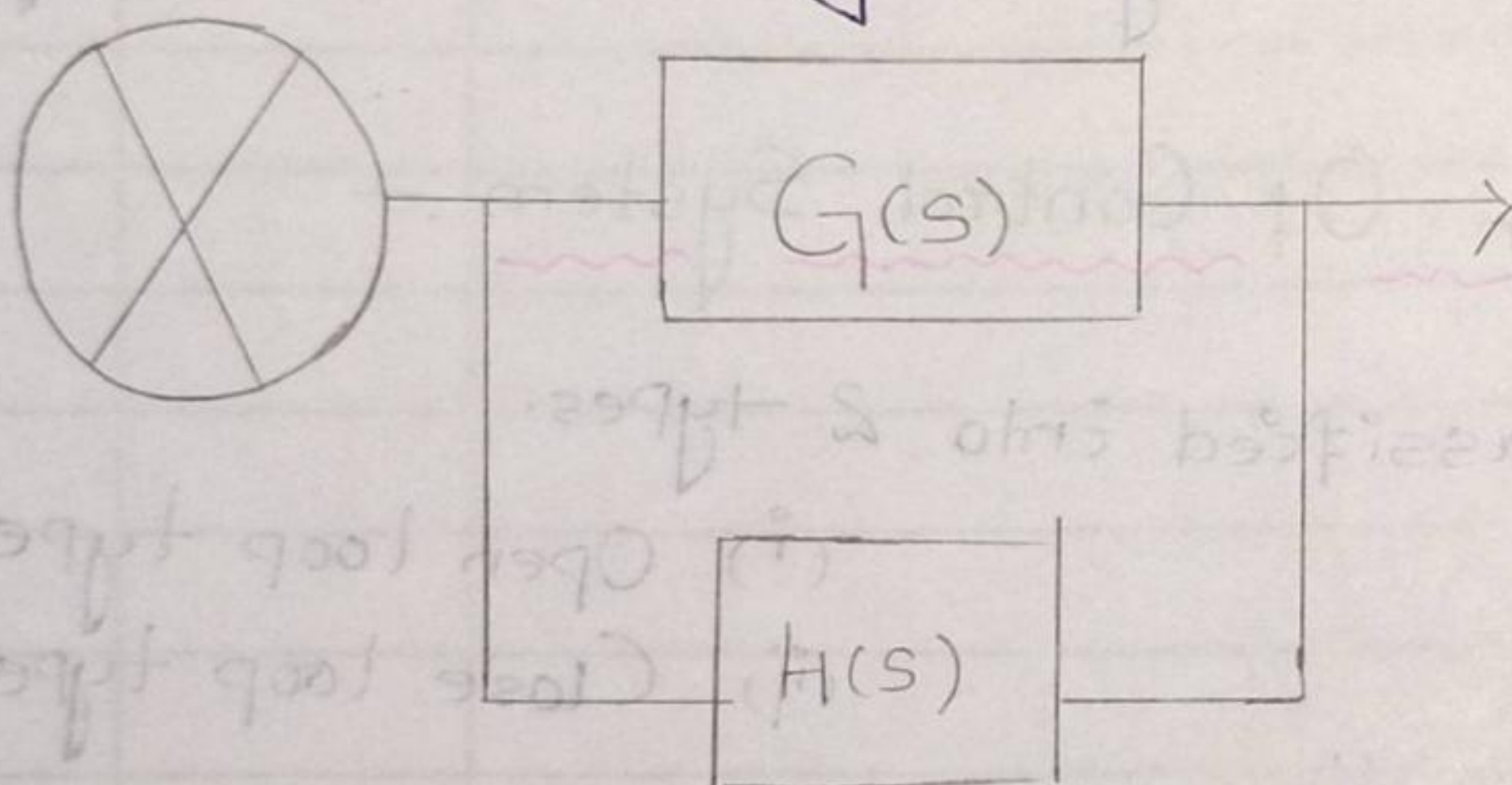
Less expensive.

II Disadvantages -

No accuracy.

Close Loop System -

In close loop Control System the output has an effect on Control action through a feedback.



Comparison between Open loop and Close loop :-

Open loop

In this system the Control action is free from output.

The Control System is also called a non-feedback Control System.

Closeloop

In this system the output mainly depends on the control act of the system.

This type of Control System is also called as feed back Control system.

The Components of this system includes a control process and Controller.

✓ The Construction of this system is simple.

✓ The Consistency is non-Reliable.

✓ The accuracy of this system mainly depends on calibration.

The Optimization in this system is not possible.

✓ The response is fast.

✓ The Calibration of the system is difficult.

The system are non-linear.

The disturbance of this system will be affected.

✓ Ex - Washing machine
Traffic lights
Tv remote
Immersion rod

The Components of this kind of system includes an amplifier, Control Process, Controller and feedback.

✓ The Construction of this system is complex.

✓ The consistency is reliable.

✓ They are accurate due to feedback.

✓ The optimization of this system is possible.

✓ The response is slow.

✓ The Calibration of the system is easy.

✓ The system are linear.

✓ The disturbance of this system will not be affected.

✓ Ex - Ac
refrigerator
Toaster

System -

A system is defined as the set of things working together as parts of mechanism or interconnecting network for a common purpose.

Control System -

It is defined as combination of elements arranged in a planned manner where an element causes an effect to produce a desired output.

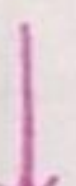
Reference
I/p



Controller



Plant



Output

$x(t) \rightarrow$ time domain

$R(s) \rightarrow$ frequency domain

$$T = \frac{1}{F}$$

$$\omega = 2\pi f$$

Control System Types -

Open loop Control System.

Close loop Control System.

Open Loop Control System:-

The system in which the output has no effect on the control action that is the control action is independent of the output as open loop control system.

Reference

input

(R)

Controller

(External Power)

Plant

(System)

Output

(C)

Ex - Tv remote

Toaster

Electric iron

Traffic light

Washing machine

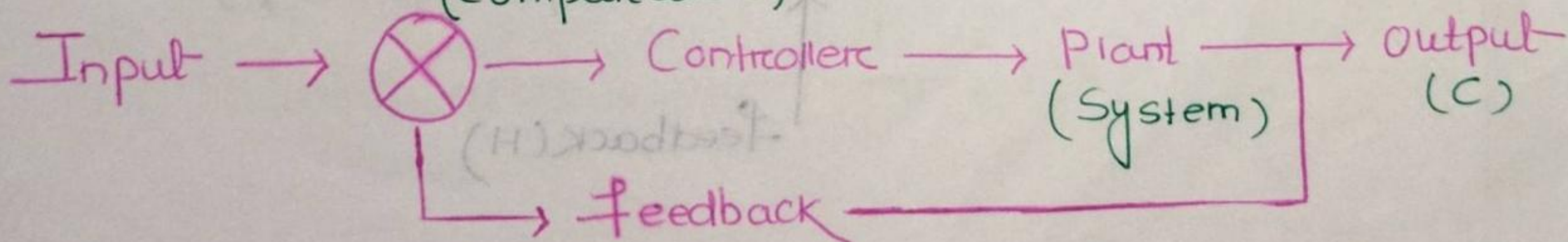
Fridge

It is otherwise called a non-feedback control system.

Close loop Control System:-

The control system in which the output has an effect on control action is called closed loop control system.

It is otherwise called feedback control system.
(Comparator)



Feedback:-

If either the output or some part of the output is returned to the input side and utilized as part of the system of input then it is called feedback.

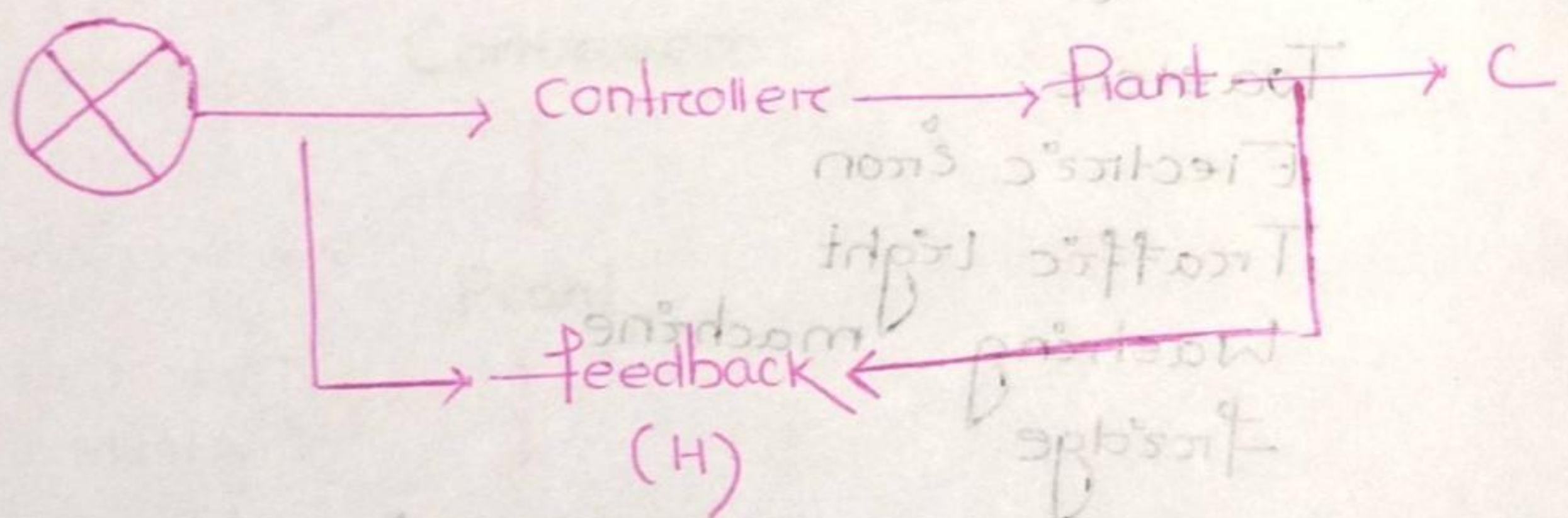
Feedback is of 2 types.

(i) Positive feedback

(ii) Negative feedback

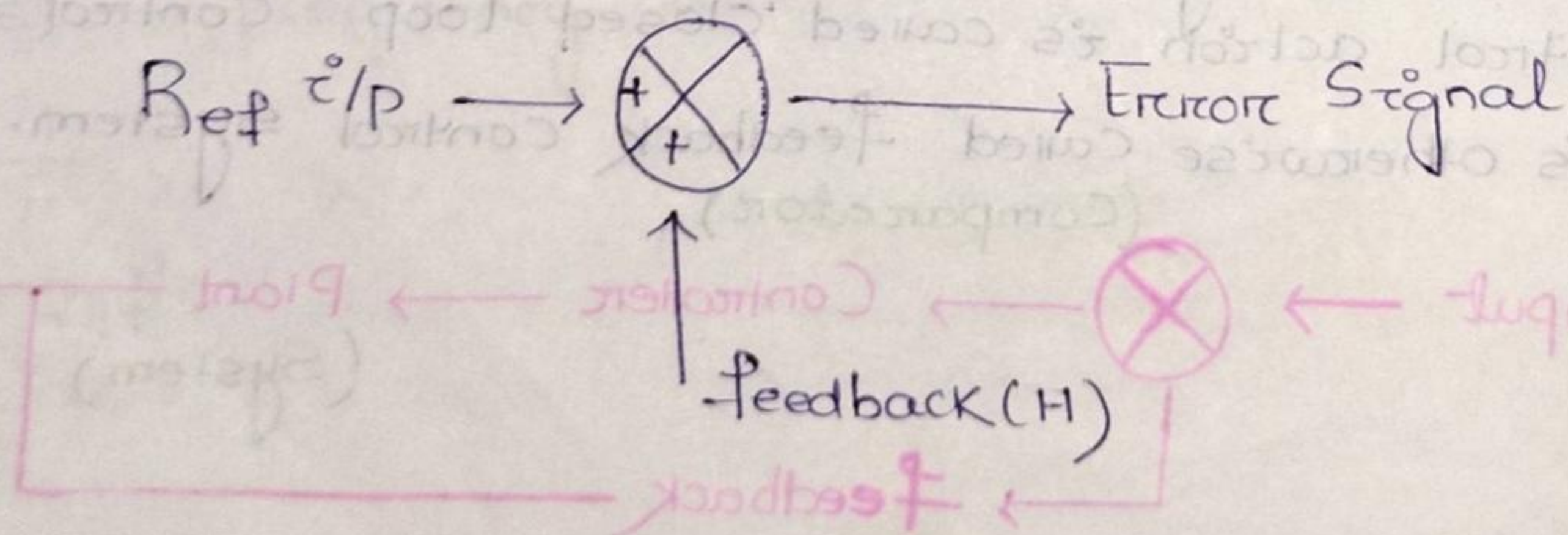
Positive feedback -

The control system in which error signal (E) is the sum of input (R) and feedback signal (H) is known as positive feedback.



$$E = R + H$$

Error signal = reference input + feedback

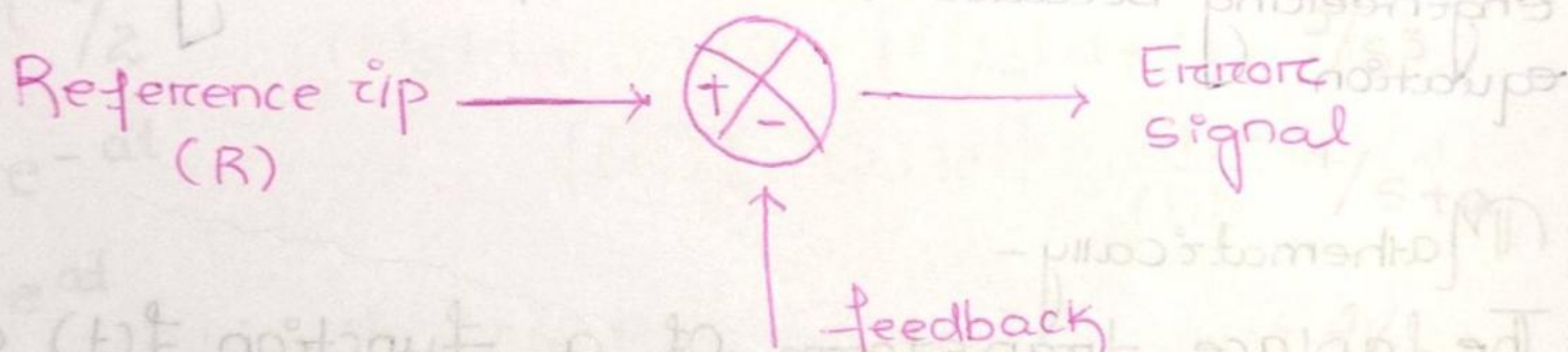
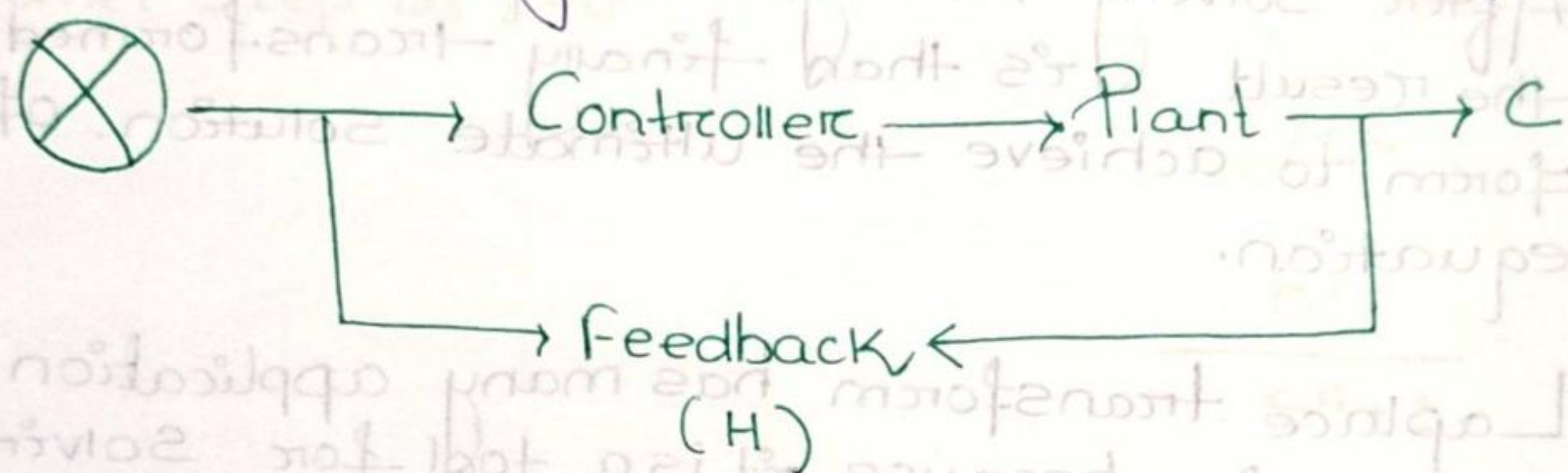


The effect of positive feedback is to increase the system gain.

Example — Electronic amplifier use as OPAMP (adder)
Loud Speaker.

Negative feedback —

The Control system in which the error signal is the difference between reference input (R) and feedback signal (H) is called negative feedback control system.



The effect of negative feedback is to reduce the system gain.

Negative feedback produces a stable circuit response and improves the stability of the system.

Ex — An amplifier used in a Subtractor.

LAPLACE TRANSFORM:

Inventor - Pierre Simon Laplace.

Laplace transform is an integral transform that converts a function of a real variable t , time to a function of a complex variable (s) complex frequency.

It is a technique for solving differential equation. Here differential equation of ~~domain~~ time domain form is first transformed to algebraic equation of frequency domain form.

After solving the algebraic equation in frequency domain, the result is then finally transformed to time domain form to achieve the ultimate solution of differential equation.

Laplace transform has many application in science & engineering because it is a tool for solving differential equation.

Mathematically -

The Laplace transform of a function $f(t)$ defined for all real number $t > 0$, is function of $F(s)$.

So,

$$F(s) = L[f(t)] = \int_0^{\infty} f(t) \cdot e^{-st} dt$$

Where $f(t) \rightarrow$ function in time domain

$f(s) \rightarrow$ function of frequency domain

$L[f(t)] \rightarrow$ Laplace transform of $f(t)$.

Inverse Laplace Transform -

$$L^{-1}[L\{f(t)\}] = L^{-1}[f(s)] = f(t)$$

Thus we can say that the time function $f(t)$ and its Laplace transform $f(s)$ are a transform pair.

It-unction Of $F(t)$:-

$$F(s) = L[f(t)]$$

- | | |
|--|---------------------------------|
| 1. $\delta(t)$ = unit impulse at $t=0$ | 1 |
| 2. $u(t)$ = unit step at $t=0$ | $1/s$ |
| 3. t | $1/s^2$ |
| 4. t^n | $\frac{n!}{s^{n+1}}$ |
| 5. $t^2/2$ | $1/s^3$ |
| 6. e^{-at} | $1/s+a$ |
| 7. e^{at} | $1/s-a$ |
| 8. $t \cdot e^{-at}$ | $1/(s+a)^2$ |
| 9. $t \cdot e^{at}$ | $1/(s-a)^2$ |
| 10. $t^n \cdot e^{-at}$ | $\frac{n!}{(s+a)^{n+1}}$ |
| 11. $\sin \omega t$ | $\frac{\omega}{s^2 + \omega^2}$ |
| 12. $\cos \omega t$ | $\frac{s}{s^2 + \omega^2}$ |

$$12. t^n \cdot e^{at}$$

$$13. e^{-at} \sin \omega t$$

$$14. e^{-at} \cos \omega t$$

$$15. \sinh at$$

$$16. \cosh at$$

$$17. u(t=T) \text{ unit step at } (t=T)$$

$$18. \text{Basic Laplace Transform Theorem—}$$

$$1) \quad L[a f_1(t) + b f_2(t)] = a F_1(s) + b F_2(s)$$

$$(a) \quad L \left[\frac{df(t)}{dt} \right] = [sF(s) - f(0)]$$

$$2) \quad L \left[\frac{d^2 f(t)}{dt^2} \right] = [s^2 F(s) - s f(0) - f'(0)]$$

$$L \left[\frac{d^3 f(t)}{dt^3} \right] = [s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)]$$

Where

$f(0), f'(0), f''(0), \dots$ are value of $f(t)$.

$\frac{df(t)}{dt}, \frac{d^2 f(t)}{dt^2}, \dots$ at $t=0$

3.

$$L \int f(t) = \left[\frac{F(s)}{s} + \frac{f^{-1}(0t)}{s} \right]$$

$$L \iint f(t) = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0t)}{s^2} + \frac{f^{-2}(0t)}{s} \right]$$

$$L \iiint f(t) = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0t)}{s^3} + \frac{f^{-2}(0t)}{s^2} + \frac{f^{-3}(0t)}{s} \right]$$

Where $f^{-1}(0t)$, $f^{-2}(0t)$, $f^{-3}(0t)$ are the value of $f(t)$, $\iint f(t)$, $\iiint f(t)$ at $t = 0t$

$$L[f(t)] = f(s) \text{ then}$$

$$L[e^{-at} f(t)] = F(s+a)$$

$$L[t \cdot f(t)] = -\frac{d}{ds} F(s)$$

Initial Value theorem -

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s \cdot F(s)$$

Final Value theorem -

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \cdot F(s)$$

The purpose of initial value theorem is to determine the initial value of the function $f(t)$.

The purpose of final value theorem is to determine the final value of function $f(t)$.

Inverse Laplace Transform -

Inverse Laplace Transform Converts a function of Complex Variable 's' to a function of real variable 't' time.

We can say that it converts algebraic equation to differential equation.

Some inverse Laplace Transform formulas -

$$\mathcal{L}^{-1} \left(\frac{1}{s} \right) = 1$$

$$\mathcal{L}^{-1} \left(\frac{1}{s-a} \right) = e^{at}$$

$$\mathcal{L}^{-1} \left[\frac{1}{s^n} \right] = \frac{t^{n-1}}{(n-1)!}, \quad n = 1, 2, 3, \dots$$

$$\mathcal{L}^{-1} \left[\frac{1}{(s-a)^n} \right] = \frac{e^{at} t^{n-1}}{(n-1)!}$$

$$\mathcal{L}^{-1} \left[\frac{1}{s^2 + b^2} \right] = \frac{1}{b} \sin bt$$

$$\mathcal{L}^{-1} \left[\frac{3}{s^2 + a^2} \right] = \cos at$$

$$\mathcal{L}^{-1} \left[\frac{1}{s+a} \right] = e^{-at}$$

$$\mathcal{L}^{-1} \left[\frac{1}{s-a} \right] = e^{at}$$

Classification of Control System -

Time Variant (R, L, C)

Time Variant (AC, traffic light)

Linear (obey Ohm's law)

Non-linear

Continuous

discontinuous

Lumped Parameters

distributed Parameters

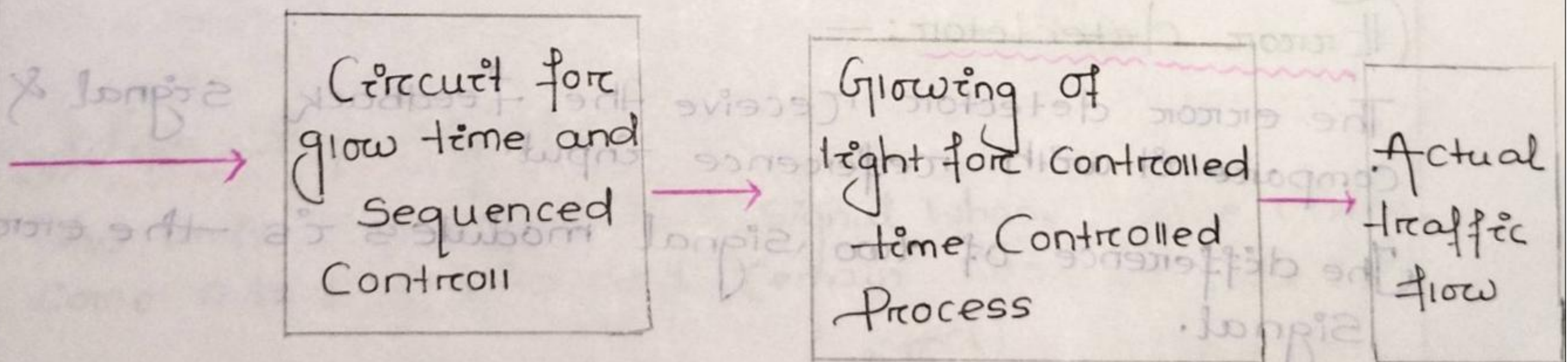
SISO

MISO

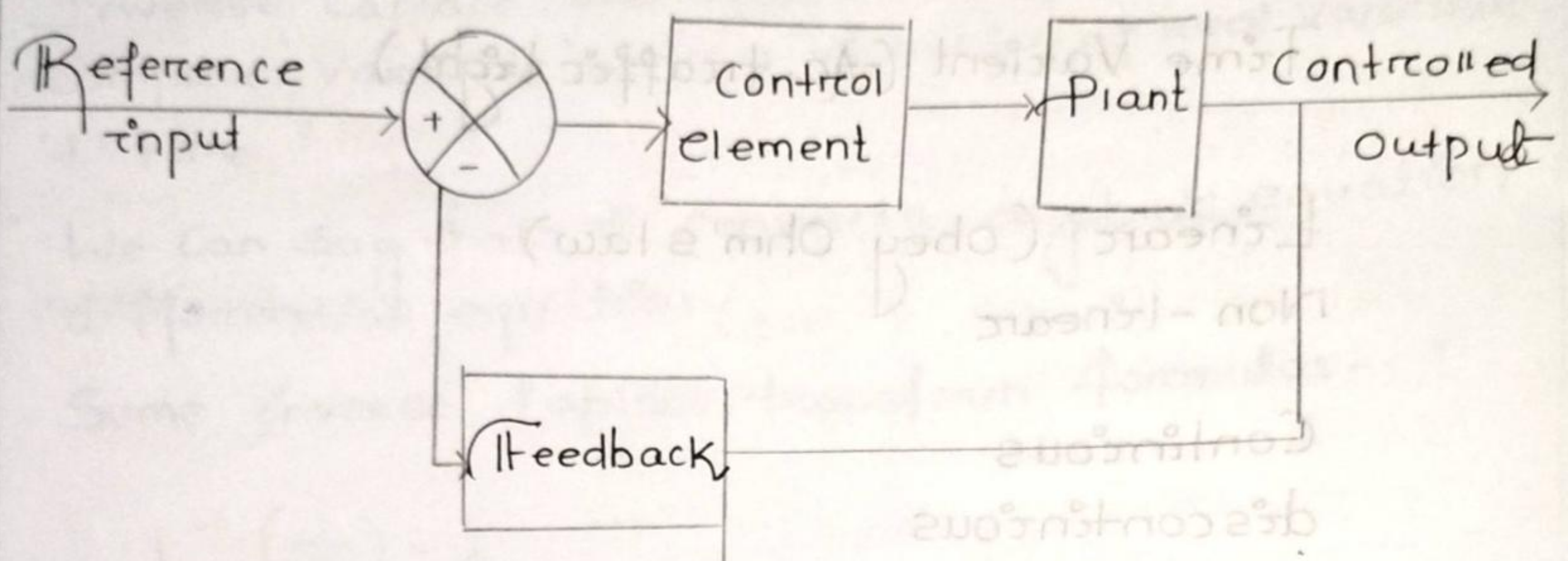
Static System

Dynamic System.

Traffic Control System :-



Automatic Control System -



Reference Input :-

It is the actual signal input to control system.

Control Variable Output Signal :-

It is control response obtained from a control system which must be maintain at a prescribe balance value.

Feedback Element :-

The element feedback the output in order to compare with this reference input.

Error detector :-

The error detector receive the feedback signal & compare it with reference input.

The difference of two signal modules is the error signal.

Control Element :-

This regulate the output according to the signal obtained from error detector.

Plant :-

It is the body or machine with a particular output to be control.

Standard Test Signal :-

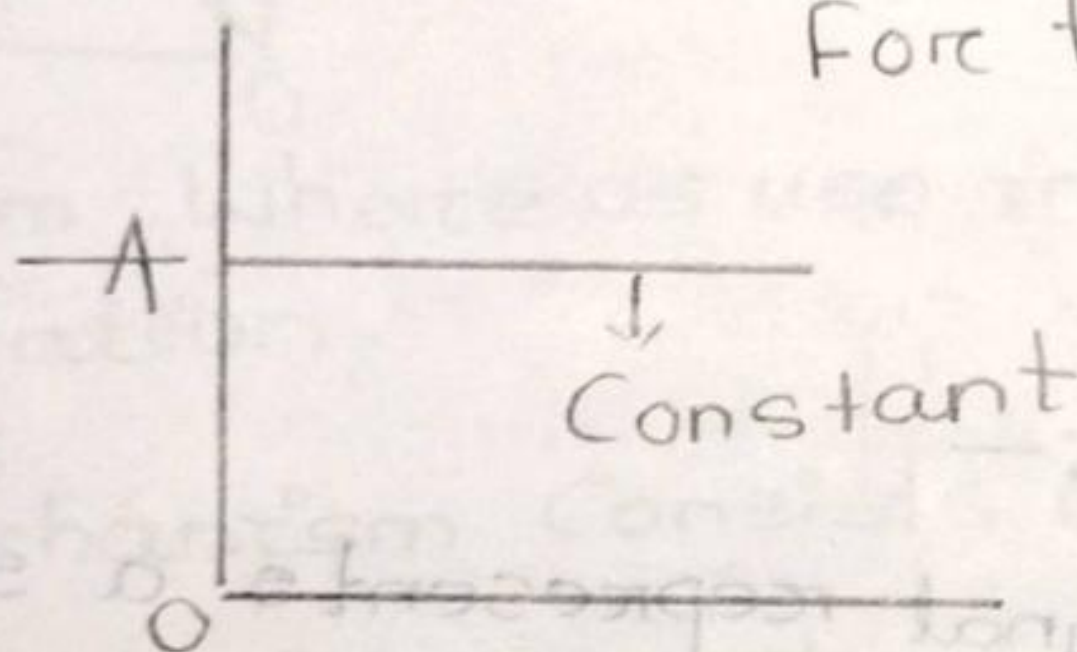
Test Signal are used to know the Performance of Control System time response of the output.

Step Signal :-

The Step Signal is a signal whose value changes from '0' to 'a' at $t=0$ and remain constant at a for $t > 0$

$$L[\pi(t)] = R(s) = L[A] = \frac{A}{s}$$

For $t > 0$



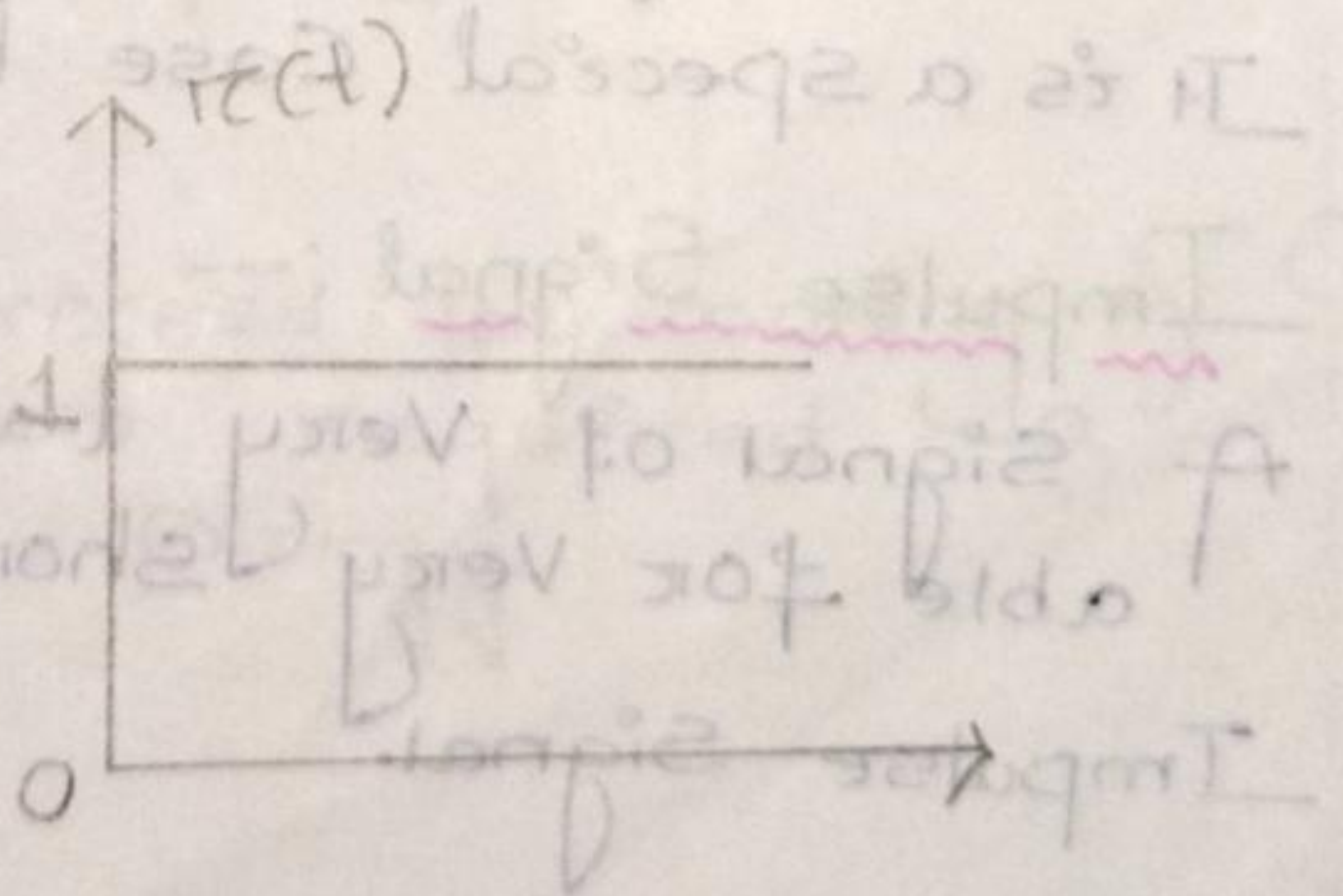
$$\pi(t) = \begin{cases} A, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Unit Step Signal :-

The unit step signal is a signal whose value change from 0 to 1 at $t=0$ and remain constant at 1 for $t > 0$.

$$\pi(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$L[\pi(t)] = R(s) = [1] \cdot \frac{1}{s}$$



Ramp Signal:-

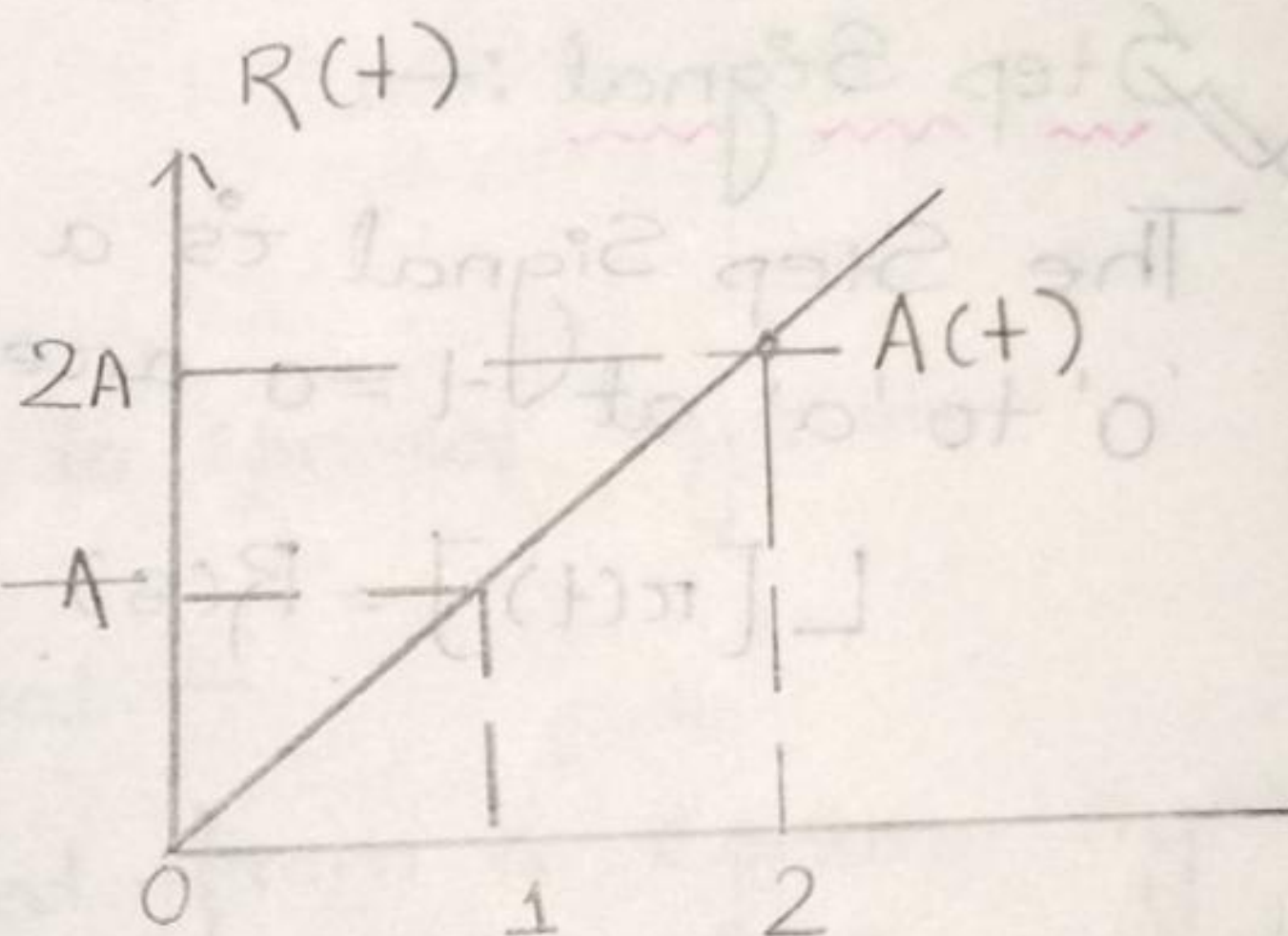
✓ Ramp Signal starts as the value of zero and increases linearly with time.

Ramp Signal is the integral of step signal.

The Ramp signal has the ability to test how the system response to a signal ~~has~~ in change ~~light~~ with respect to time.

$$r(t) = \begin{cases} A(t), & t > 0 \\ 0, & t < 0 \end{cases}$$

$$\begin{aligned} R(s) &= L[r(t)] \\ &= L[A(t)] \\ &= \frac{A}{s^2} \end{aligned}$$



Parabolic Signal:-

The parabolic signal represents a signal that one order first than ramp function.

The Parabolic signal is the integral as a ramp signal.

Area under ~~parabolic~~ signal is ramp signal is equal to parabolic signal.

Parabolic signal is a constant acceleration input to the system.

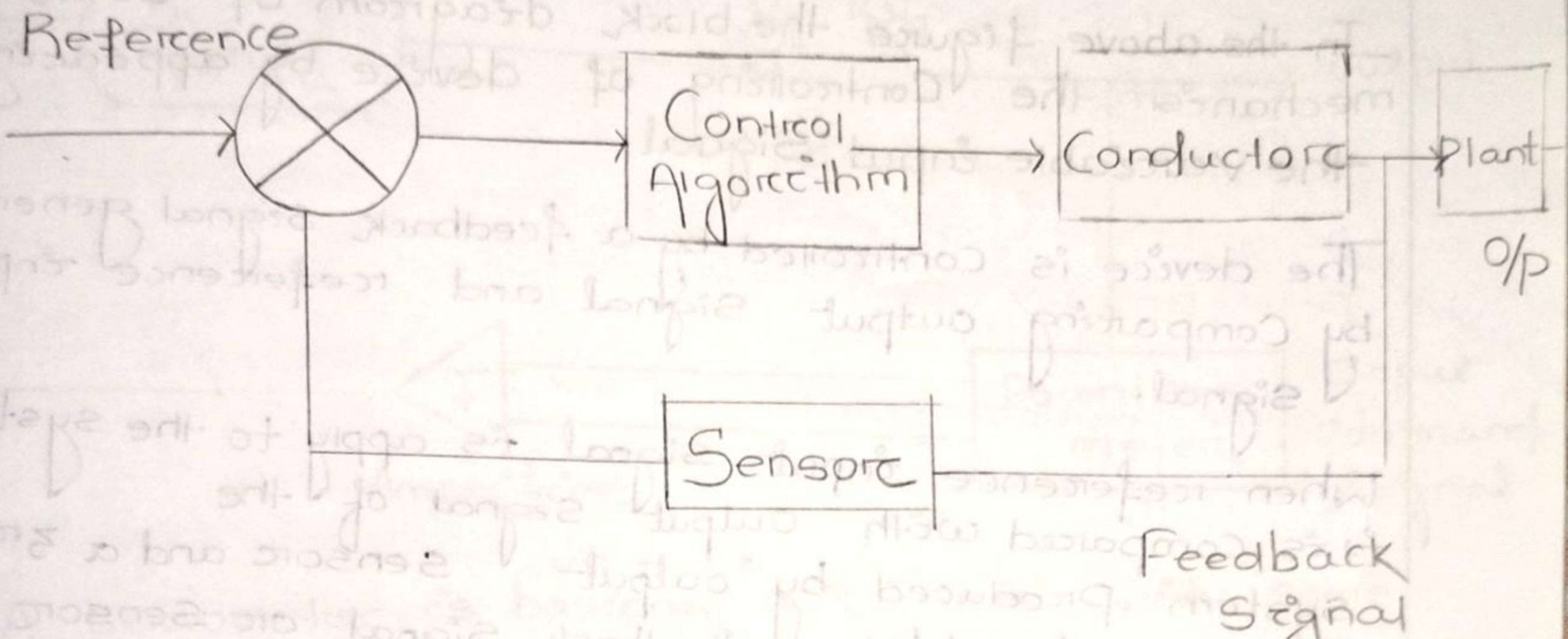
It is a special case Parabolic signal.

Impulse Signal:-

A signal of very large magnitude which is available for very short duration is called

Impulse signal.

SERVO MECHANISM :-



Servo mechanism is an close loop Control system. A servo mechanism can be called as Servomechanism when use for error Sensing of -ve feedback to correct the performance of a mechanism.

Servo mechanism Where as use in fire Control, firing, machine navigation.

The servo mechanism Consists of 3 basic Component.

- Control device.
- Output Sensor
- Feedback Signal

Control Algorithm :-

It is defined as a mathematically representation of the control action to prefer

Actuator -

It is a device that is process for moving & controlling mechanism. or a system required a Control Signal or Source Energy.

Note:-

In the above figure the block diagram of Servo-mechanism the Controlling of device by applying the Variable input Signal.

The device is controlled by a feedback signal generated by comparing output signal and reference input signal.

When reference input signal is applied to the system it is compared with output signal of the system produced by output sensor and a 3rd signal produced by feedback signal or sensor signal.

This feedback signal acts as an input signal of controlled device.

The Primary Servo mechanism cannot maintain the output of system at desired level in the presence of disturbance.

WORKING OF SERVO MECHANISM:-

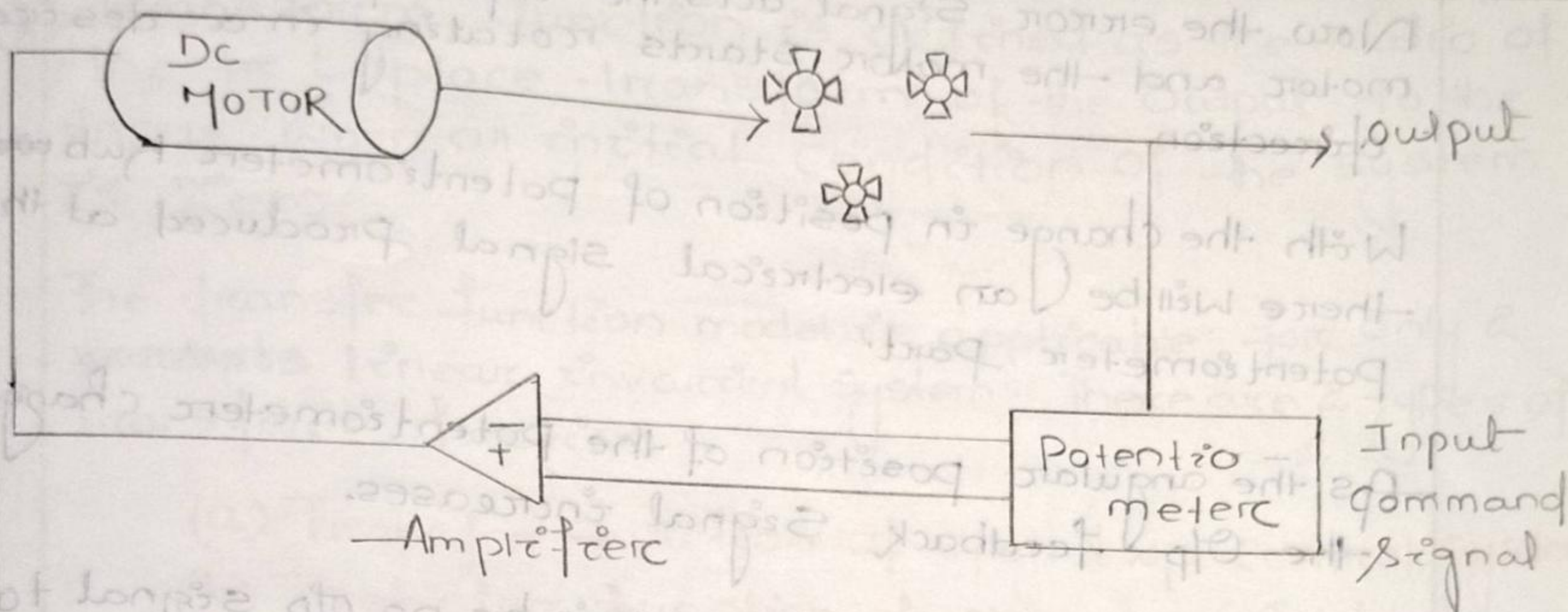
A Servo motor is basically a dc motor with special components that are-

Dc motor

Potentiometer

Gear Arrangement

Intelligent Circulation



A servometer is basically a DC motor with some special components that are :-

1. A DC motor
2. A potentiometer
3. A gear arrangement
4. An intelligent circuitry

At initial position or servomotor shaft, the position of the potentiometer is that there is no electrical signal generated at the potentiometer.

This output part of the potentiometer is connected with one of the input terminals of the error detector amplifier. Then the difference between these two signals i.e. the input signal from external power source and the feedback signal from potentiometer will be amplified in the error detector amplifier and feeds the DC motor.

Now the error signal acts the input power to DC motor and the motor starts rotating in a desired direction.

With the change in position of potentiometer knob there will be an electrical signal produced at the potentiometer part.

As the angular position of the potentiometer changes the OP feedback signal increases.

After this condition there will be no OP signal to the motor is zero at that position the motor stops rotating.

Advantages -

Higher torque

Higher speed

Control speed

Speed is proportional to applied voltage.

Disadvantages -

More complex

Higher cost for feedback use.

Maintenance is difficult.

Application -

1. It was first used in military fire and marine navigation equipment.
2. It is used in automatic machine tools, satellite tracking antennas and remote control air planes.
3. Autofocus Camera.

TRANSFORM FUNCTION

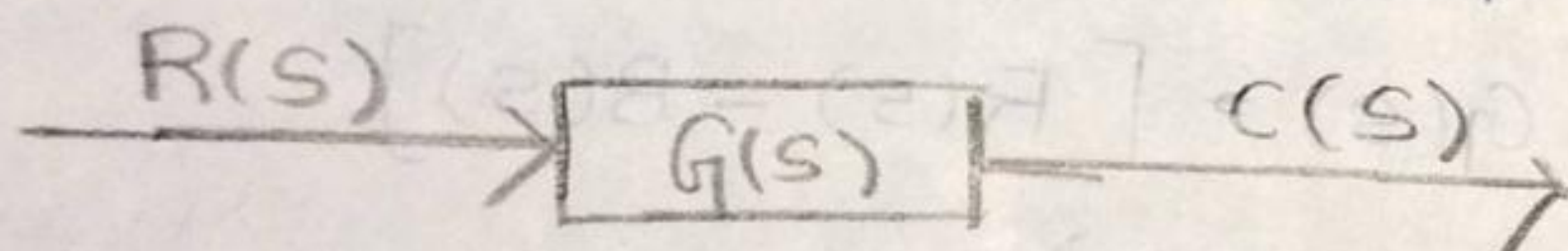
Q. Transform function is defined as the ratio of the Laplace transform of the output to the input, when all initial condition of the system is zero.

The transfer function model is applicable for only 2 ~~variables~~ linear invariant system. There are 2 types of Transform function.

- (a) Transfer function of open loop control system
- (b) Transfer function of close loop control system.

Transfer function of open loop control system :-

The transfer function = $\frac{\text{Laplace transform output}}{\text{Laplace transform input}}$ with initial zero condition.



Transfer function = $R(s)$ = reference input

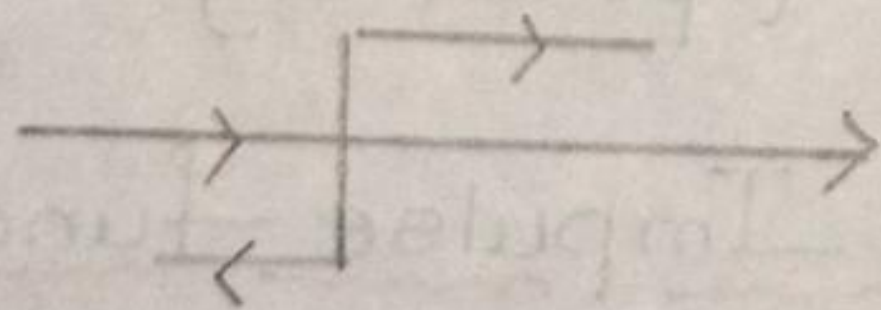
$C(s)$ = output

$G(s)$ = Transfer function.

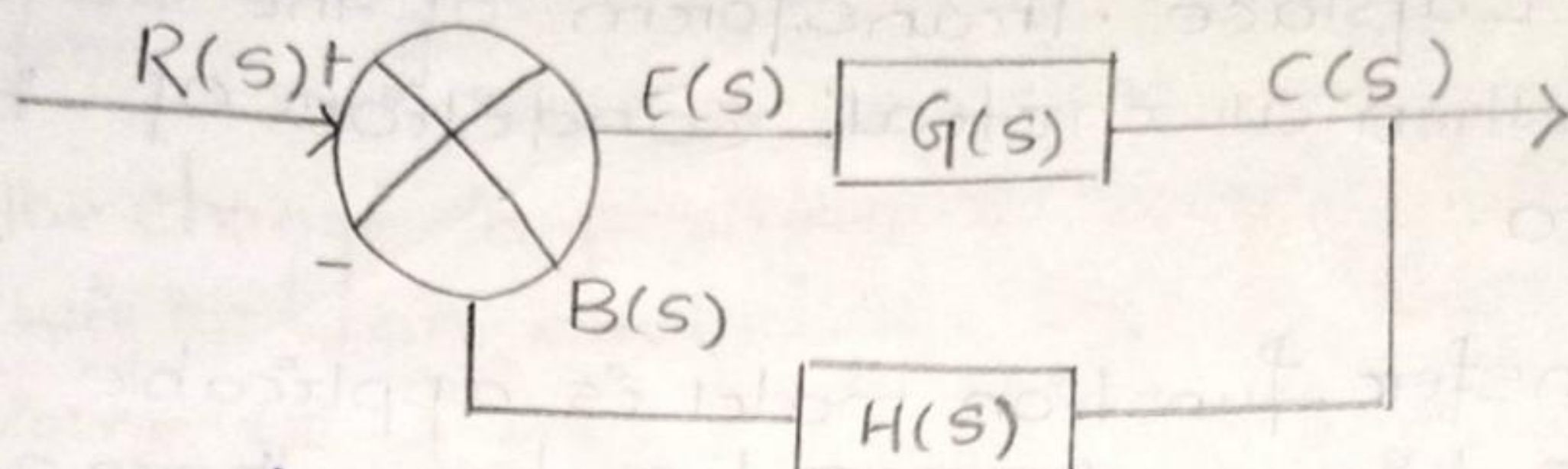
$$T(f) = \frac{C(s)}{R(s)}$$

The flow of system variable from one block to another block is represented by arrow.

The takeoff point of a signal is represented by



Transfer function of close loop Control system:-



$R(s)$ = reference input

$C(s)$ = output Control

$B(s)$ = feedback signal

$H(s)$ = feedback path of transfer function

$E(s)$ = actuating error signal

$$T.F = \frac{\text{Output}}{\text{input}} = \frac{C(s)}{R(s)}$$

$$\Rightarrow C(s) = E(s) \cdot G(s)$$

$$\Rightarrow E(s) = R(s) - B(s)$$

$$C(s) = G(s) \cdot [R(s) - B(s)]$$

$$B(s) = C(s)H(s)$$

$$C(s) = G(s) \cdot [R(s) - C(s)H(s)]$$

$$C(s) = G(s)R(s) - G(s)C(s)H(s)$$

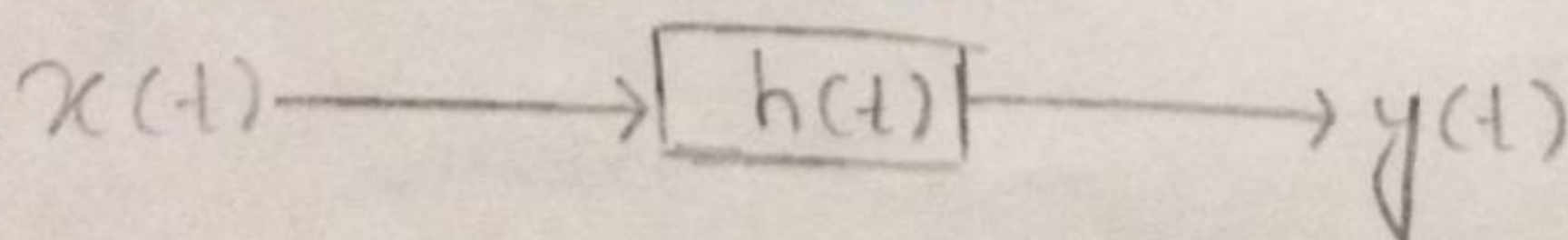
$$\Rightarrow C(s)[1 + G(s)H(s)] = G(s)R(s)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad [\text{Negative}]$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} \quad [\text{positive}]$$

Transfer function and Impulse function:-

The response of the system function of an impulse is called as input of signal.



$$y(t) = h(t)x(t)$$

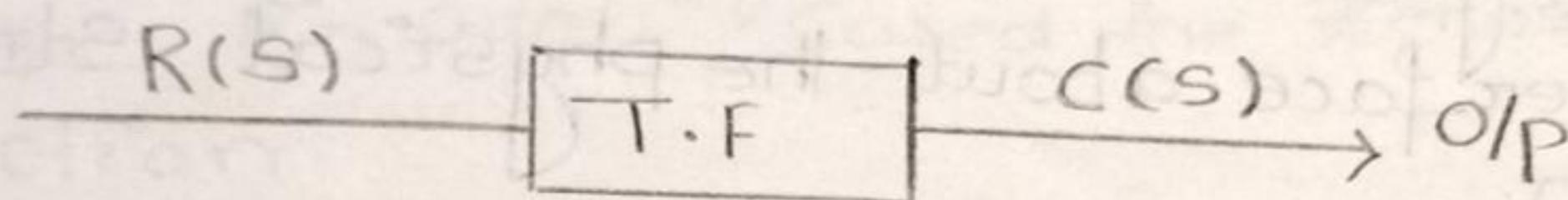
$$y(\omega) = x(\omega) \times h(\omega) \quad [\because \text{fourier Transfer function}]$$

$$h(\omega) = \frac{y(\omega)}{x(\omega)}$$

If we consider inverse fourier transfer function then $f^{-1}[h(\omega)] = h(t)$

Purely impulse transform function related the transform of impulse response is nothing but transfer function is ratio of output transform and input transform

Properties Of Transfer function :-



Mathematically model the differential equation that related the O/p and the I/p in the system.

Independent of the magnitude and nature of the input.

It does not provide any information about physical structure of the system.

Transfer function of a physical difference can be identical.

If the transfer function is known as the O/p reference can be studies or various input to understand the nature of the system.

Advantages Of Transfer function :-

If transfer function of the system is known the response of the system to any input can be determine very easily.

Transfer function is the mathematical model and give gain of the system.

It involves the terms as simple algebraic expression and no differential terms are present.

Poles and Zeros of the system can be determined of the knowledge of transfer function of the system.

Disadvantages Of Transfer function -

Transfer function doesnot takes into account the initial condition of the system.

Transfer function can be defined by only for linear system.

No interface about the physical structure of the system.

It is applicable only for SISO system.

To find frequency respond we need to shift the system in pure domain with mathematically relationship.

Poles & Zeros Transfer function:-

The transfer function of control system can be expressed in the form of B-ratio of the polynomial

$$G(s) = \frac{C(s)}{R(s)}$$

$$G(s) = \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}$$

The N^r and D^r can be factorises to Z and P .

$$G(s) = \frac{K(s-z_1)(s-z_2) \dots (s-z_n)}{(s-p_1)(s-p_2) \dots (s-p_m)}$$

Where $K = \frac{a_0}{b_0}$ is known as the gain factor of transfer function.

Poles Of Transfer function:-

In the transfer function if s is equal to $p_1, p_2, p_3, \dots, p_n$ the value of transfer function infinity. As $p_1, p_2, p_3, p_4, \dots, p_n$ are the poles of the transfer function.

Zeros Of Transfer function:-

In the transfer if s is put equal to $z_1, z_2, z_3, \dots, z_n$ the value of transfer function becomes zero. As $z_1, z_2, z_3, \dots, z_n$ are called the zeros of the transfer function.

It is possible as either poles or zeros such poles and zeros are called multiple poles or multiple zeros.

Otherwise non - Considering poles or zeros are called simple poles or simple zeros.

The poles or zeros occur due to the presence of respective factor in the $D(s)$ or $N(s)$ transfer function.

Poles is $\times$ and zero is o . The set symbols are used where pole and zero are to be shown on real axis and imaginary axis.

If the $z > p$ then the value of transfer function is found to be infinity (∞) or $s = \infty$.

Hence it is concluded that there exists poles of the transfer function at ∞ .

An the order of such a pole $z > p$.

If $P > Z$ then the value of the transfer function found to zero or $s = \infty$ hence it is concluded that there exists a zero of transfer function at ∞ .

An Order of such a zero $P > Z$.

Question - 1

Obtain the pole and zero plot of the following function.

$$f(s) = \frac{s(s+2)}{(s+1)(s+3)}$$

Zero

$$s+2=0$$

$$s = -2$$

$$s = 0$$

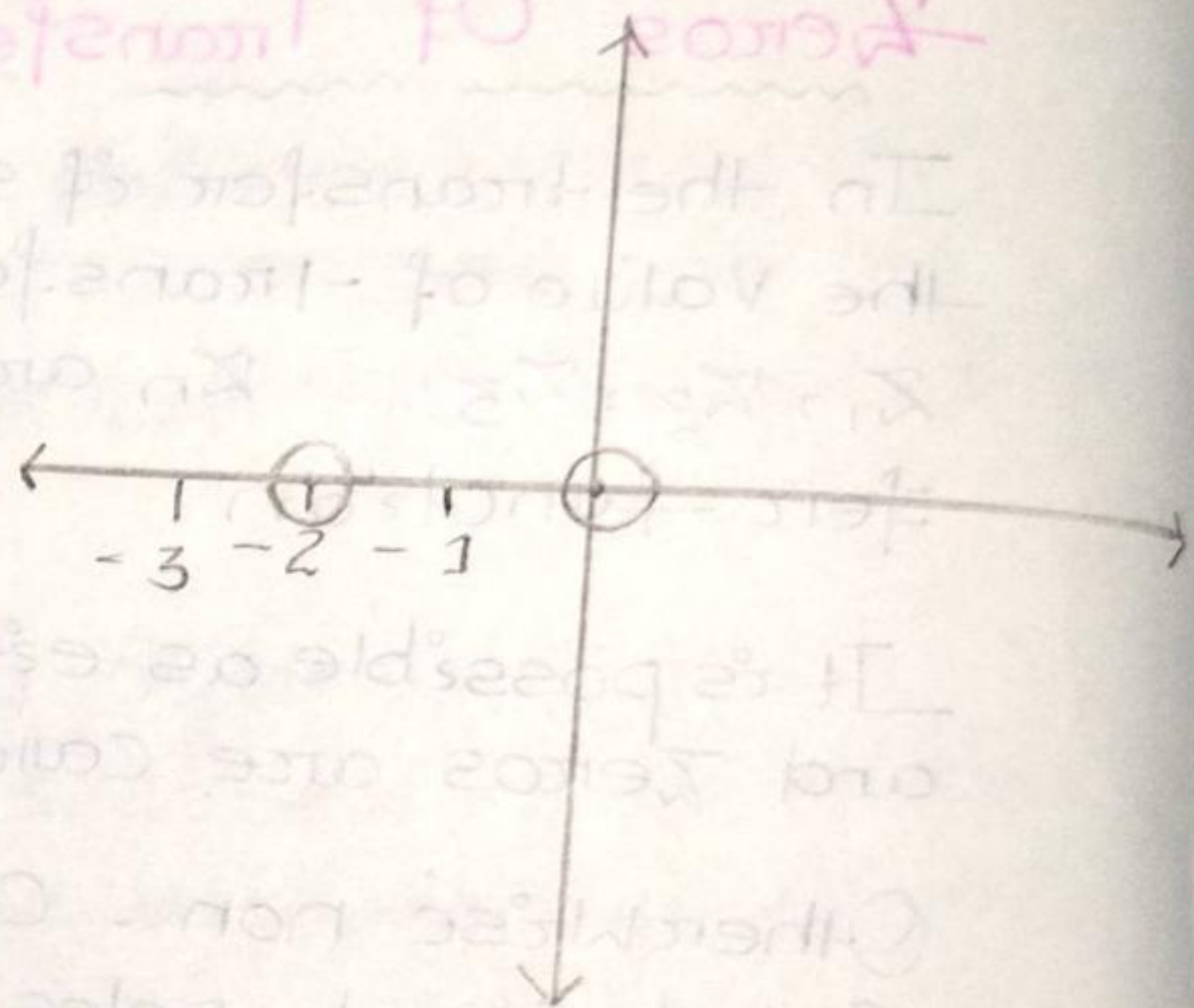
Poles

$$s+1=0$$

$$\Rightarrow s = -1$$

$$s+3=0$$

$$\Rightarrow s = -3$$



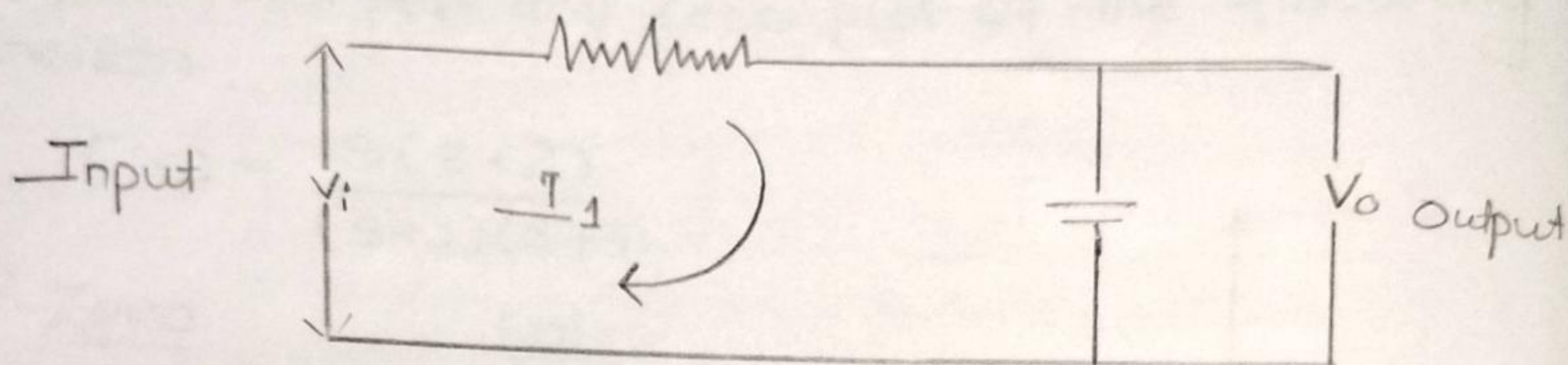
Question - 2

$$\frac{s(s+1)(s+3)}{(s+4)(s+6)}$$

Simple Problem Of Transfer Function Network:-

The transfer function of an electrical system consisting of resistance inductance and capacitance is determined by taking the Laplace transform of output to input.

The transfer of the R-C network -



$$\text{Transfer function } G(s) = \frac{C(s)}{R(s)}$$

Applying KVL in I_1

$$V_i(t) = R i(t) + \frac{1}{C} \int i(t) dt$$

Initial Condition

$$V_i^0 = R i^0 + \frac{i^0}{C}$$

$$V_i(t) = R i(t) + \frac{1}{C} \int i(t) dt \quad \text{--- (i)}$$

Applying KVL in loop 2 mesh 2

$$0 = \frac{1}{C} \int i(t) dt \quad \text{--- (ii)}$$

Now the Laplace transform on both sides

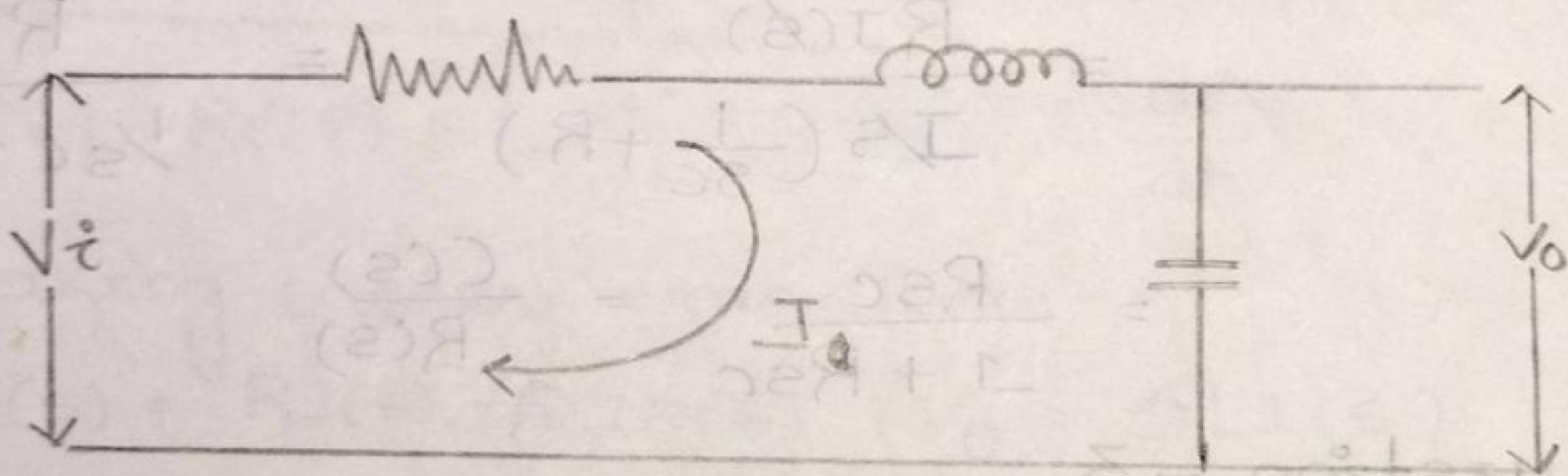
$$V_i(s) = R i(s) + \frac{1}{sC} I(s) \quad \text{--- (iii)}$$

$$V_o = \frac{1}{sC} I(s) \quad \text{--- (iv)}$$

$$\text{T.F} = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC} I(s)}{R I(s) + \frac{1}{sC} I(s)}$$

ELECTRICAL NETWORK

Question no-1



$$V_s(t) = R i(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i(t) \cdot dt$$

$$V_o(t) = \frac{1}{C} \int i(t) dt$$

By taking Laplace transform

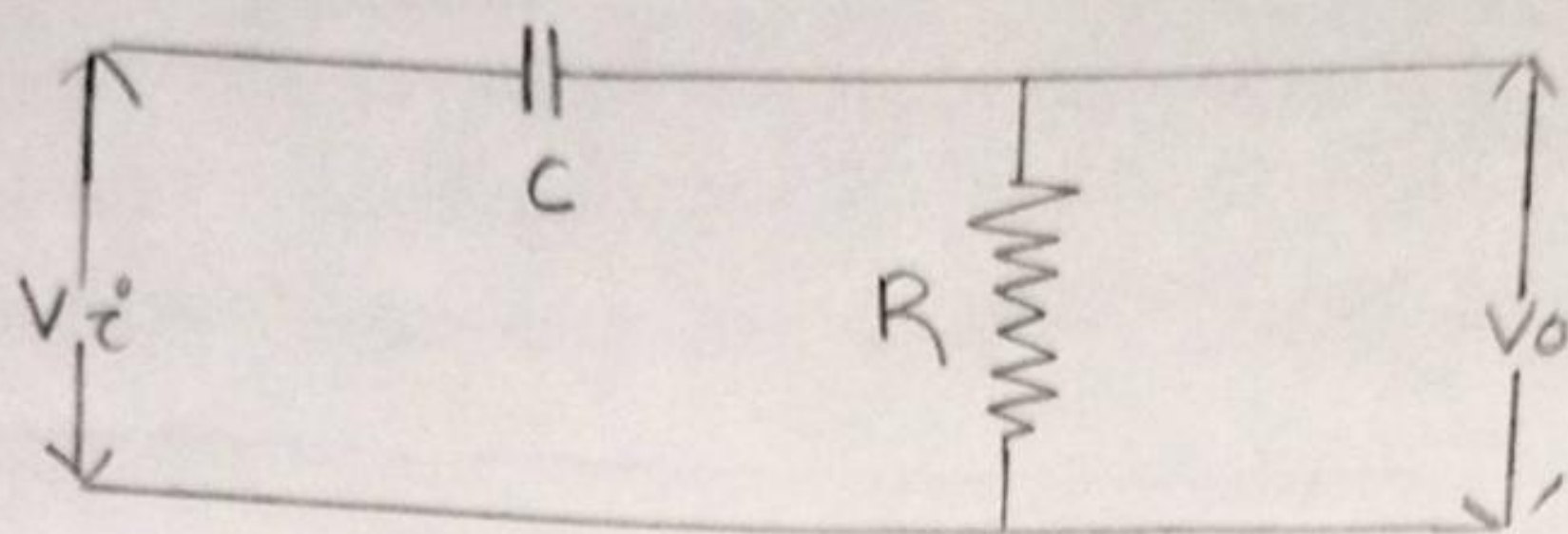
$$V_I(s) = R I(s) + sL I(s) + \frac{1}{sC} I(s)$$

$$V_o(s) = \frac{1}{sC} I(s)$$

Transfer function -

$$\begin{aligned} \frac{V_o(s)}{V_s(s)} &= \frac{\frac{1}{sC} I(s)}{(R + sL + \frac{1}{sC}) I(s)} \\ &= \frac{1/sC}{Rsc + s^2Lc + 1} = \frac{1}{Rs} \end{aligned}$$

Question - 2



$$V_i(t) = \frac{1}{C} \int i(t) dt + R i(t)$$

$$V_o(t) = R i(t)$$

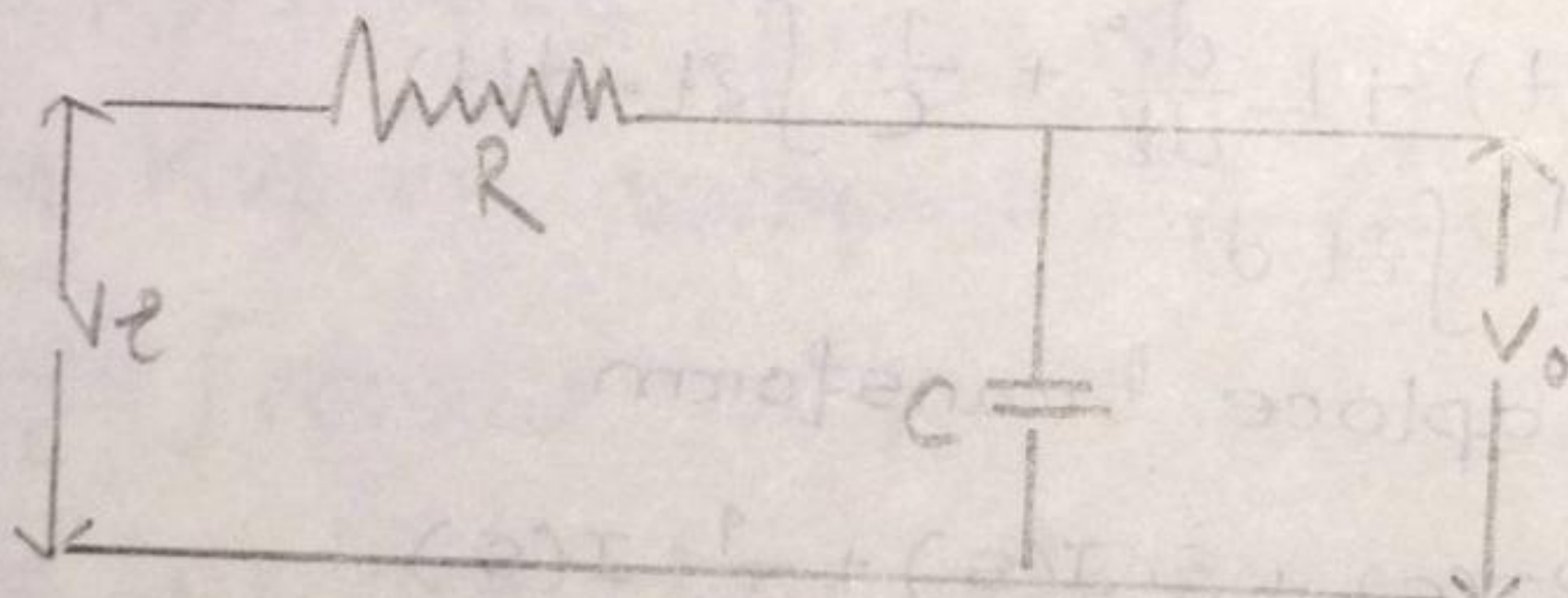
By taking Laplace transform -

$$V_i(s) = \frac{1}{sC} I(s) + R I(s)$$

$$V_o(s) = R I(s)$$

$$\begin{aligned} \text{Transfer function} &= \frac{R I(s)}{\frac{1}{sC} I(s) + R I(s)} \\ &= \frac{R I(s)}{I(s) \left(\frac{1}{sC} + R \right)} = \frac{R}{\frac{1}{sC} + R} \\ &= \frac{R s C}{1 + R s C} = \frac{C(s)}{R(s)} \end{aligned}$$

Question - 3



$$V_i(t) = R i(t) + \frac{1}{C} \int i(t) dt$$

$$V_o(t) = \frac{1}{C} \int i(t) dt$$

By taking Laplace transform

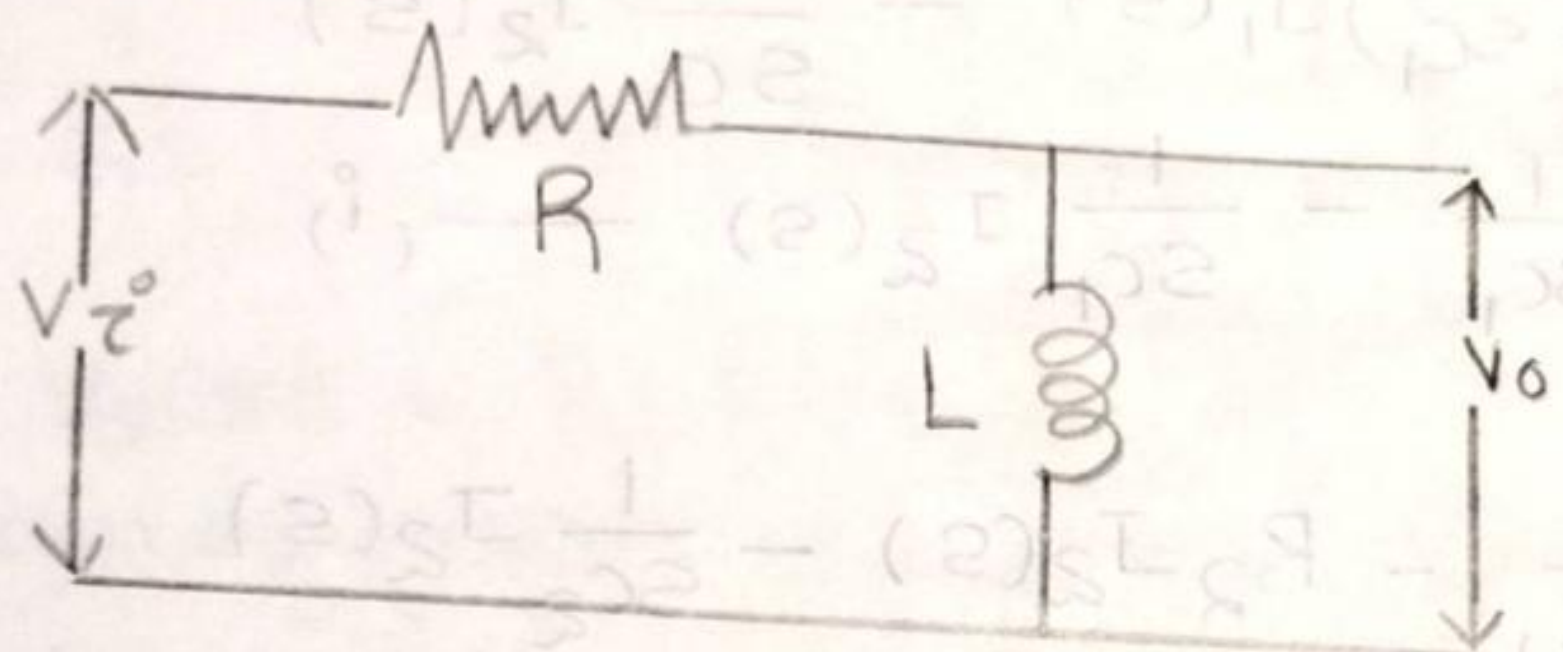
$$V_i(s) = R I(s) + \frac{1}{C(s)} I(s)$$

$$V_o(s) = \frac{1}{C(s)} I(s)$$

$$\begin{aligned} \text{Transfer function} &= \frac{\frac{1}{C(s)} I(s)}{R I(s) + \frac{1}{C(s)} I(s)} = \frac{\frac{1}{C(s)} I(s)}{\left(R + \frac{1}{C(s)}\right) I(s)} \\ &= \frac{1/C(s)}{R + \frac{1}{C(s)}} = \frac{1/C(s)}{R C(s) + 1} \end{aligned}$$

$$= \frac{1}{R C(s) + 1} = \frac{C(s)}{R(s)}$$

Question no-4



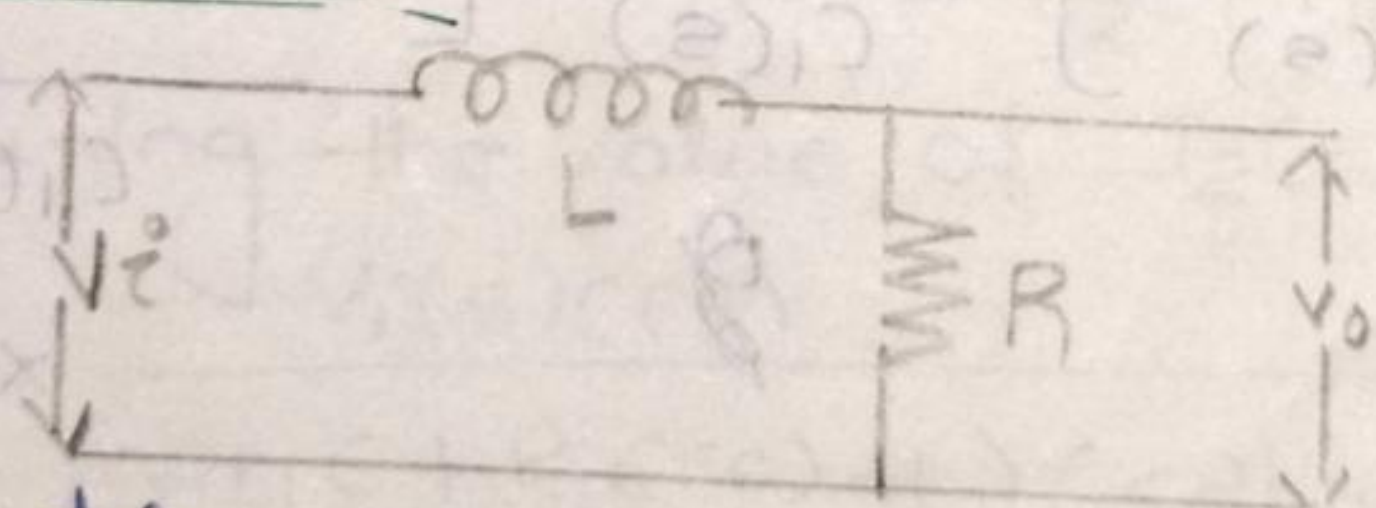
$$V_i(t) = R i(t) + L \frac{di}{dt}, \quad V_o = L \frac{di}{dt}$$

By taking Laplace transform -

$$V_i(s) = R I(s) + L I(s), \quad V_o = S L I(s)$$

$$\begin{aligned} \text{Transfer function} &= \frac{S L I(s)}{R I(s) + S L I(s)} = \frac{S L I(s)}{I(s) [R + S L]} = \frac{S L}{R + S L} \\ &= \frac{C(s)}{R(s)} \end{aligned}$$

Question no-5



$$V_i(t) = L \frac{di}{dt} + R i(t), \quad V_o = R i(t)$$

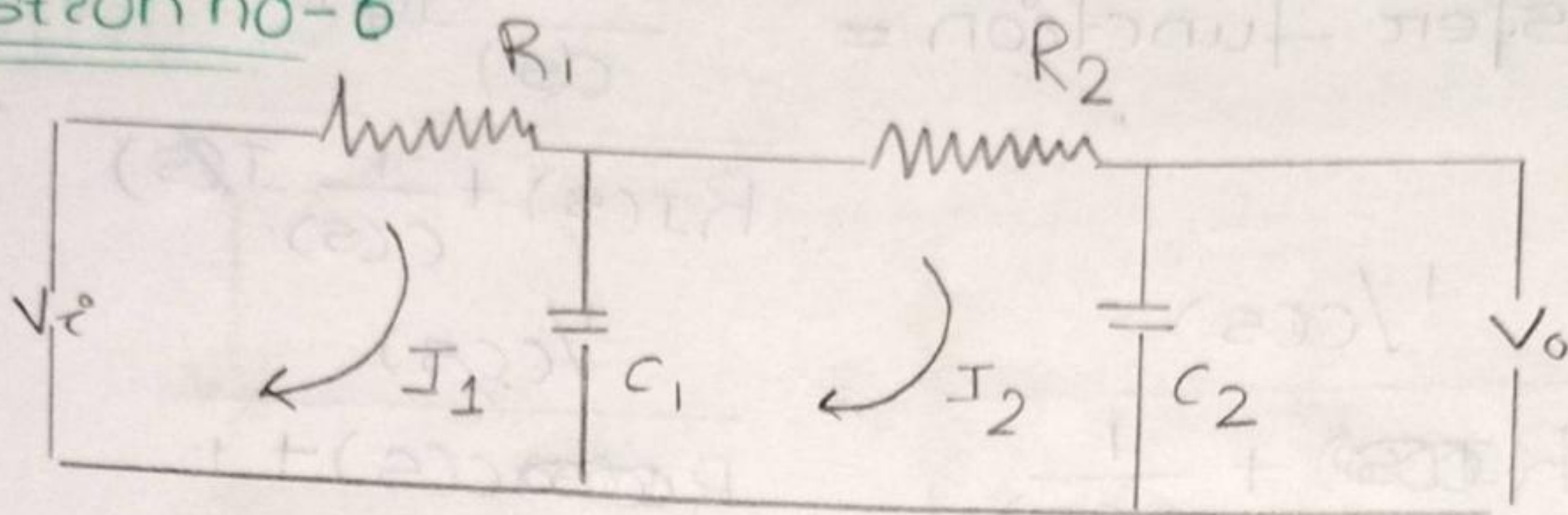
By taking Laplace transform -

$$V_i(s) = S L I(s) + R I(s)$$

$$V_o(s) = R I(s)$$

$$\begin{aligned} \text{T.F} &= \frac{R I(s)}{S L I(s) + R I(s)} = \frac{R I(s)}{(S L + R) I(s)} = \frac{C(s)}{R(s)} \end{aligned}$$

Question no-5



By applying KVL in Loop-1

$$\Rightarrow V_i(s) = R_1 I_1(s) + \frac{1}{sC_1} (I_1 s - I_2 s)$$

$$= R_1 I_1(s) + \frac{-I_1 s}{sC_1} + \frac{I_2 s}{sC_1}$$

$$= \left(R_1 + \frac{1}{sC_1} \right) I_1(s) - \frac{1}{sC_1} I_2(s)$$

$$I_1(s) = \frac{R_1 + \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1} - \frac{1}{sC_1}} I_2(s) \quad (1)$$

Loop-2

$$[-I_2(s) - I_1(s)] \frac{1}{sC_1} - R_2 I_2(s) - \frac{1}{sC_2} I_2(s)$$

$$I_1(s) = C_1(s) \left[R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s} \right] I_2(s) \quad (2)$$

Putting the value of $I_1(s)$ in eqn(1) and eqn(2)

$$V_i(s) = \left(R_1 + \frac{1}{C_1 s} \right) + \frac{1}{C_1 s} - \left(R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s} \right) - \frac{1}{C_1 s} I_2(s)$$

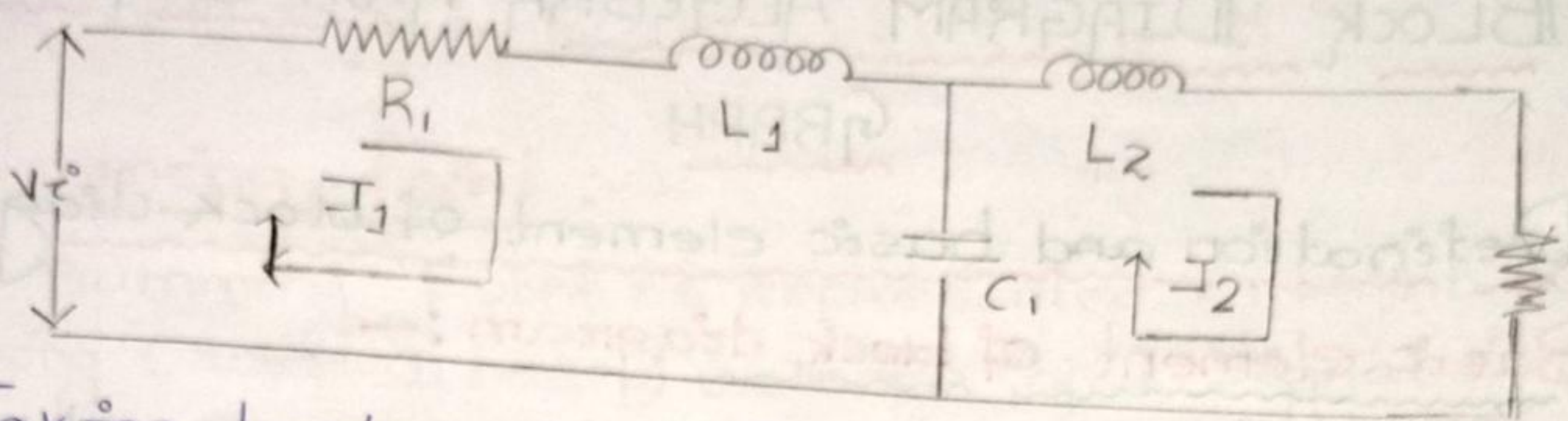
$$V_i(s) = \left(R_1 + \frac{1}{C_1 s} \right) + \frac{1}{C_1 s} - \left(R_2 + \frac{1}{C_1 s} + \frac{1}{C_2 s} - \frac{1}{C_1 s} \right) I_2 s$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC_2} I_2(s)}{\left[\frac{R_1 C_1(s) + 1}{C_1(s)} + \frac{1}{C_1(s)} - \left[\frac{R_2 C_1(s) C_2(s) + C_2(s) + 1}{C_1(s)} \right] \right]}$$

$$C_1(s) \cdot C_2(s)$$

$$\times \frac{1}{C_2} (s)$$

Question no-7



Taking Laplace transform of network using Loop analysis we can directly write the equation in matrix form -

$$\begin{bmatrix} R_1 + sL_1 + \frac{1}{Cs} & -\frac{1}{Cs} \\ -\frac{1}{Cs} & R_2 + sL_2 + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_i(s) \\ 0 \end{bmatrix} \quad (1)$$

$$V_o(s) = R_2 I_2(s) \quad (2)$$

From equation (1) we can write $I_2(s)$ as

$$I_2(s) = \frac{\begin{vmatrix} R_1 + sL_1 + \frac{1}{Cs} & V_i(s) \\ -\frac{1}{Cs} & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + sL_1 + \frac{1}{Cs} & -\frac{1}{Cs} \\ -\frac{1}{Cs} & R_2 + sL_2 + \frac{1}{Cs} \end{vmatrix}}$$

$$= \frac{V_i(s) / Cs}{(R_1 + sL_1 + \frac{1}{Cs})(R_2 + sL_2 + \frac{1}{Cs})(-\frac{1}{Cs})^2}$$

$$= \frac{V_i(s) / Cs}{(R_1 Cs + s^2 LC + 1)(R_2 Cs + s^2 LC + 1)}$$

$$= \frac{V_i(s) / s}{(s^2 LC + R_1 Cs + 1)(s^2 LC + R_2 Cs + 1)}$$

Substituting the value of $I_2(s)$ in eqn (2)

$$R_2 \left[\frac{V_i(s) Cs}{(s^2 L_1 C + R_1 Cs + 1)(s^2 L_2 C + R_2 Cs + 1)} \right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 Cs}{(s^2 L_1 C + R_1 Cs + 1)(s^2 L_2 C + R_2 Cs + 1)}$$

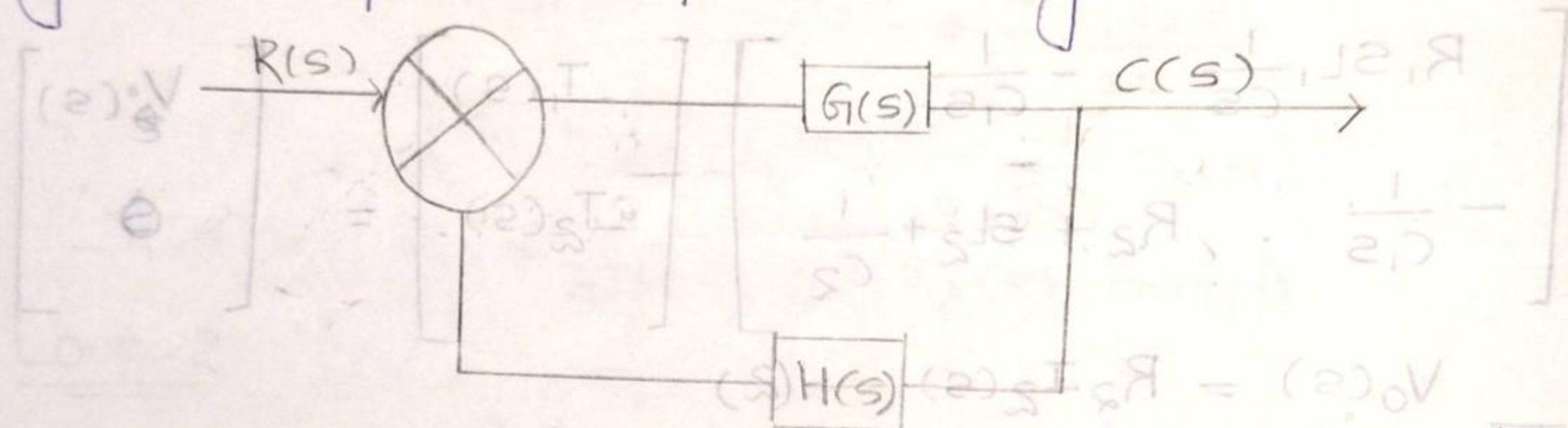
CHAPTER-4

BLOCK DIAGRAM ALGEBRA AND SIGNAL FLOW GRAPH

Definition and basic element of block diagram:-

Basic element of block diagram:-

The basic element of block diagram are a block. The Summing and take up point. Consider the block diagram of close loop Control system.



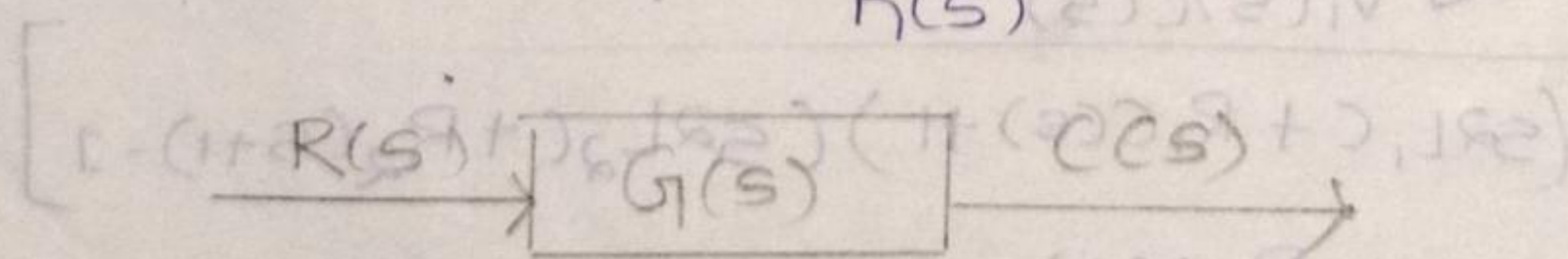
The above block diagram consists of 2 block having Transfer function $G(s)$ and $H(s)$

It also having one Summing Point and one takeoff Point arrow indicate the direction of the flow of signal.

Block:-

The transfer function of a Component is represented by the block.

Block has single I/p and single O/p. The following figure show a block having Input $R(s)$ output $C(s)$ that $T.F = G(s) = \frac{C(s)}{R(s)}$



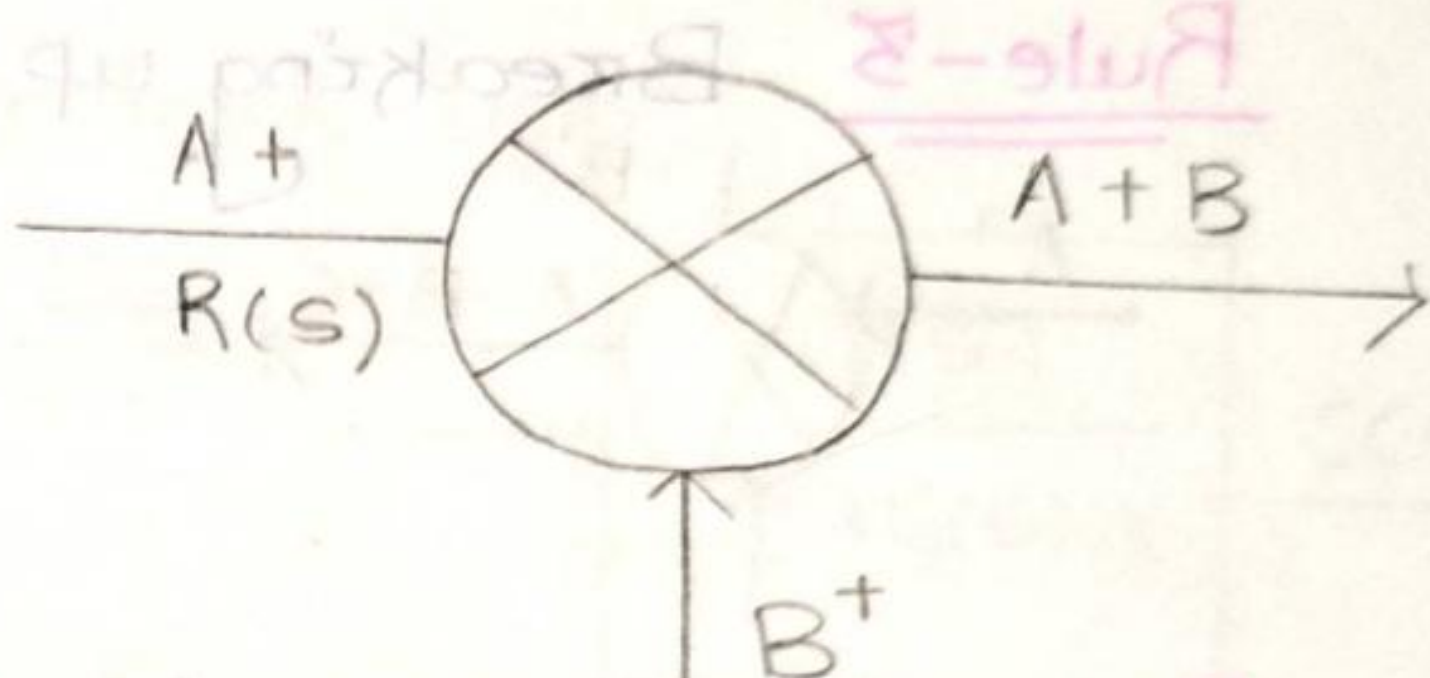
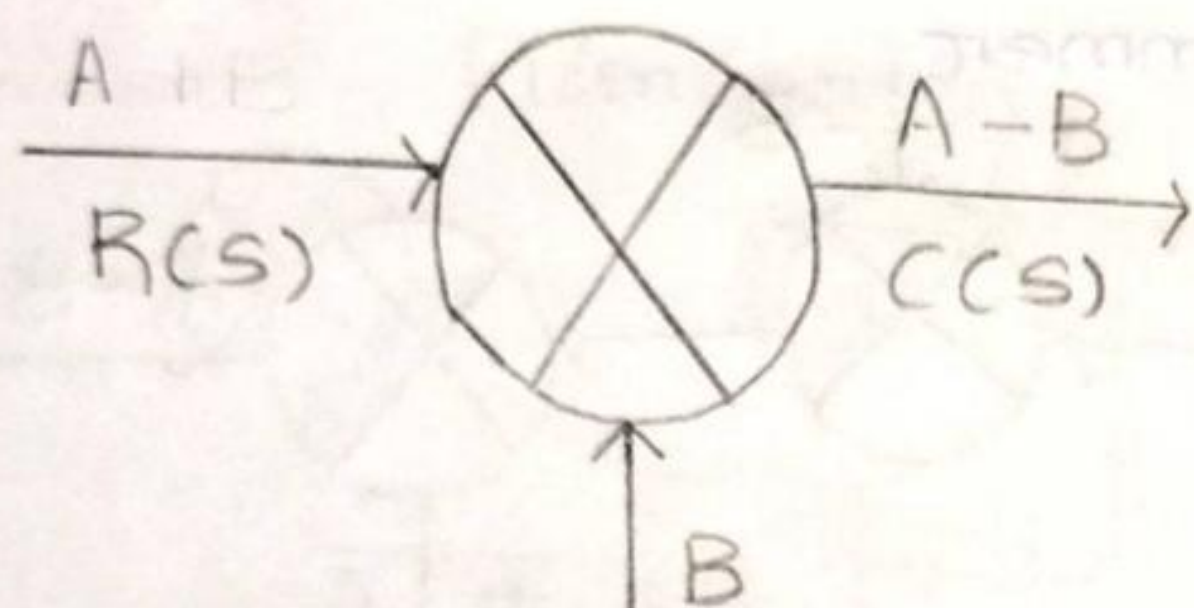
$$T.F = G(s) = \frac{C(s)}{R(s)}$$

$$\Rightarrow C(s) = G(s)R(s)$$

Output of block is obtained by multiplying T.F of the block with input $R(s)$

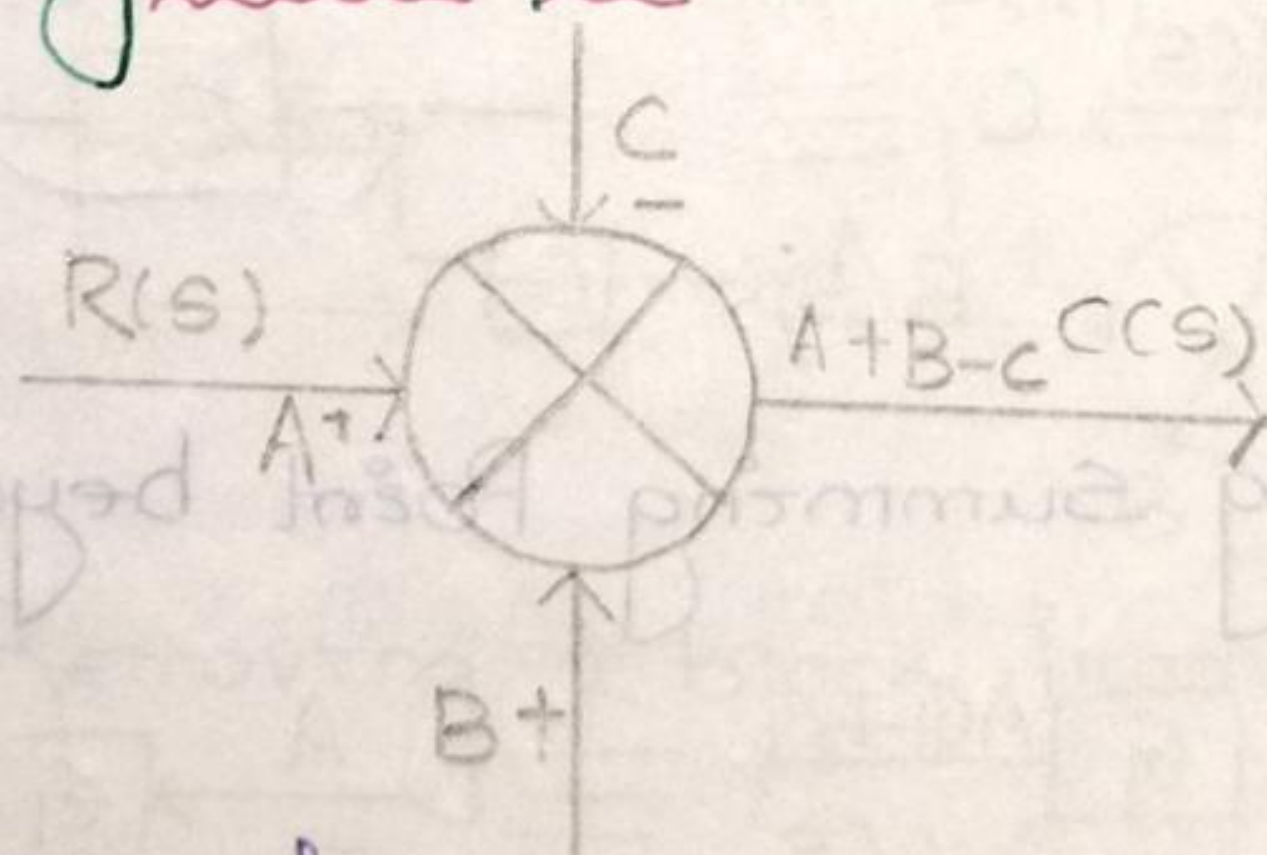
Summing Point :-

The Summing Point is represented by a circle having cross. It has 2 or more input signals and single output. It produces the algebraic sum of the input. It also performs something or subtraction or combination of a input base on the polarity of the input.



The following figure shows the Summing Point with 2 input and one output. Hence the input A and B are having +ve sign and -ve sign so the summing point produces the output $C(s)$ addition and subtraction.

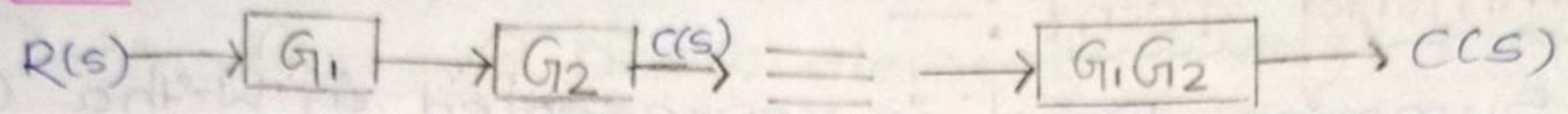
3-input Single Output :-



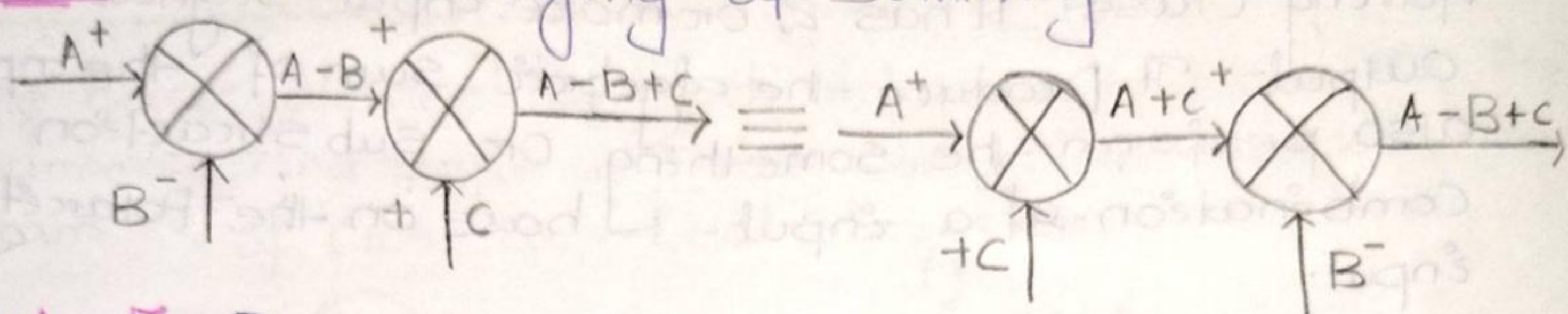
The following figure shows the summing point with 3 input and one output. Hence here the input A and B +ve sign and C having -ve sign. So the summing point produces the output of $C(s)$.

BLOCK DIAGRAM REDUCTION TECHNIQUE :-

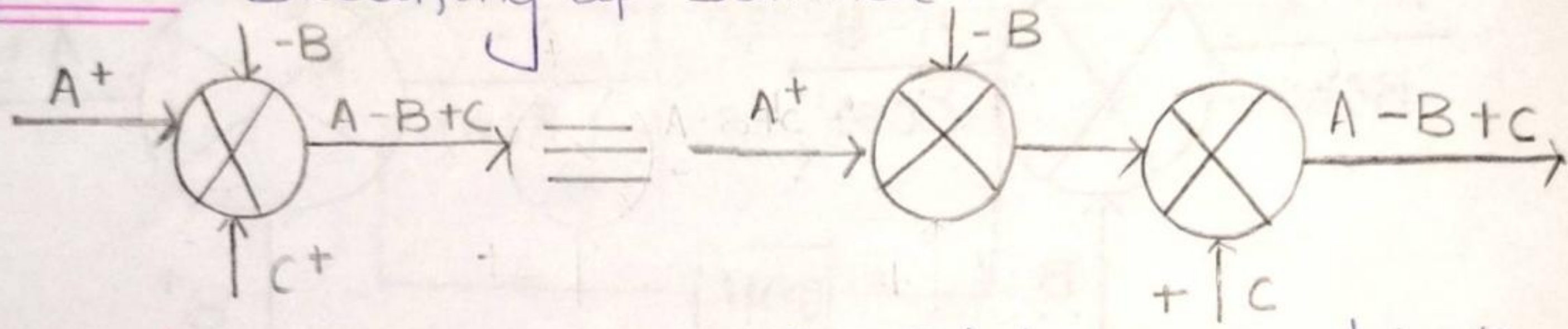
Rule-1 Block Connected in Cascode.



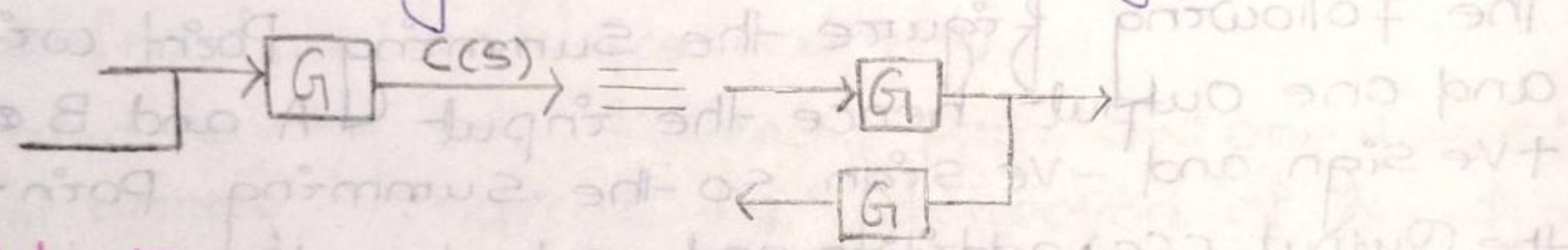
Rule-2 Interchanging of Summing Point.



Rule-3 Breaking up Summer.

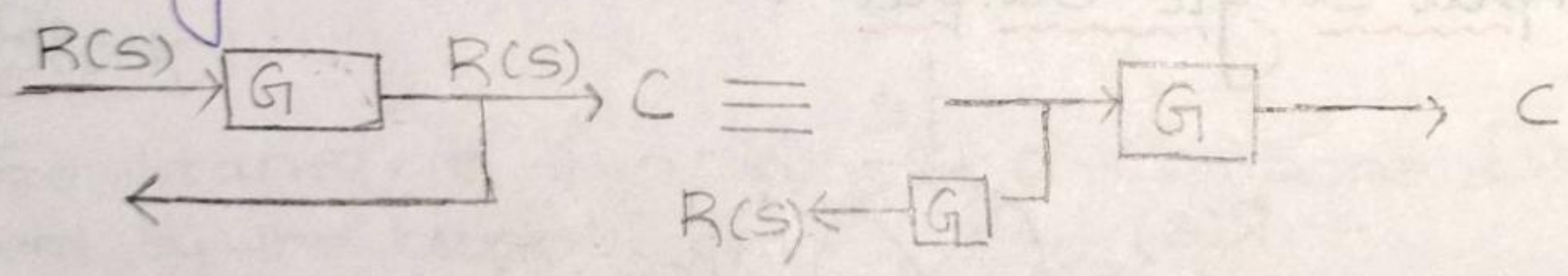


Rule-4 Moving a take-off Point beyond a block.

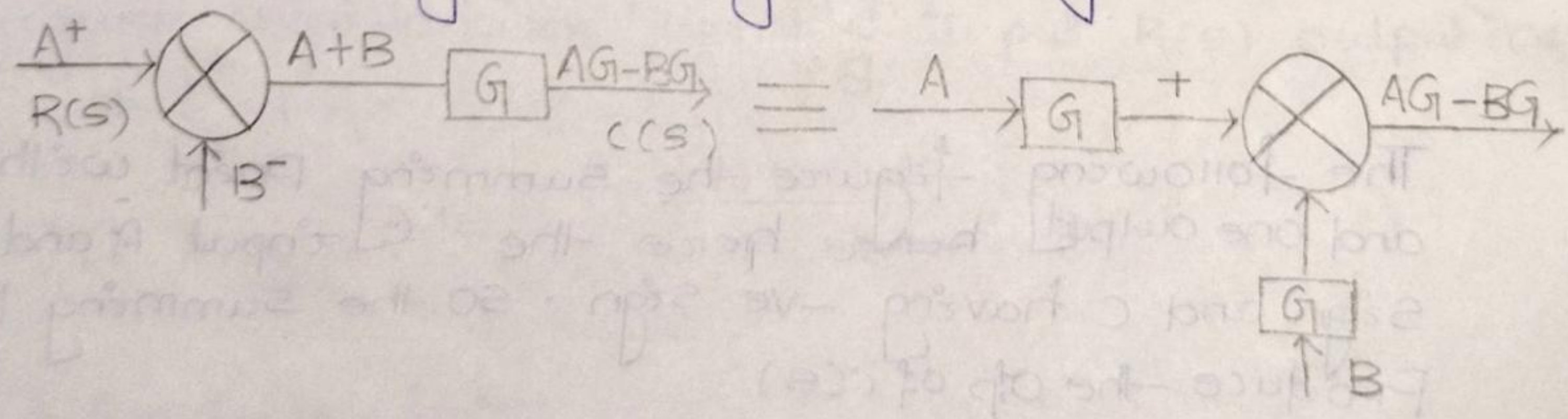


Rule-5

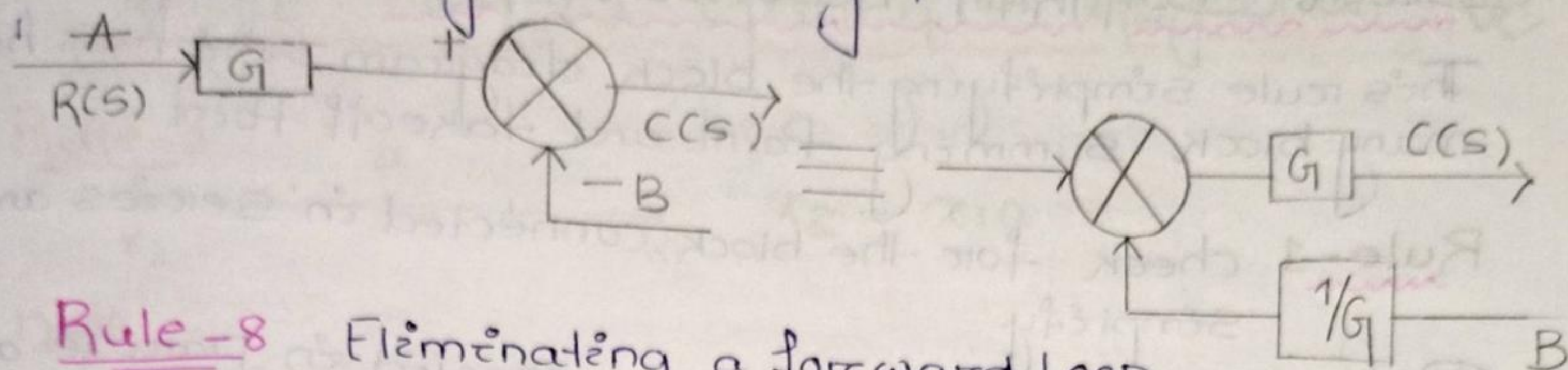
Moving a take-off Point through of a block.



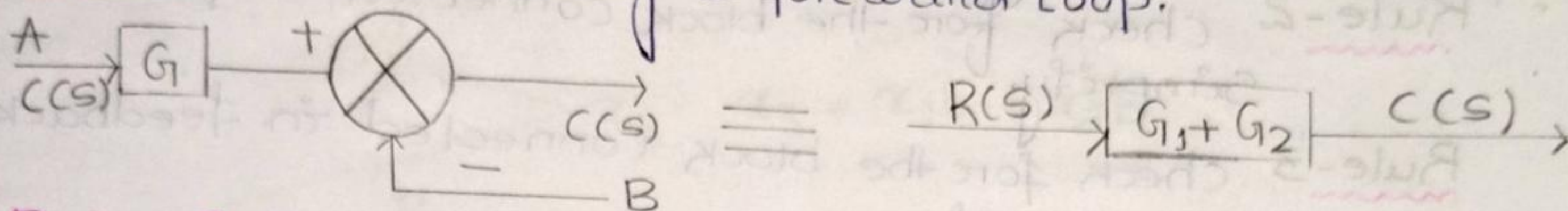
Rule-6 Moving Summing Point beyond a block.



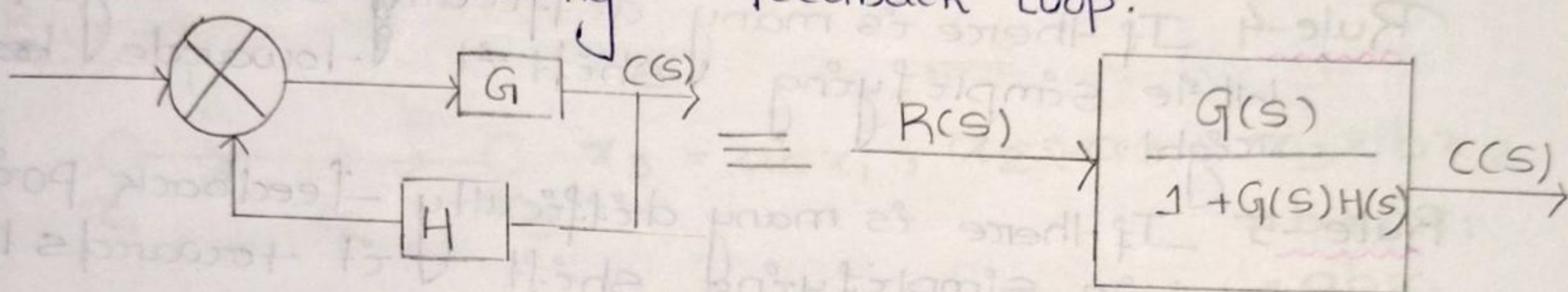
Rule-7 Moving a Summing point ahead of block



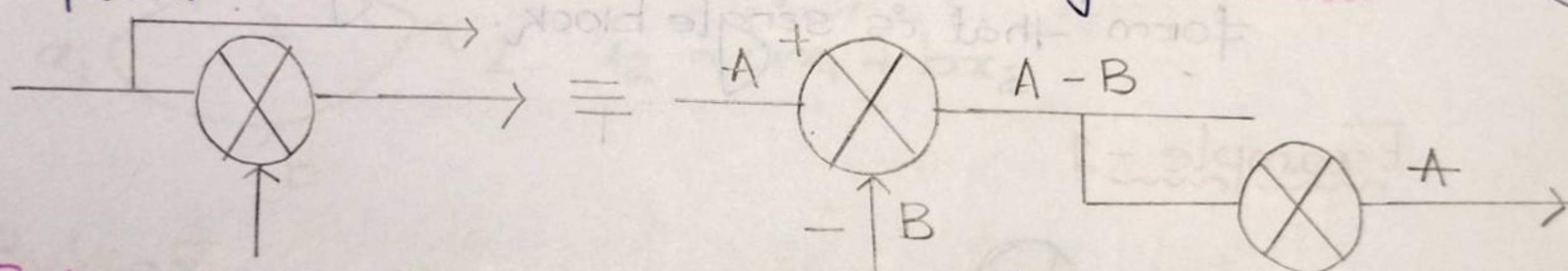
Rule-8 Eliminating a forward loop.



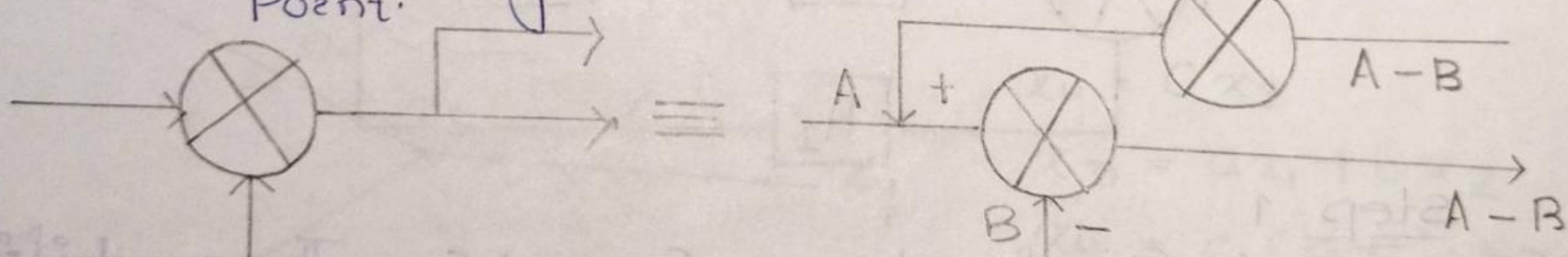
Rule-9 Eliminating a feedback loop.



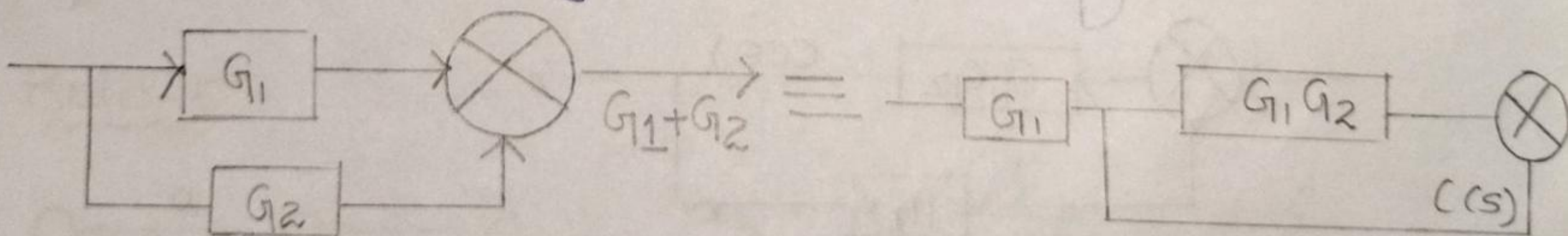
Rule-10 Moving a take-off point beyond of a summing point.



Rule-11 Moving take-off point ahead of summing point.



Rule-12 Removing a block from a forward path.



BLOCK DIAGRAM REDUCTION RULE:

This rule simplifying the block diagram which is having many block summing point and takeoff point.

Rule-1 check for the block connected in series and simplify.

Rule-2 check for the block connected in parallel and simplify.

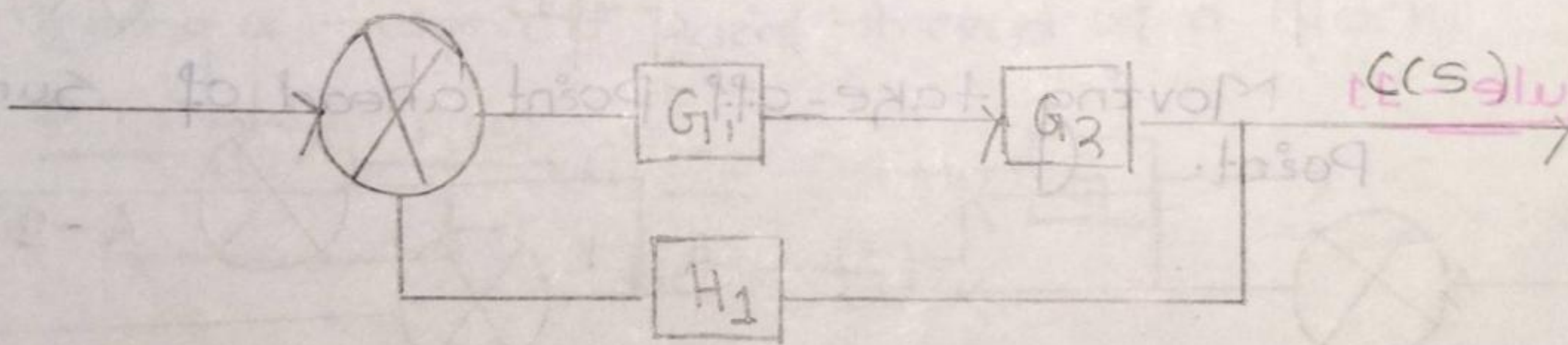
Rule-3 check for the block connected in feedback loop and simplify.

Rule-4 If there is many difficulty in simplifying summing point while simplifying shift it towards right.

Rule-5 If there is many difficulty in simplifying feedback point while simplifying shift it towards left.

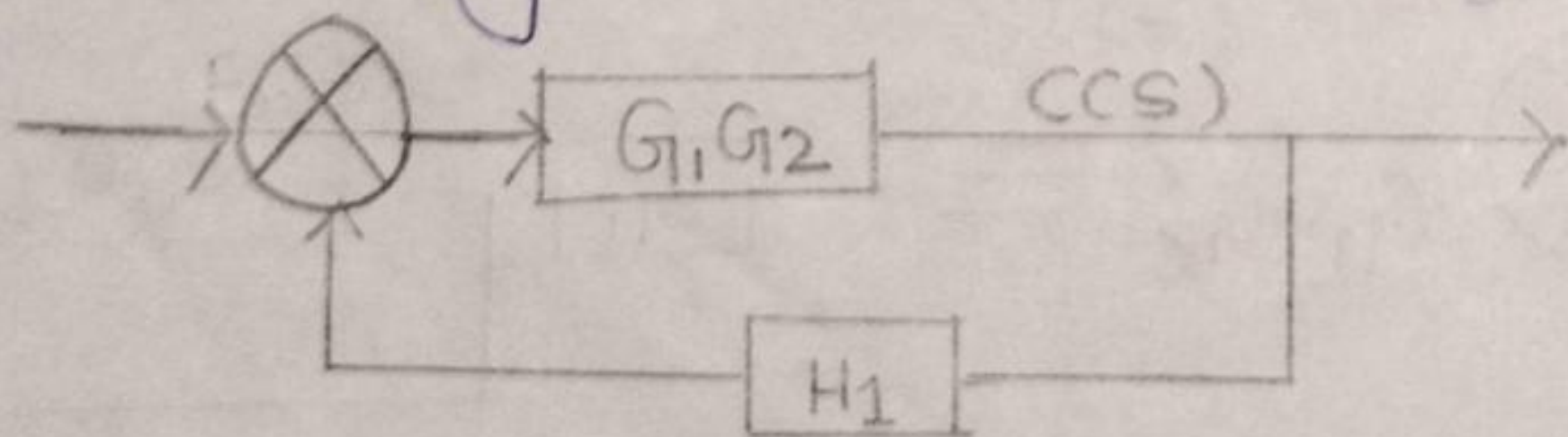
Rule-6 Repeat the above step till you get the simple form that is single block.

Example - 1

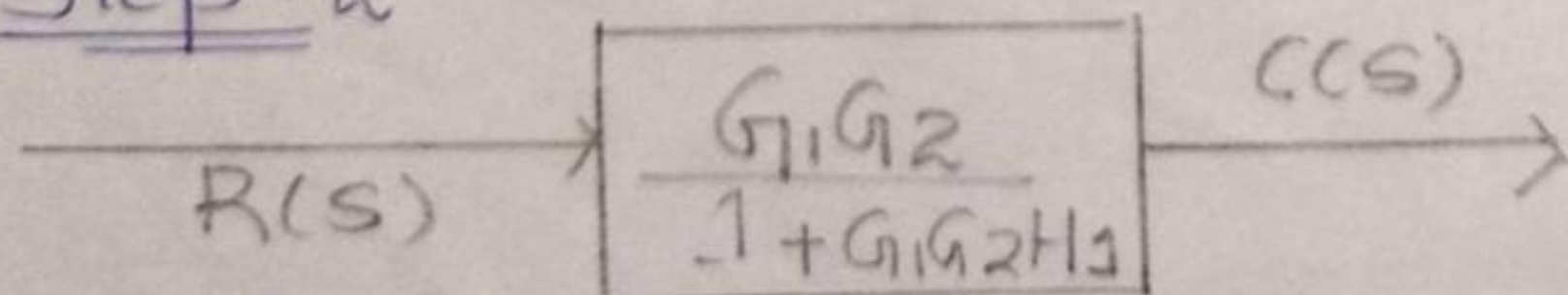


Step - 1

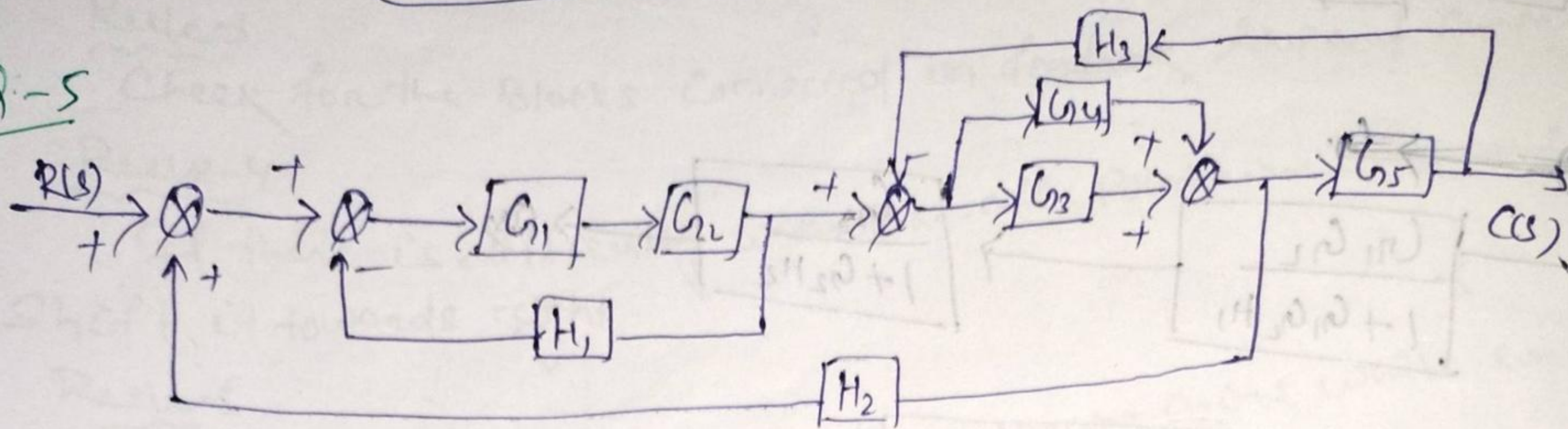
use rule (1) for blocks G_1 and G_2 . The modified block diagram.



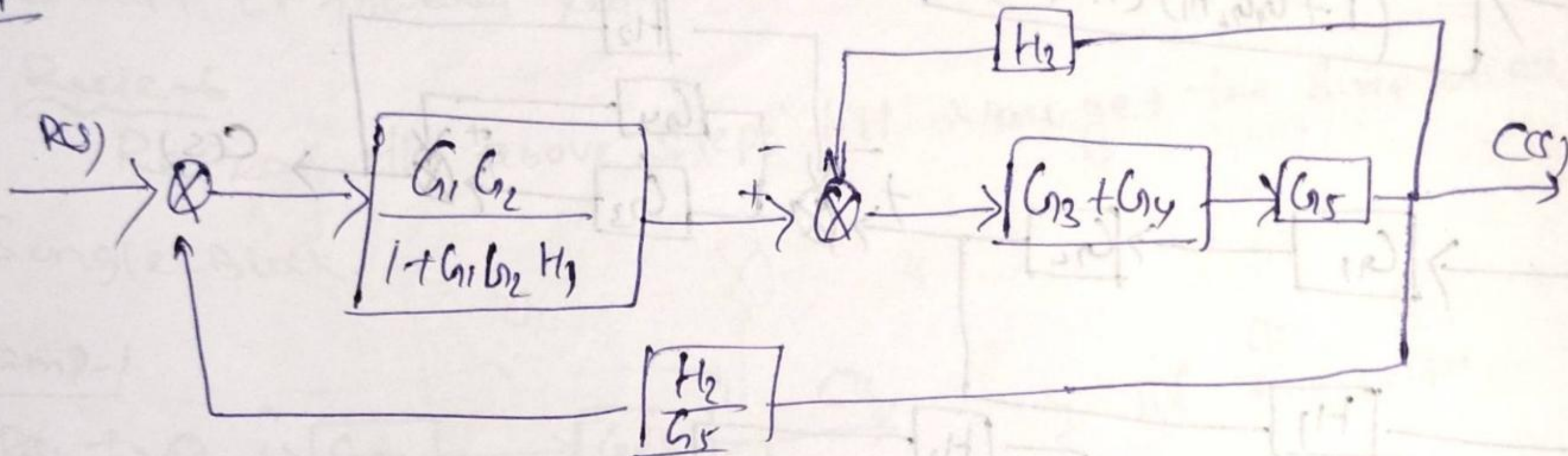
Step - 2



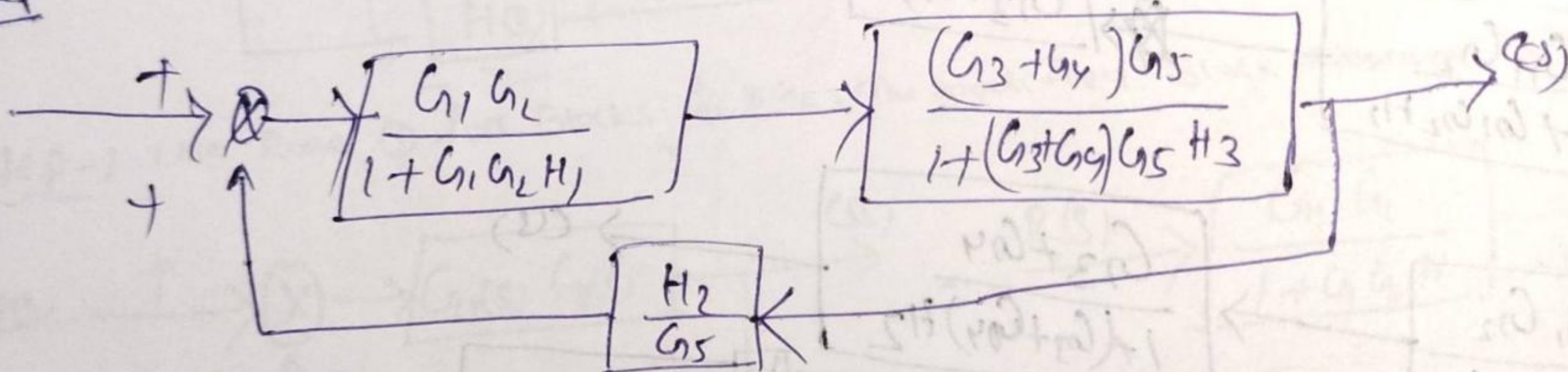
Q-5



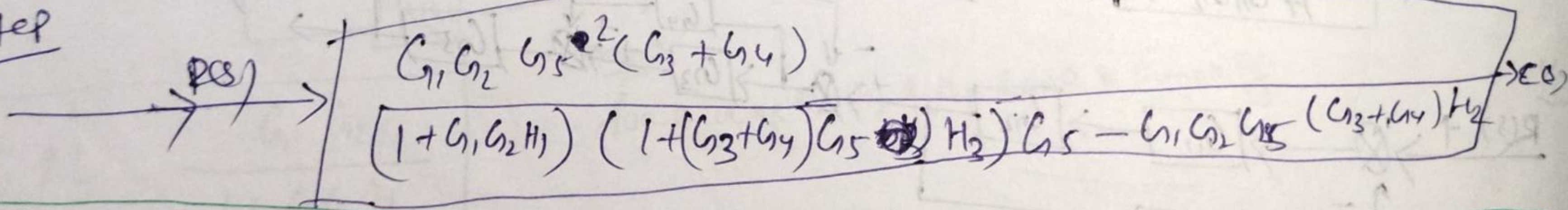
Step



Step



Step



Note

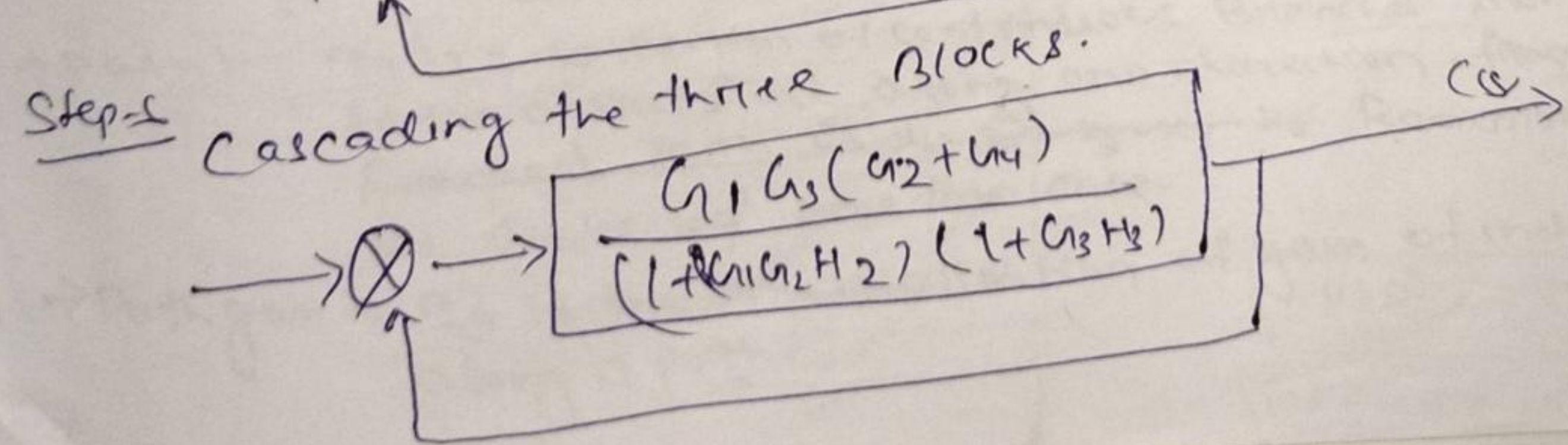
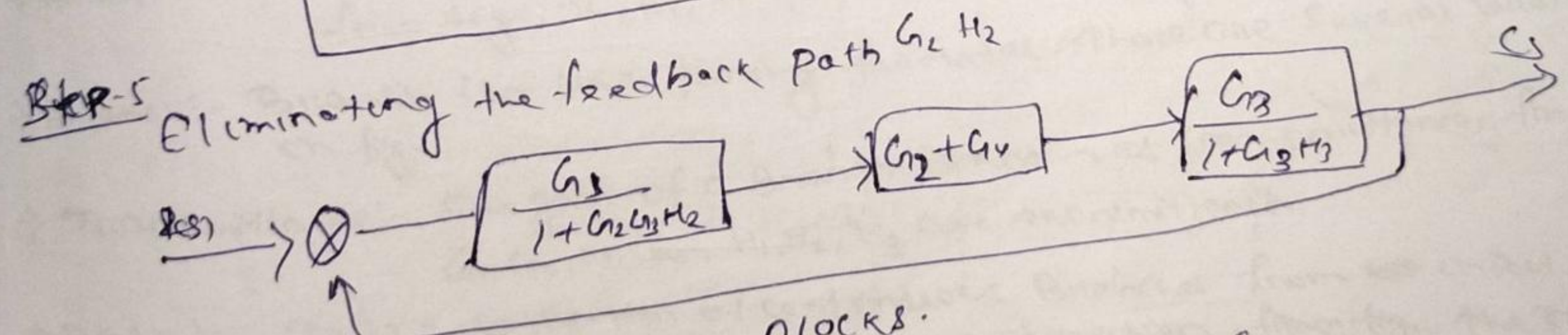
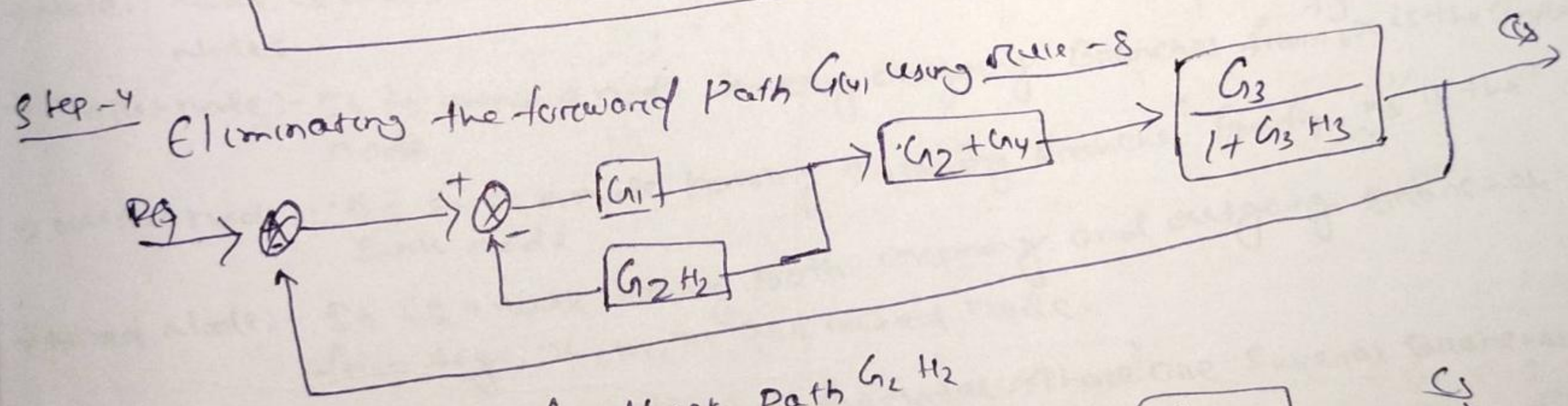
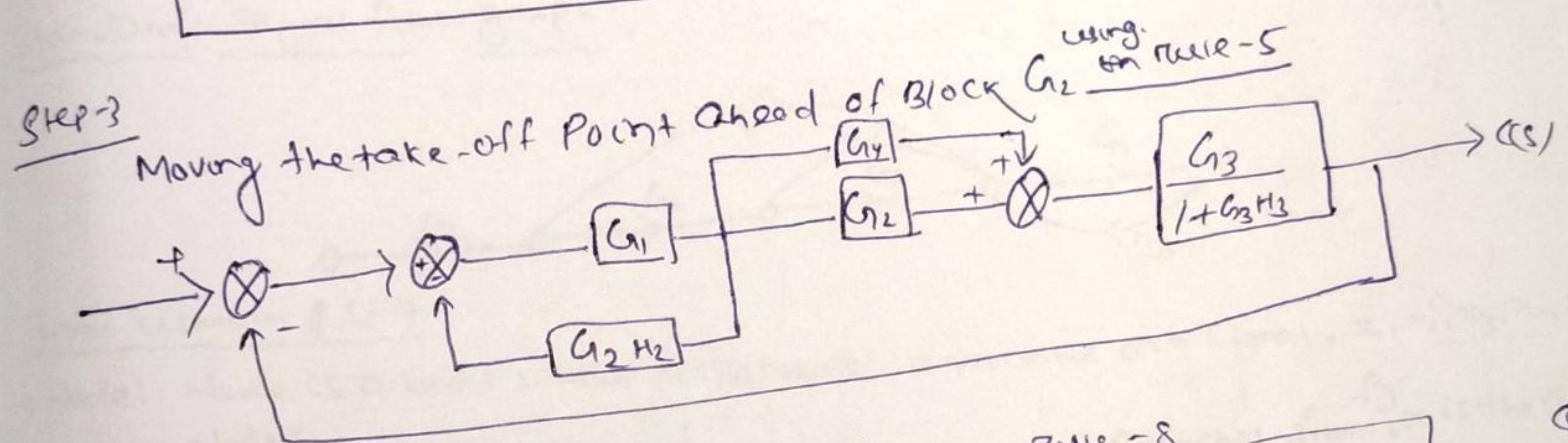
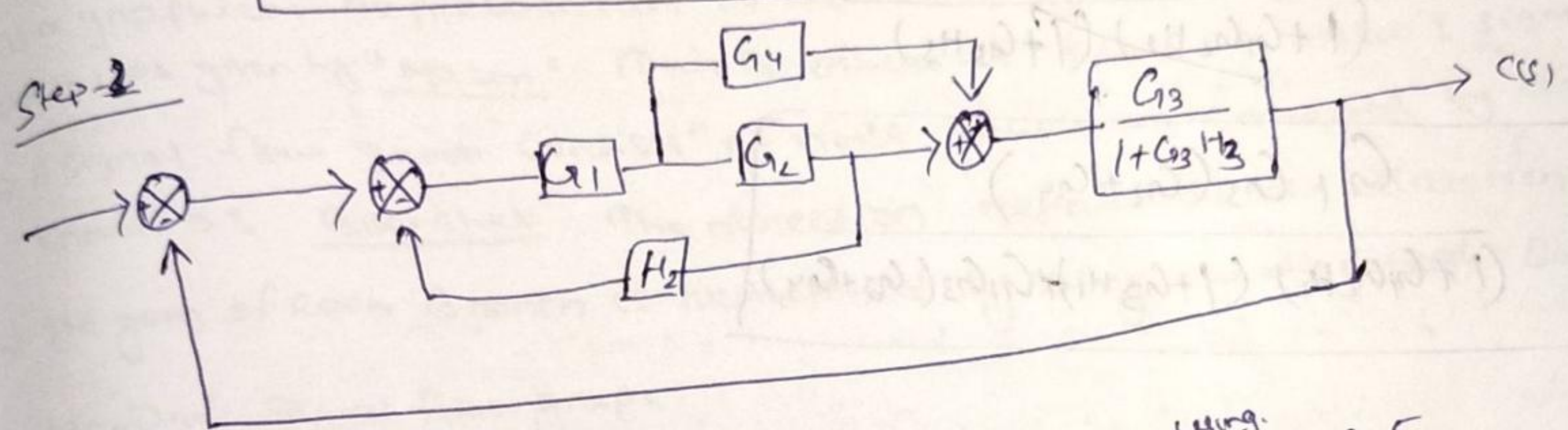
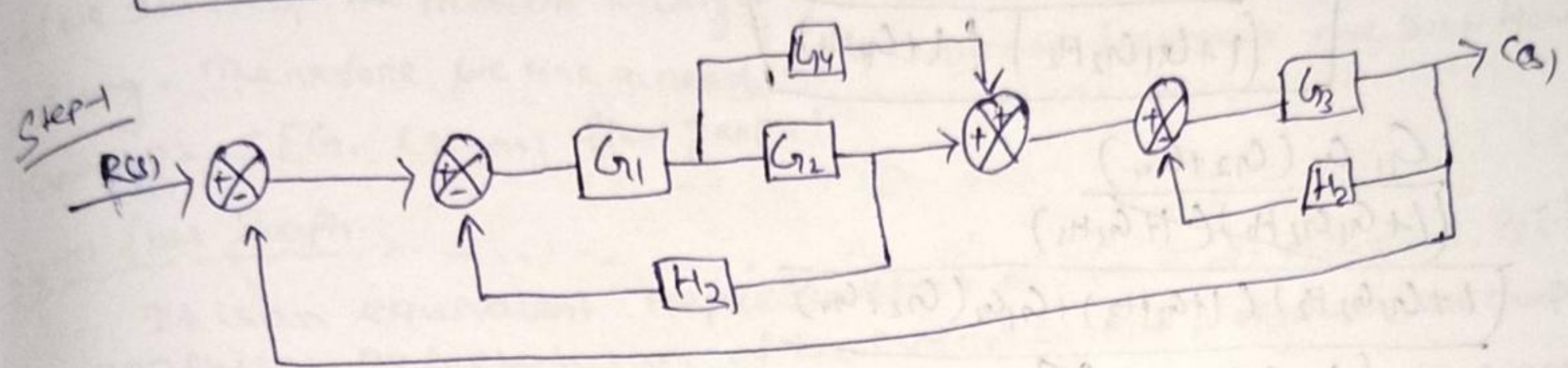
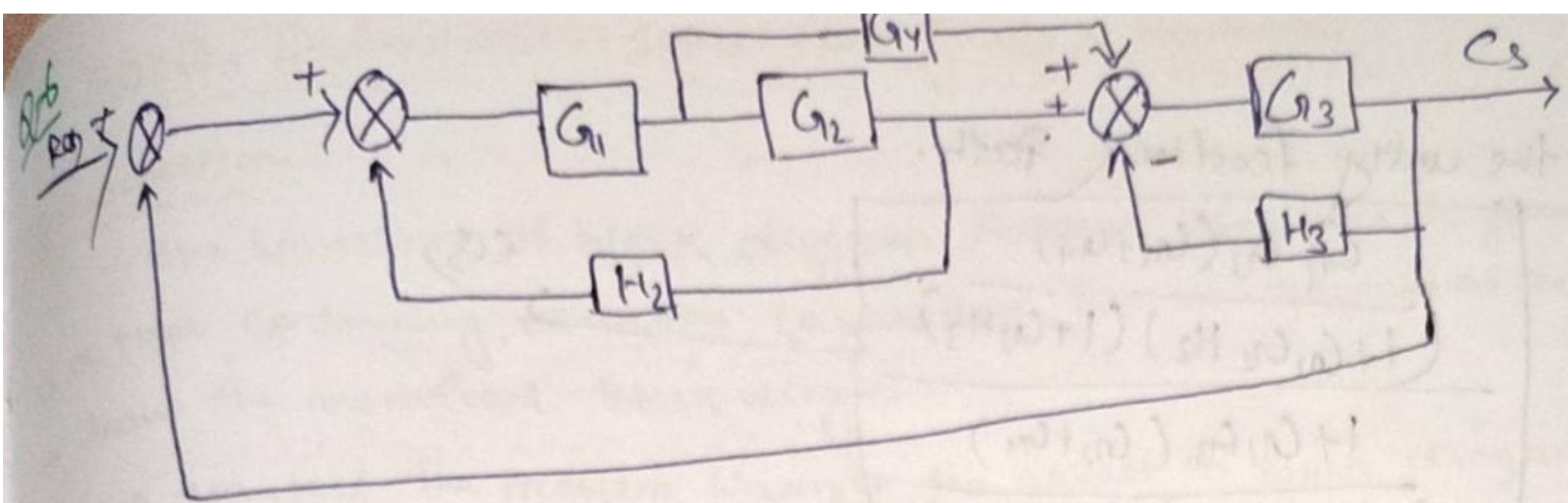
Follow these steps in order to calculate the T.F of Block diagram having multiple Input

Step 1 - find the T.F of Block diagram by considering one input at a time and make the remaining inputs as zero

Step 2 Repeat Step 1 for remaining input.

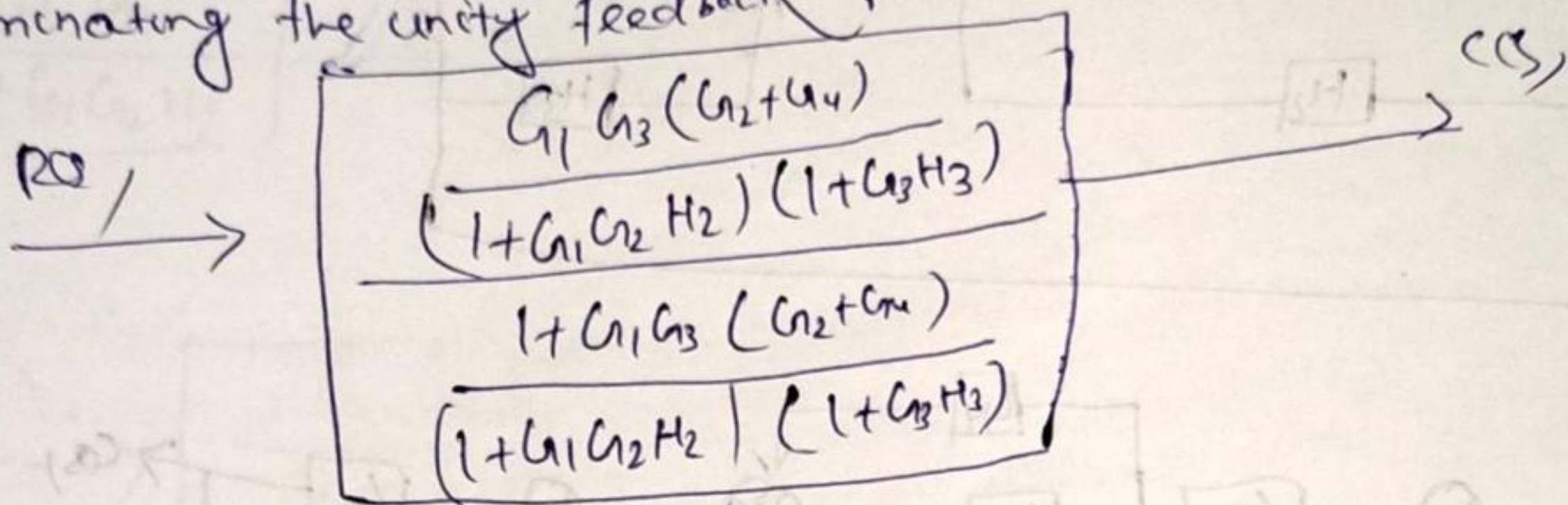
Step 3 Get the overall T.F by adding all these T.F

The block diagram reduction process takes more time for complicated system. Bcz we have to draw the block diagram after each step. So to overcome this drawback we use signal flow graphs. In the next two chapters, we will discuss about the concepts related to signal flow graph, i.e. how to represent SFG from given block diagram & calculation of T.F just by using gain formula without doing any reduction.



Step-7

Eliminating the unity feedback path.

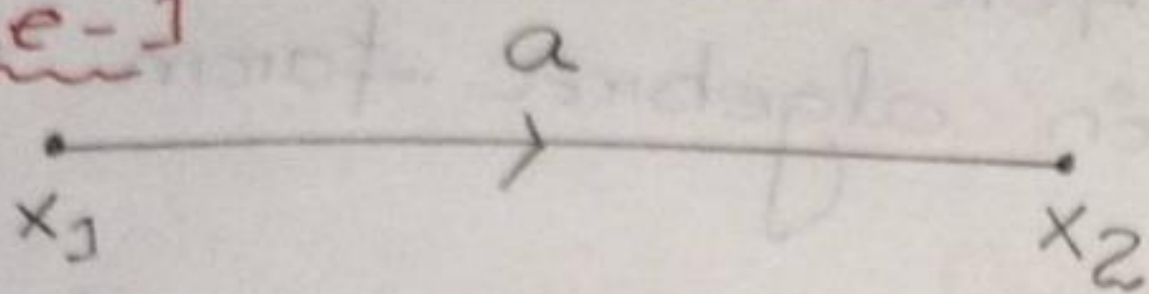


$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G_1 G_3 (G_2 + G_4)}{(1 + G_1 G_2 H_2) (1 + G_3 H_3) + G_1 G_3 (G_2 + G_4)}$$

$$= \frac{G_1 G_3 (G_2 + G_4)}{(1 + G_1 G_2 H_2) (1 + G_3 H_3) + G_1 G_3 (G_2 + G_4)}$$

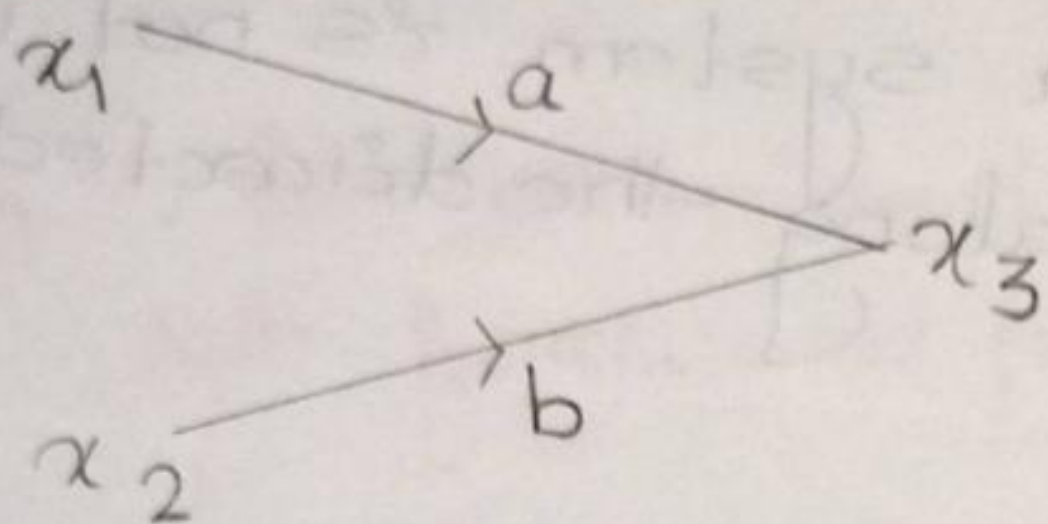
Single Flow GRAPH:- Algebra

Rule-1



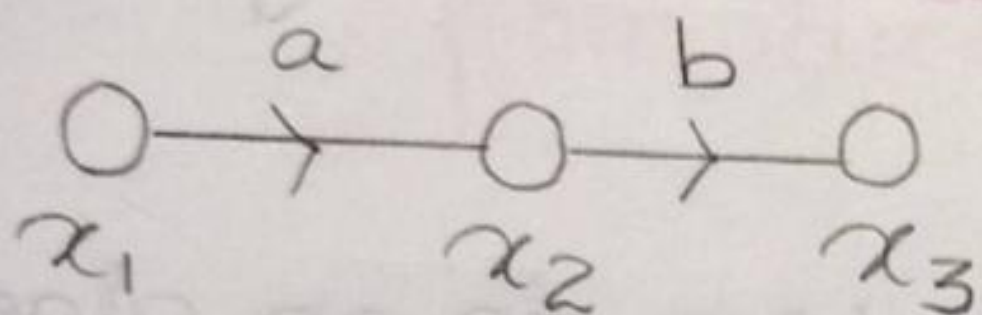
$$x_2 = x_1 a$$

Rule-2



$$x_3 = x_1 a + x_2 b$$

Rule-3

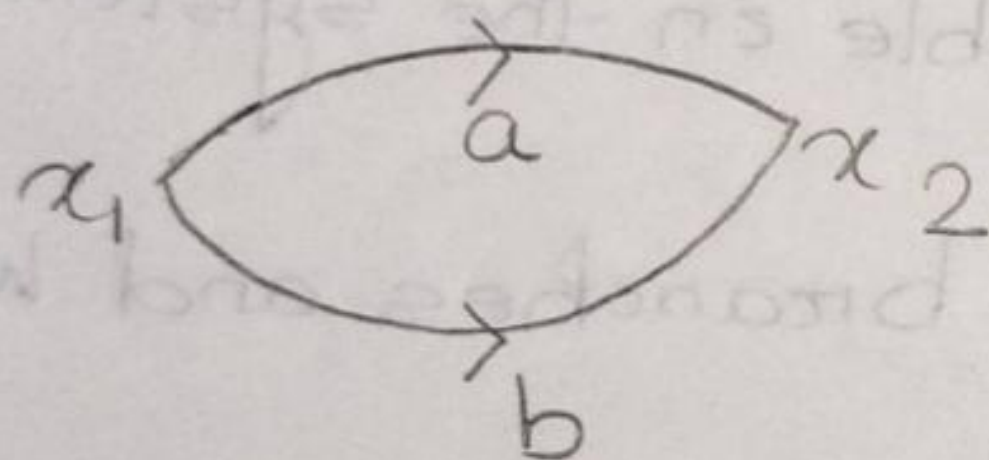


$$x_3 = abx_1, \quad x_2 = ax_1, \quad x_3 = bx_2$$

$$= b \times ax_1$$

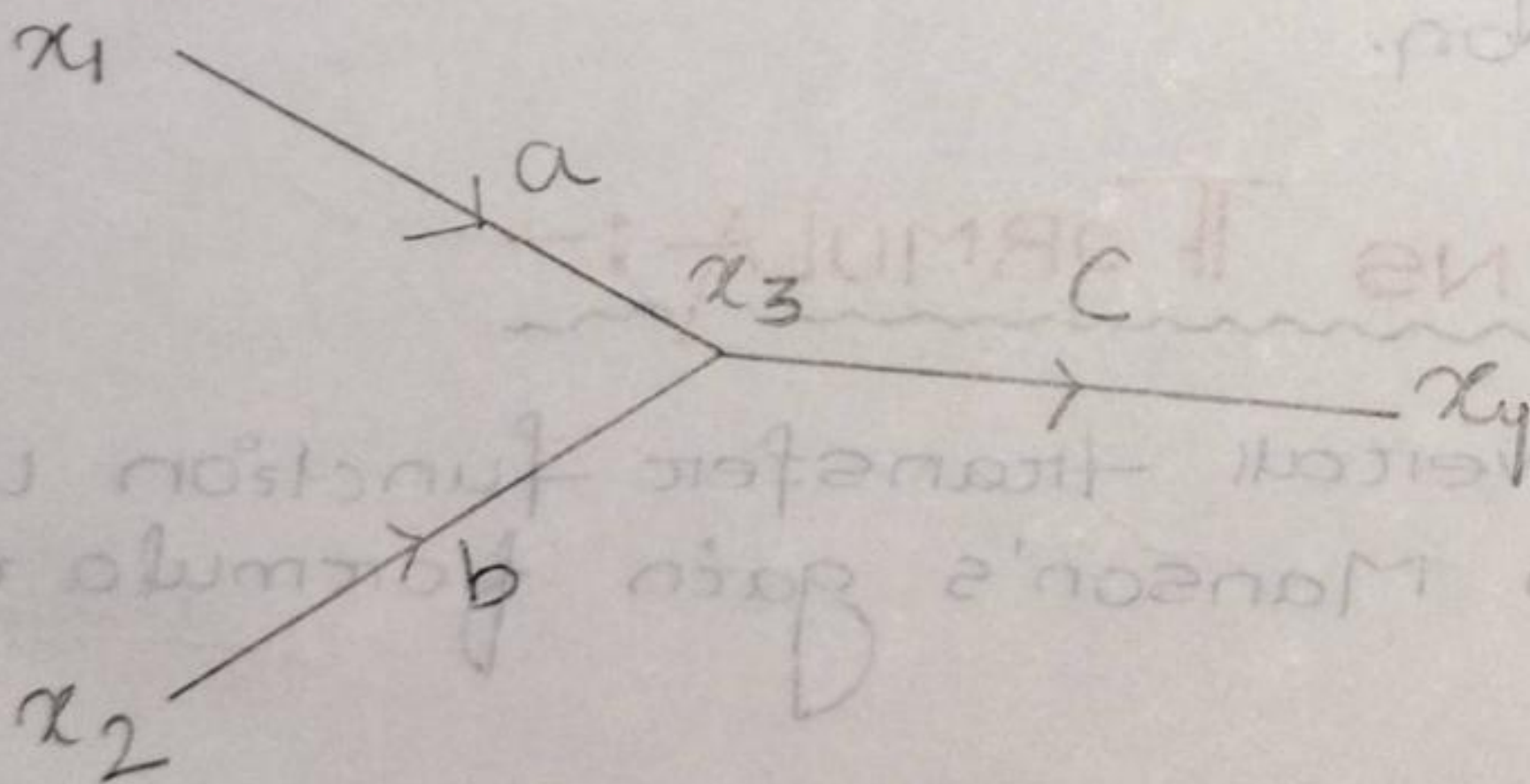
$$= abx_1$$

Rule-4



$$x_2 = ax_1 + bx_2$$

Rule-5



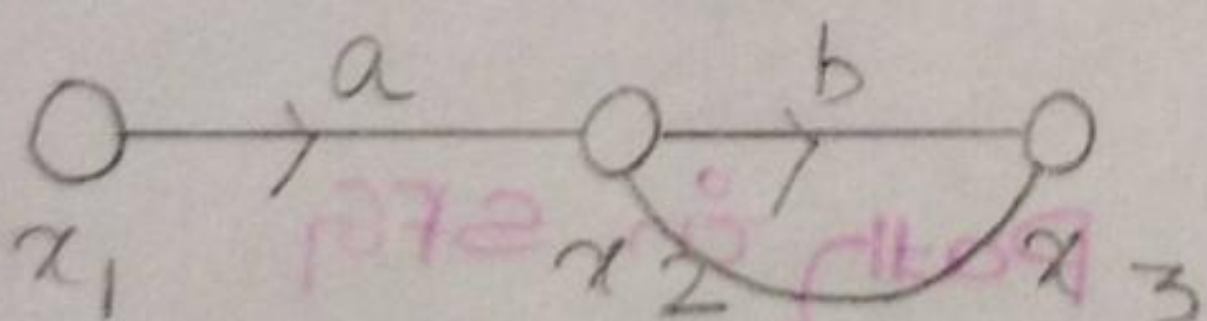
$$x_4 = cx_3$$

$$x_3 = ax_1 + bx_2$$

$$x_4 = c(ax_1 + bx_2)$$

$$= cx_3$$

Rule-5



$$x_3 = x_2 \times \frac{b}{1-bc}$$

$$x_2 = ax_1 \times \frac{b}{1-bc}$$

$$= \frac{bax_1}{1-bc}$$

Properties Of Single flow Graph -

- ⇒ It is applicable only for two linear system.
- ⇒ The expression must in algebraic form to draw the signal flow graph.
- ⇒ Nodes are used to represent the variable.
- ⇒ The signal flow graph of a system is not unit.
- ⇒ The signal travels along the directed line but not in the reverse way.

Procedure for Construction Of Signal flow graph :-

- ⇒ Determine the system equation in an algebraic form.
- ⇒ Represent the system variable in the system by a node.
- ⇒ Connect the node by proper branches and write the transmittance.
- ⇒ Use Manson's gain formula find out the overall transfer function.

MANSON'S GAINS FORMULA :-

To find out the overall transfer function using signal flow graph Manson's gain formula is used that is

$$\sum_{k=1}^n \frac{P_k \Delta_k}{\Delta}$$

P_k = k^{th} forward path in SFG

Δ = determinant of the graph

$$\Delta - 1 = (\text{Sum of individual loop gains}) + (\text{sum of gain product of two non touching loop}) - (\text{sum of gain product three non-touching loop}) + (\text{sum of gain product four non-touching loop})$$

Δ_k = Determinant of the graph after ~~omitting~~ k^{th} forward path

1 - Loop gain which are not touching to k^{th} forward path.

Two non-touching Loop

$$L_{21} = (-H_1)(H_2)$$

$$L_{22} = (-H_2G_1) - H_3$$

$$\Delta = 1 - (L_1 + L_2 + L_3)(L_{21} + L_{22})$$

$$\Delta = 1 - (H_1 - H_2G_1 - H_3)(-H_1H_2)(-H_2G_1)(-H_3)$$

Transfer Function

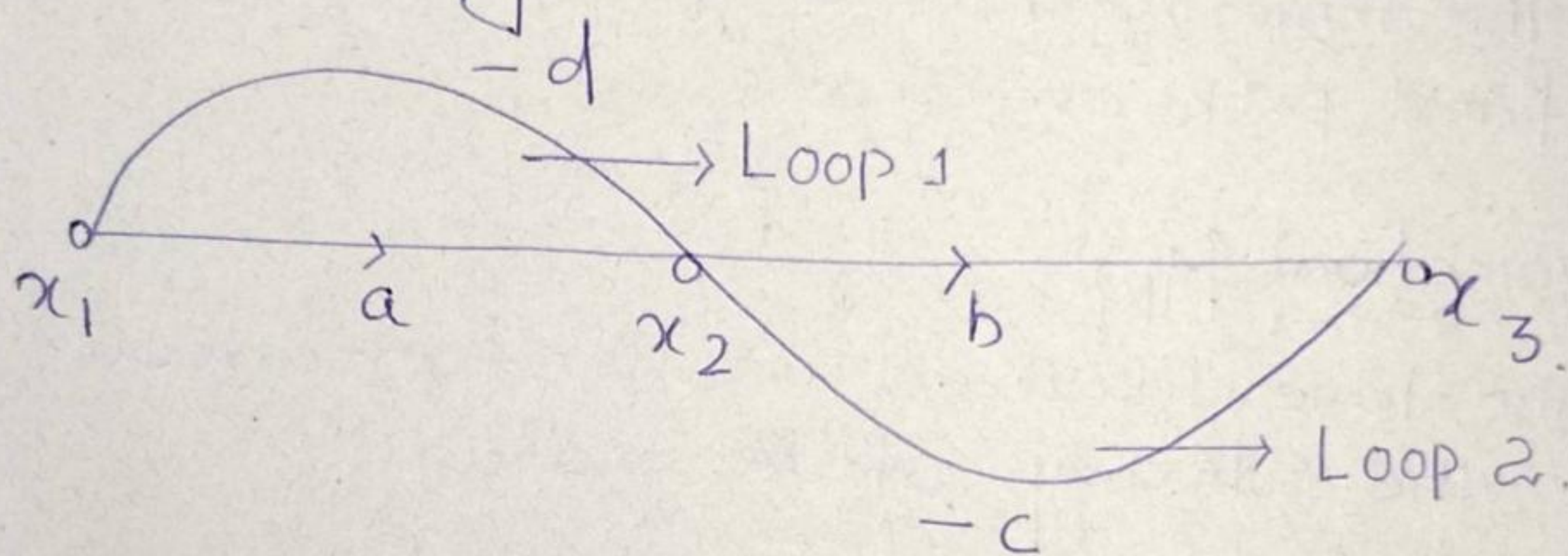
$$\frac{Y}{X} = \frac{G_1G_2 \times 3}{1 + H_1 + H_2 + L_3 + H_1H_2G_1H_3}$$

* Basic properties of SFG -

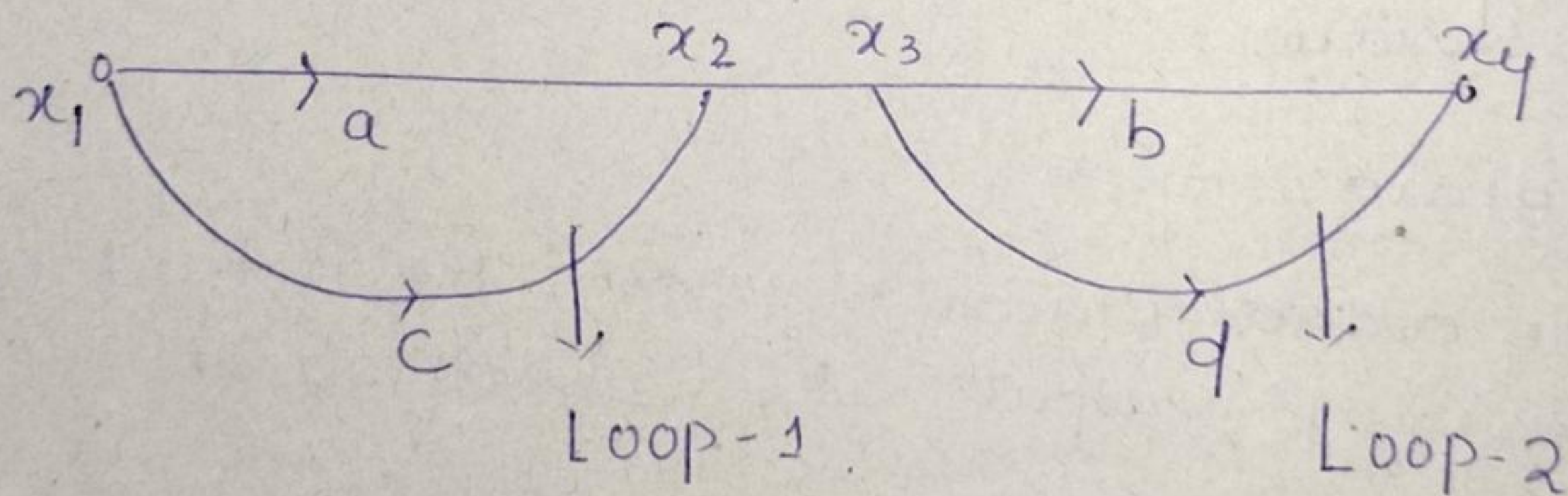
→ SFG requires forward path from input node and Output node with some gain

→ In SFG individual loops are formed and they have feedback.

→ The loops may not be touched

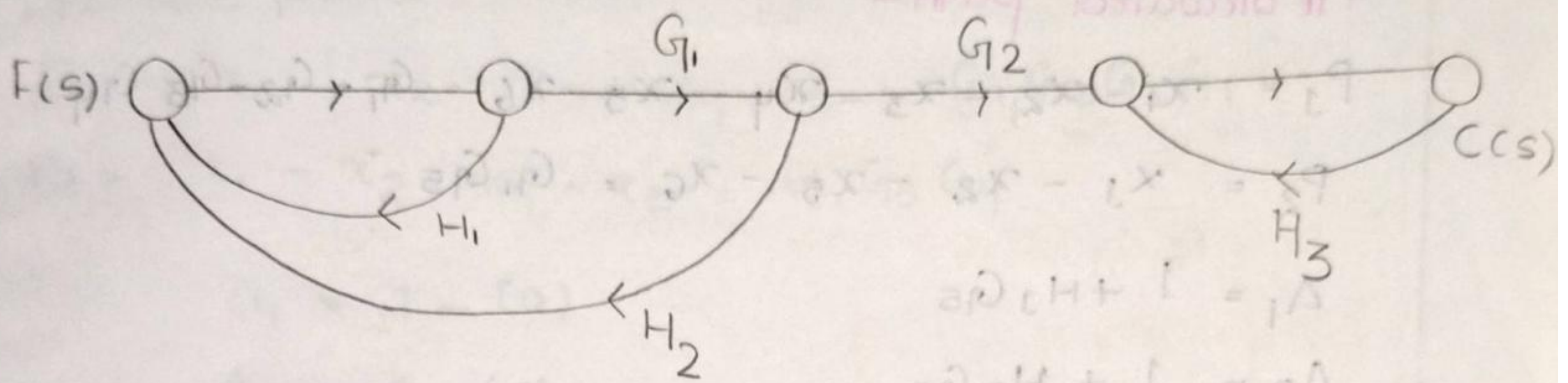


→ Two loops are non-touching



→ The closed loop T.f in a SFG can be calculated by using Mason's gain formula.

$$\frac{C(s)}{R(s)} = \sum_{k=1}^n \frac{P_k \Delta_k}{\Delta}$$



Forward path -

$$P_1 = G_1 G_2$$

$$\Delta_1 = 1 - (0)$$

Individual loop -

$$L_1 = -H_1$$

$$L_2 = -H_2 G_1$$

$$L_3 = -H_3$$

Two non-touching loop -

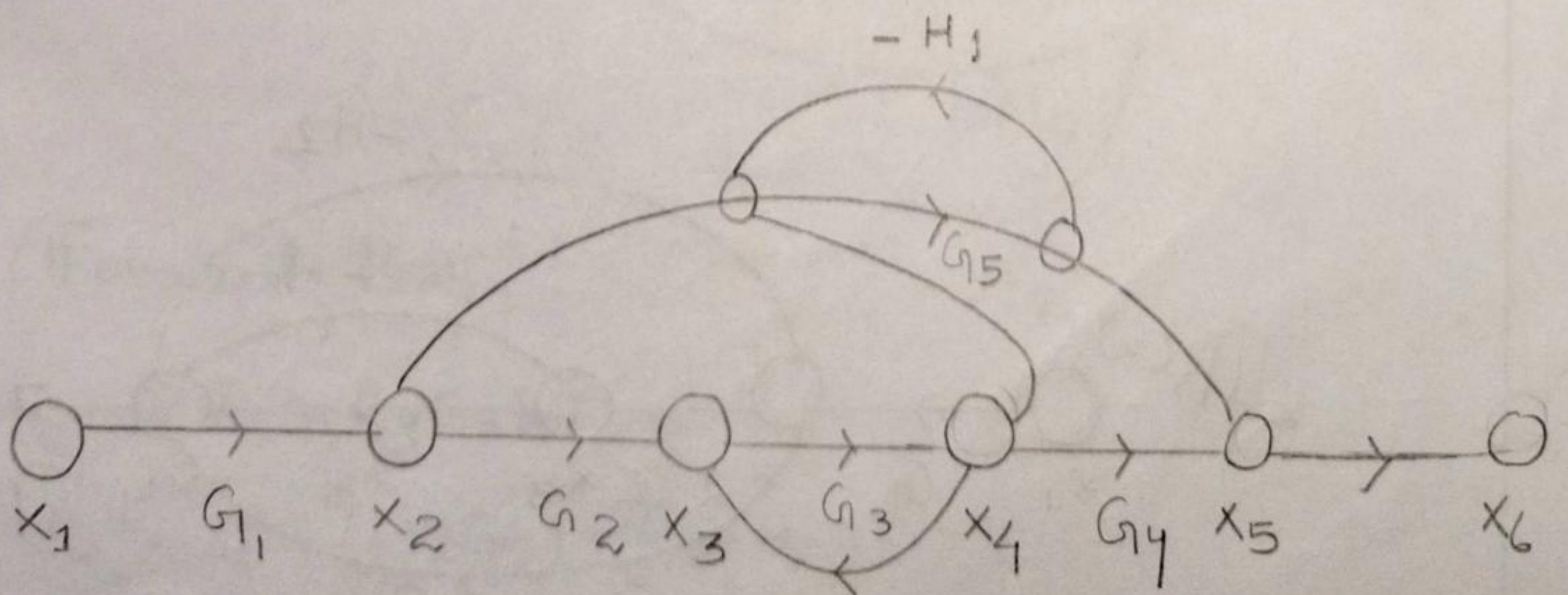
$$L_{21} = (-H_1)(H_3)$$

$$L_{22} = (-H_2 G_1) - H_3$$

$$\Delta = 1 - (L_1 + L_2 + L_3)(L_{21} + L_{22})$$

$$= 1 - (H_1 - H_2 G_1 - H_3)(-H_1 H_3)(-H_2 G_1)(-H_3)$$

$$\text{Trans form function} = \frac{G_1 G_2 \times 1}{1 + H_1 + H_2 + H_3 + H_1 H_3 + G_1 H_2 H_3}$$



Forward path -

$$P_1 = x_1 - x_2 - x_3 - x_4 - x_5 - x_6 = G_1 \cdot G_2 \cdot G_3 \cdot G_4$$

$$P_2 = x_1 - x_2 - x_5 - x_6 = G_1 \cdot G_5$$

$$\Delta_1 = 1 + H_1 G_5$$

$$\Delta_2 = 1 + H_2 G_3$$

Individual Loop -

$$L_1 = -H_1 G_5$$

$$L_2 = -H_2 G_3$$

Two non-touching loop -

$$L_{21} = G_5 H_1 (-H_2 G_3)$$

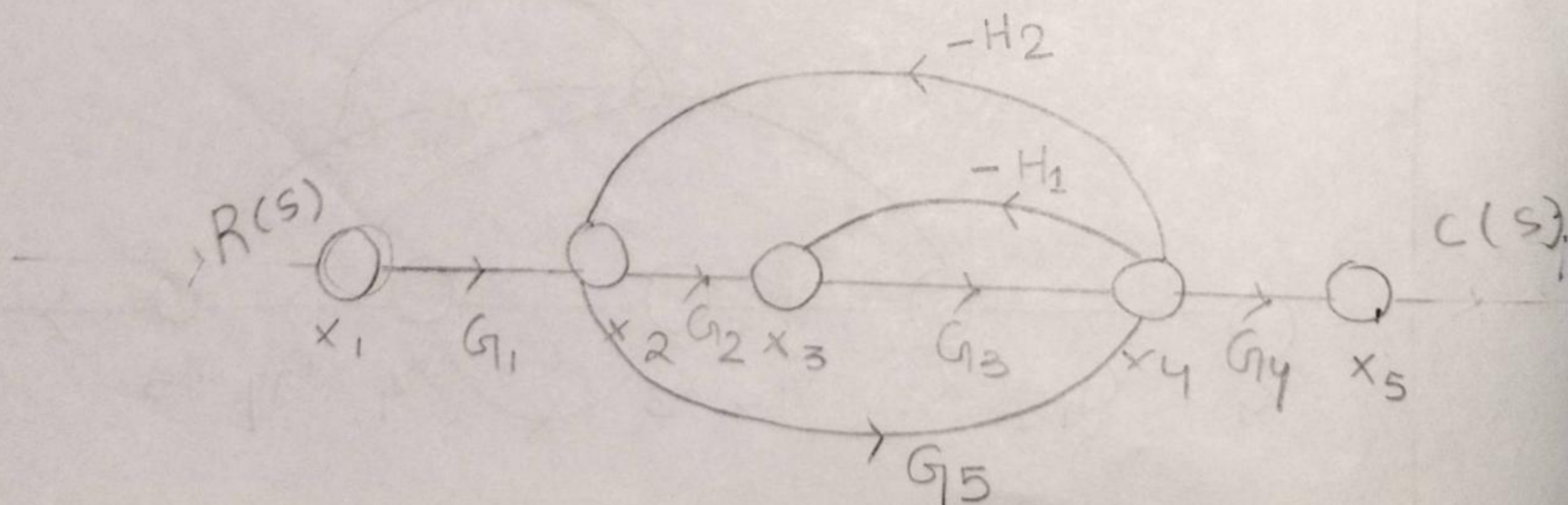
$$= G_5 G_3 H_1 H_2$$

Transfer function = $\frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

$$\Delta = (1 - (L_1 + L_2) + L_{21})$$

$$G_1 G_2 G_3 G_4 (1 + H_1 G_5) + G_1 G_5 (1 + H_2 G_3)$$

$$1 + G_1 H_1 + H_2 G_3 + G_5 G_3 H_1 H_2$$



Forward Path -

$$P_1 = x_1 - x_2 - x_3 - x_4 - x_5 = G_1 G_2 G_3 G_4$$

$$P_2 = x_1 - x_2 - x_4 - x_5 = G_1 G_5 G_4$$

$$\Delta_1 = 1 - (0)$$

$$\Delta_2 = 1 - (0)$$

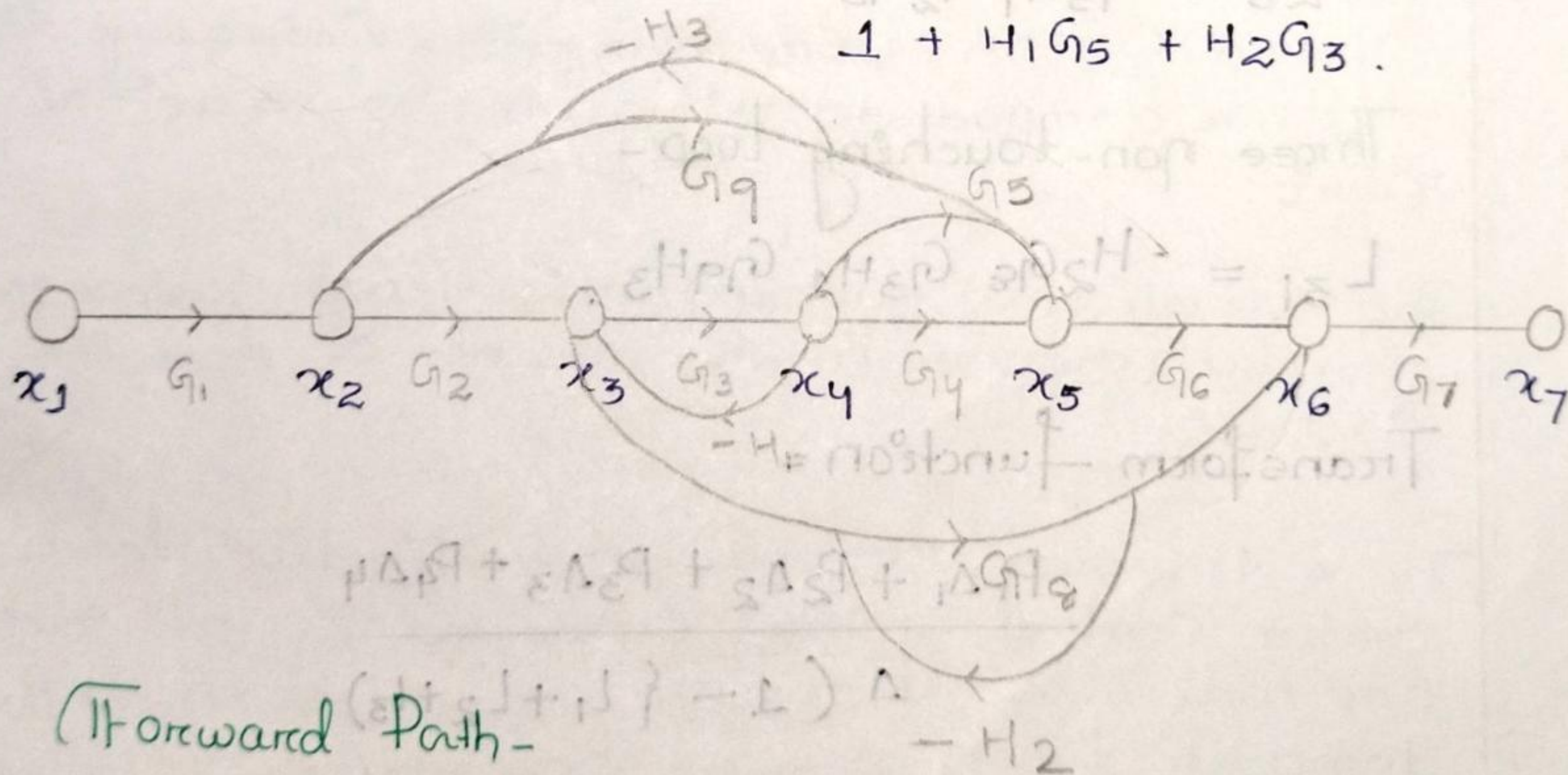
Individual Loop -

$$L_1 = -H_1 G_3$$

$$L_2 = -H_2 G_2 G_3$$

$$\Delta = 1 - (L_1 + L_2)$$

$$\begin{aligned} \text{Transfer function} &= \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} \\ &= \frac{G_1 G_2 G_3 G_4 \times 1 + G_1 G_2 G_4}{1 + H_1 G_5 + H_2 G_3} \end{aligned}$$



Forward Path -

$$P_1 = G_1 G_2 G_3 G_4 G_6 G_7$$

$$\Delta_1 = 1 - (-H_2 G_9) + (G_8 - H_2)$$

$$P_2 = G_1 G_2 G_3 G_5 G_6 G_9$$

$$P_3 = G_1 G_9 G_6 G_7$$

$$\Delta_2 = 1 + G_8 H_2$$

$$\Delta_3 = 1 + G_3 H_3 + G_8 H_2$$

$$P_4 = G_2 G_1 G_8 G_7$$

$$\Delta_4 = 1 + H_3 G_9$$

Individual Loop-

$$L_1 = -G_9 H_3$$

$$L_2 = -H_2 G_8$$

$$L_3 = -G_3 H_1$$

Two non-touching Loop-

$$L_{21} = G_8 H_2 + H_3 G_9$$

$$L_{22} = G_3 H_1 H_3 G_9$$

$$L_{23} = G_3 H_1 H_2 G_8$$

Three non-touching loop-

$$L_{31} = -H_2 G_8 G_3 H_1 G_9 H_3$$

Transform function =

$$P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4$$

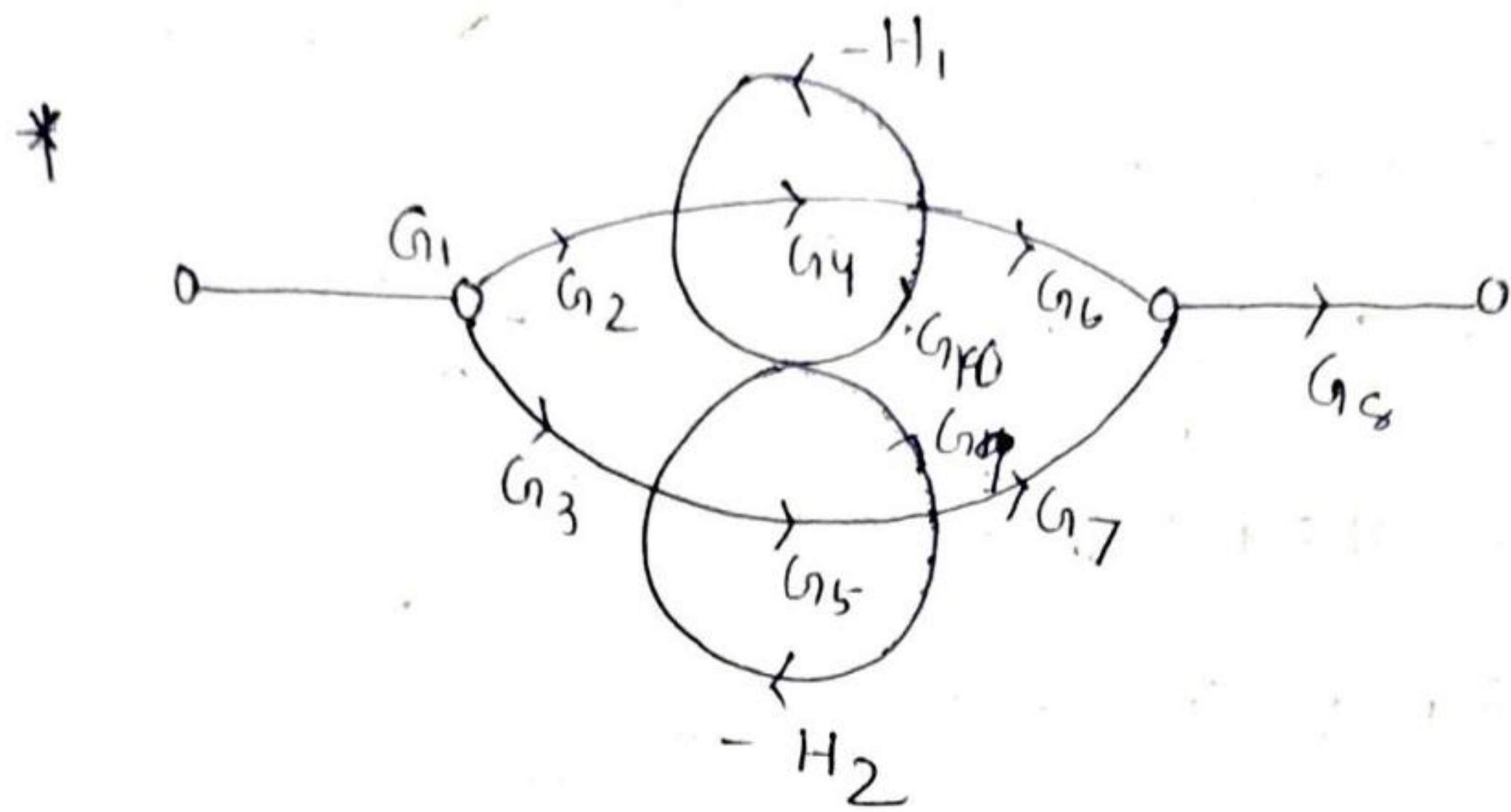
$$\Delta (1 - \{L_1 + L_2 + L_3\})$$

$$+ (L_{21} + L_{22} + L_{23})$$

$$- (L_{31})$$

Conversion Block Diagram to SFG

- i) Represent the output and input with separate row.
- ii) Take the takeoff point at the ~~row~~ node.
- iii) Take the summing point as node.
- iv) If the takeoff is before the summing point represent takeoff point and summing point with 2 separate nodes.
- v) If the takeoff point behind the summing point consider both as summing point.



Forward path-

$$P_1 = G_1 G_2 G_4 G_6 G_8$$

$$P_2 = G_1 G_2 G_6 G_{10} G_8$$

$$P_3 = G_1 G_3 G_5 G_7 G_8$$

$$P_4 = G_1 G_3 G_9 G_7 G_8$$

Individual loop-

$$L_1 = -H_1 G_4$$

$$L_2 = -H_2 G_5 - G_{10} H_1$$

$$L_3 = -G_{10} H_1 - H_2 G_5$$

$$L_4 = -G_9 - G_9 H_2$$

$$L_1 L_3 = H_1 G_4 G_5 H_2$$

$$L_1 L_4 = H_1 G_4 G_9 H_2$$

$$L_2 L_3 = G_{10} H_1 G_5 H_2$$

$$L_2 L_4 = G_{10} H_1 G_9 H_2$$

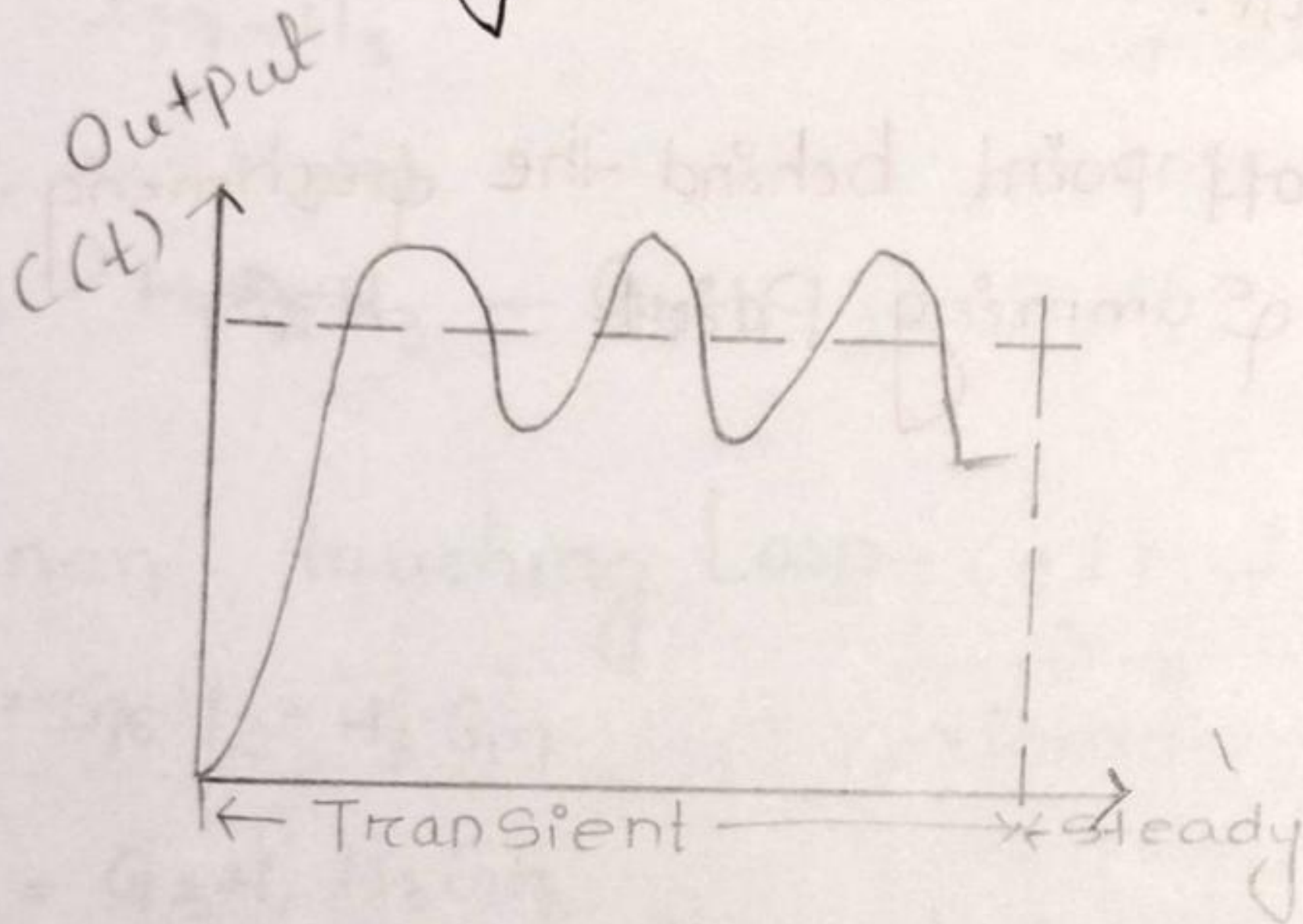
TIME RESPONSE ANALYSIS

It is a variation of output with respect to time in order to obtain the output with respect to time a system required input.

Input function is known as excitation function.

There are 2 type of time response function.

1. Transient
2. Steady state.



The transient response of a Second Order Control System can be underdamped, Critically damped, Overdamped depending on the value of damping ratio.

The reduction in damping ratio tends towards underdamped response in the absence of damping Substend Oscillation.

The transient response specification for a Second Order Control System subjected to unit step function are maximum overshoot, undershoot, natural & damping frequency of oscillation.

Steady State Error:

When output deviated from steady state value the actual output does not match exactly with steady state value.

The total time response =

$$C(t) = C_{ss}(t) + C_{Ts}(t)$$

↓
Accuracy of system

↓
Speed and nature response

Physically Reliable Excitation Function:

The excitation function which can be shown graphically.

They are -
Step function
Ramp function
Parabolic function
Impulse function.

Damping:

It is the opposing signal on based on the differential signal.

It may be mechanical damping (friction)

Electrical damping (resistance)

Magnetic field damping (reluctance)

A system without damping is a not good system.

It is represented by damping ratio δ .

The system can be divided into 3 type.

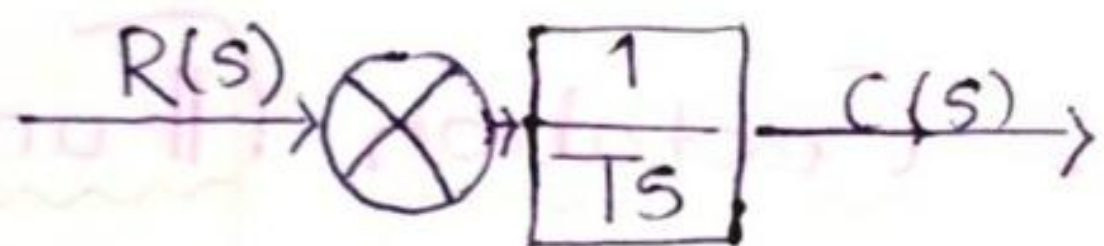
(a) Under damping ($\delta < 1$)

(b) Over damping ($\delta > 1$)

(c) Critically damping ($\delta = 1$)

Time response of 1st order system-

Let us consider a first order system.



$$\text{Overall T.f} = \frac{C(s)}{R(s)}$$

$$= \frac{1/Ts}{1 + 1/Ts \times 1}$$

$$\Rightarrow \frac{1}{Ts+1}$$

Where is known as transfer function

Respond to unit step input

For a unit step input

$$R(t) = u(t) = 1$$

$$L^{-1}\{R(t)\} = L^{-1}(1) = 1/s$$

$$R(s) = 1/s$$

The output response is given by

$$T.f = \frac{C(s)}{R(s)}$$

$$= \frac{1}{1+Ts}$$

$$\Rightarrow C(s) = R(s) \frac{1}{1+Ts}$$

$$= \frac{1}{s} \left[\frac{1}{1+Ts} \right] = \frac{1}{s} \times \frac{1}{1+Ts}$$

Breaking R.H.s of equation into P.f

$$C(s) = \frac{1}{s} - \frac{1}{1+sT}$$

$$= \frac{1}{s} - \frac{1}{s + 1/T}$$

Taking ~~Laplace~~ Inverse Laplace transform on both side

$$L^{-1}[C(s)] = L^{-1} \left[\frac{1}{s} - \frac{1}{s + 1/T} \right]$$

$$C(t) = 1 - e^{-t/T}$$

$$C(t) = 1 - e^{-t/T}$$

It is the 1st Order system.

The error is given by

$$e(t) = r(t) - c(t)$$

$$= 1 - (1 - e^{-t/T})$$

$$e(t) = e^{-t/T}$$

The steady state error is

$$e_{ss} = \lim_{t \rightarrow \infty} e^{-t/T}$$

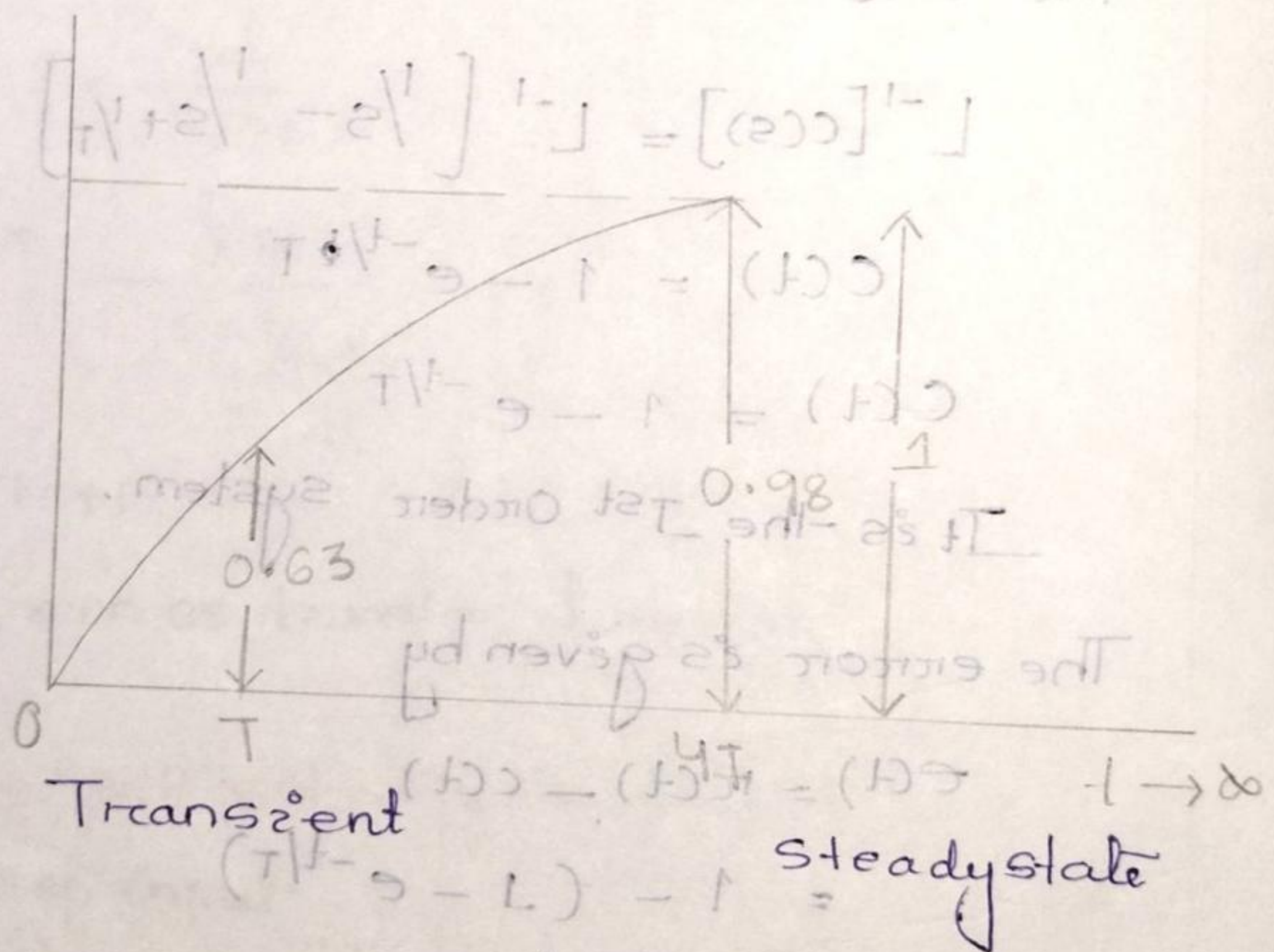
$$= 0$$

$$t = \infty$$

The time response is shown in figure -
 It is seen that the output rises exponentially from zero value to the final value of unity. The initial group of the curve path

$$\frac{dC(t)}{dt} \bigg|_{t=0} = \frac{1}{T} e^{-t/T} \bigg|_{t=0} = \frac{1}{T}$$

Where T is known as the time constant of the system.



The time constant indicates how fast the system tends to reach the final value.

The speed of response can be quantitatively dependent on the time for output to become a particular percentage of its final value.

A Large time Constant Correspond to a ~~so~~ sluggish system and spot time Constant to first response.

Time response to the unit Ramp function -

For unit ramp input $R(t) = t$

$$\mathcal{L}^{-1}\{t\} = 1/s^2$$

$$\Rightarrow R(s) = \frac{1}{s^2}$$

$$\text{The output response } C(s) = \frac{1}{s^2} \left[\frac{1}{1+Ts} \right]$$

using Partial fraction

$$\Rightarrow C(s) = \frac{1/T}{s^2(s+1/T)}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1/T}$$

Comparing the numerators

$$\frac{1}{T} = As\left(s+\frac{1}{T}\right) + B\left(s+\frac{1}{T}\right) + Cs^2 \quad \text{--- eqn (i)}$$

Putting $s=0$

$$\Rightarrow \frac{1}{T} = B \frac{1}{T}$$

$$\Rightarrow B = 1$$

Putting $s = -1/T$ in eqn (i)

$$\Rightarrow \frac{1}{T} = A \times \frac{1}{T} \left(\frac{1}{T} + \frac{1}{T} \right) + B \left(\frac{1}{T} + \frac{1}{T} \right) + C \left(\frac{1}{T} \right)^2$$

$$\Rightarrow \frac{1}{T} = A \times \frac{1}{T} \times \frac{2}{T} + \frac{2B}{T} + 0 \frac{1}{T^2}$$

$$\Rightarrow \frac{1}{T} = \frac{1}{T} (A \times 2 + 2B + C/T)$$

$$\Rightarrow 2A + 2B + \frac{C}{T} = 1$$

Comparing the co-efficient of s^2 on both the sides
 Taking inverse Laplace transform of eqn

$$A + C = 0$$

$$\Rightarrow A = -C$$

$$\Rightarrow A = -T$$

$$\Rightarrow C(s) = -\frac{T}{s} + \frac{1}{s^2} + \frac{T}{s + 1/T}$$

Taking inverse Laplace transform of eqn

$$C(t) = L^{-1} \left\{ -\frac{T}{s} + \frac{1}{s^2} + \frac{T}{s + 1/T} \right\}$$

$$= t + T(e^{-t/T} + 1)$$

$$= t - T(1 - e^{-t/T})$$

$$C(t) = t - T(1 - e^{-t/T})$$

The error signal $e(t) = R(t) - C(t)$

$$\Rightarrow t - t + T(1 - e^{-t/T})$$

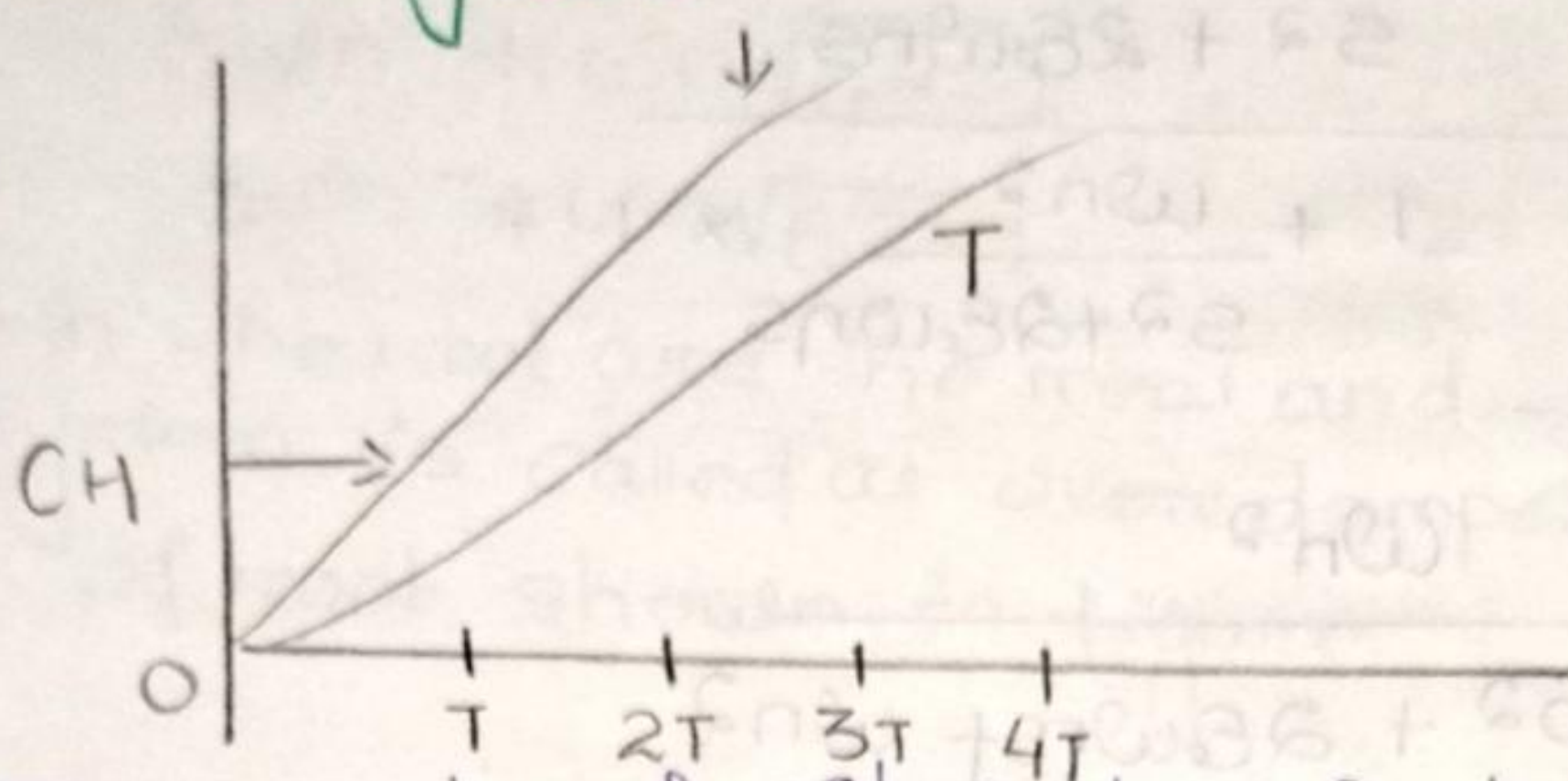
$$\Rightarrow T(1 - e^{-t/T})$$

and steady state error

$$R_{ss} = \lim_{t \rightarrow \infty} e(t)$$

$$\Rightarrow e_{ss} = T$$

First Order System-



Time response to of 1st Order Control system subjected for unit ramp input function.

The first Order system under consideration with track the unit ramp input with a steady state error 'T' which equal to the time constant of the system as shown in the figure.

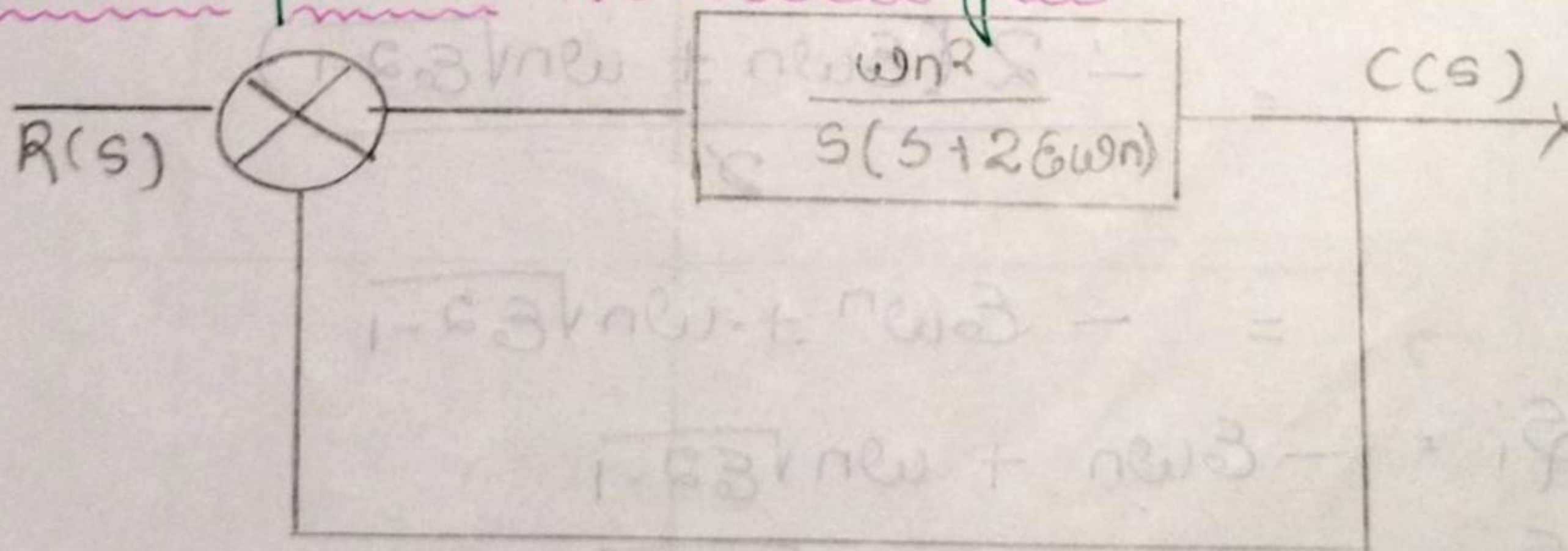
Reducing the system time constant therefore not only improve its speed of response but also steady state error to ramp input.

Conclusion:-

First Order system is characteristics by the unit constant. per step input of first Order system produce Zero Steady State errors. 1st Order system acceleration input produce steady state error.

The acceleration input cannot be apply.

Time response 2nd Order system:-



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} \times \frac{1 + \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}}{1 + \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}}$$

$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Where,

ζ = Called the damping ratio of the system

ω_n = Called underdamped natural frequency.

The characteristics equation is given by

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

The root of the characteristics eqn is given by

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\zeta\omega_n \pm 2\omega_n\sqrt{\zeta^2 - 1}}{2}$$

$$= \frac{-2\zeta\omega_n \pm 2\omega_n\sqrt{\zeta^2 - 1}}{2}$$

$$= \frac{-2\zeta\omega_n \pm 2\omega_n\sqrt{\zeta^2 - 1}}{2}$$

$$= \frac{-2(\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1})}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

Case-1 (When $\zeta > 1$) Overdamped

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$

Hence both the roots are real and -ve and also distinct. Thus the system is called as overdamped system. The location of roots shown in figure.

Case-2 (When $\zeta = 1$) Critically underdamped

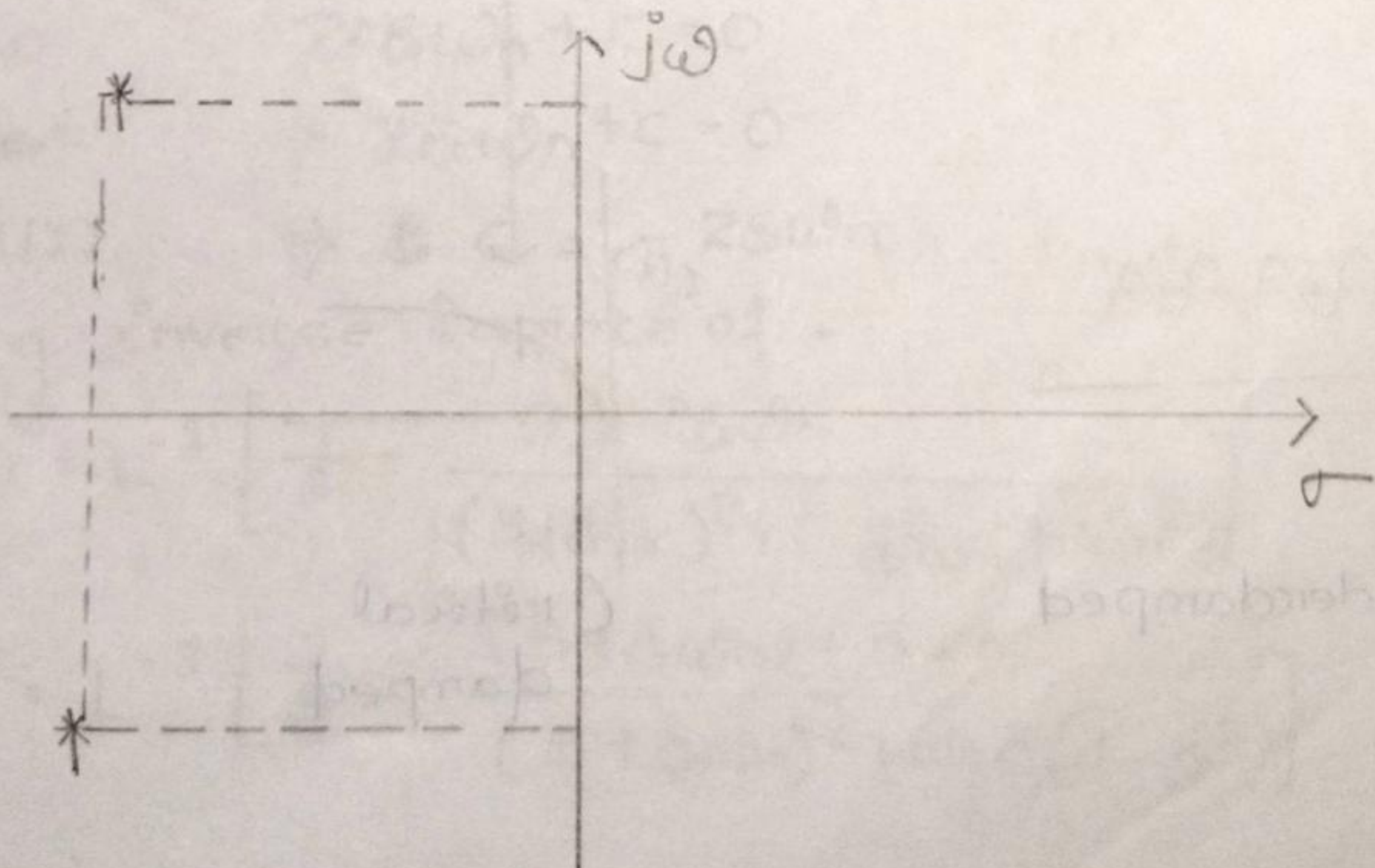
Here both the roots are -ve and equal. This system is critically underdamped system. The location of roots is shown in figure.

Case-3 (When $\zeta < 1$) underdamped

$s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$ Where $\zeta < 1$, $\zeta^2 - 1$ is negative

$$s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$$

Here both the roots are complex with -ve real part and the system is called underdamped system. This system is very important for practical point of view.

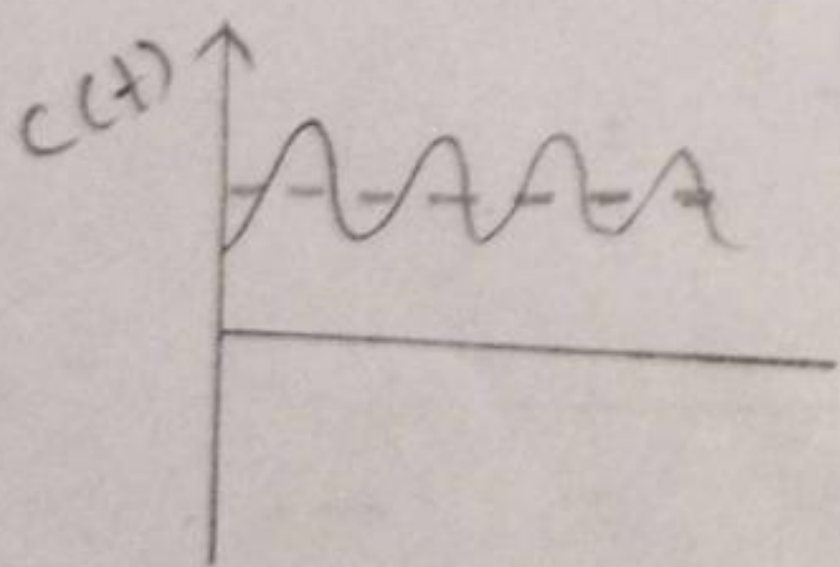
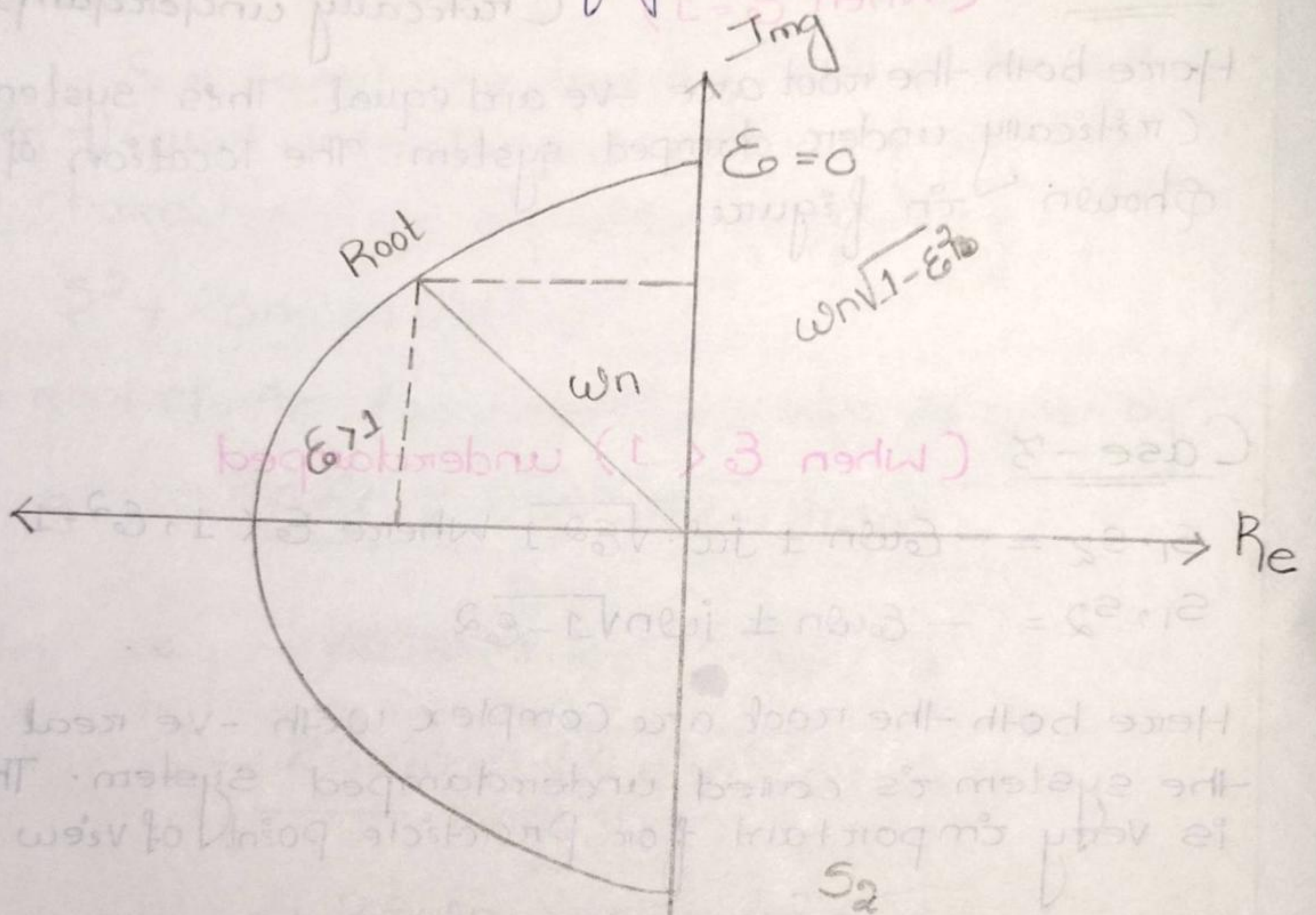


Case-4

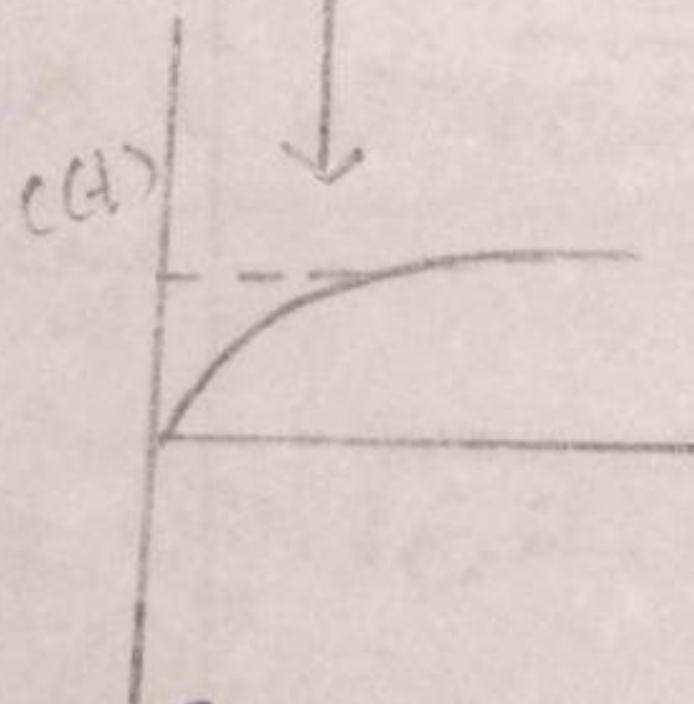
$$s_1, s_2 = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2} = \pm j \omega_n$$

In this case both the roots are imaginary and the system called as under damped system. In this under damped system we get sustained oscillation of output.

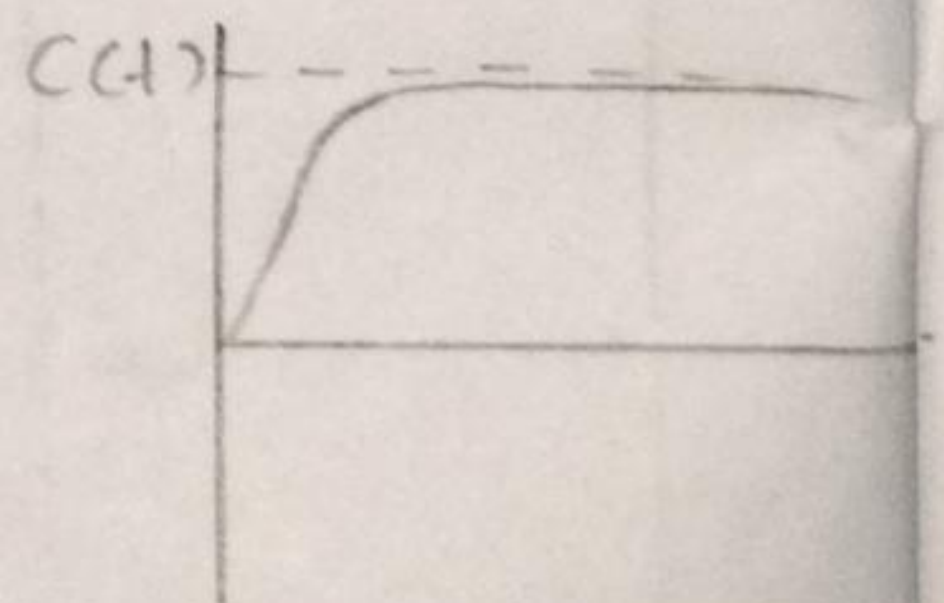
The location of roots of characteristics respond and time response shown in figure.



Underdamped



Critical damped



Overdamped

Time response of underdamped second order system for unit step input —

For unit step input $R(t) = u(t)$

$$R(s) = \frac{1}{s}$$

The overall transfer function = $\frac{C(s)}{R(s)}$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$C(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Comparing the numerator —

$$\omega_n^2 = A(s^2 + 2\zeta\omega_n s + \omega_n^2) + (Bs + C)s$$

Putting $s = 0$

$$\omega_n^2 = A\omega_n^2$$

$$\Rightarrow A = 1$$

Comparing Co-efficient of s^2 , $0 = A + B$

$$\Rightarrow B = -A$$

$$\Rightarrow B = -1$$

Comparing Co-efficient of s

$$A = 0$$

$$2A\zeta\omega_n + C = 0$$

$$\Rightarrow C = -2\zeta\omega_n$$

$$\Rightarrow C = -2\zeta\omega_n$$

$$\Rightarrow C = -2\zeta\omega_n$$

Taking inverse Laplace of —

$$C(t) = L^{-1} \left[\frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + (\omega_n^2 - \zeta^2\omega_n^2)} \right]$$

$$= L^{-1} \left[\frac{1}{s} - \frac{(s + \zeta\omega_n) + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} \right]$$

Assum,

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

Where ω_d = damped natural frequency

$$= L^{-1} \left[\frac{1}{s} - \frac{(s + \xi \omega_n)}{(s + \xi \omega_n)^2 + (\omega_d)^2} - \frac{\xi \omega_n}{(\xi \omega_n)^2 + (\omega_d)^2} \right]$$

$$= L^{-1} \left[\frac{1}{s} - \frac{(s + \xi \omega_n)}{(s + \xi \omega_n)^2 + (\omega_d)^2} - \frac{\xi \omega_n}{\omega_n \sqrt{1 - \xi^2}} \times \frac{\omega_d}{(s + \xi \omega_n)^2 + (\omega_d)^2} \right]$$

$$= L^{-1} \left[\frac{1}{s} - \frac{(s + \xi \omega_n)}{(s + \xi \omega_n)^2 + (\omega_d)^2} - \frac{\xi}{\sqrt{1 - \xi^2}} \times \frac{\omega_d}{(s + \xi \omega_n)^2 + (\omega_d)^2} \right]$$

$$C(t) = 1 - e^{-\xi \omega_n t} \left[\cos \omega_d t + \frac{\xi}{\sqrt{1 - \xi^2}} \sin \omega_d t \right] \quad (\text{for } t \geq 0)$$

$$= 1 - e^{-\xi \omega_n t} \left[\frac{\sqrt{1 - \xi^2} \cos \omega_d t + \xi \sin \omega_d t}{\sqrt{1 - \xi^2}} \right]$$

Assuming a triangle as shown in figure.

From triangle:

$$\cos \phi = \frac{\xi \omega_n}{\omega_n}$$

$$= \xi$$

$$\sin \phi = \frac{\omega_n \sqrt{1 - \xi^2}}{\omega_n}$$

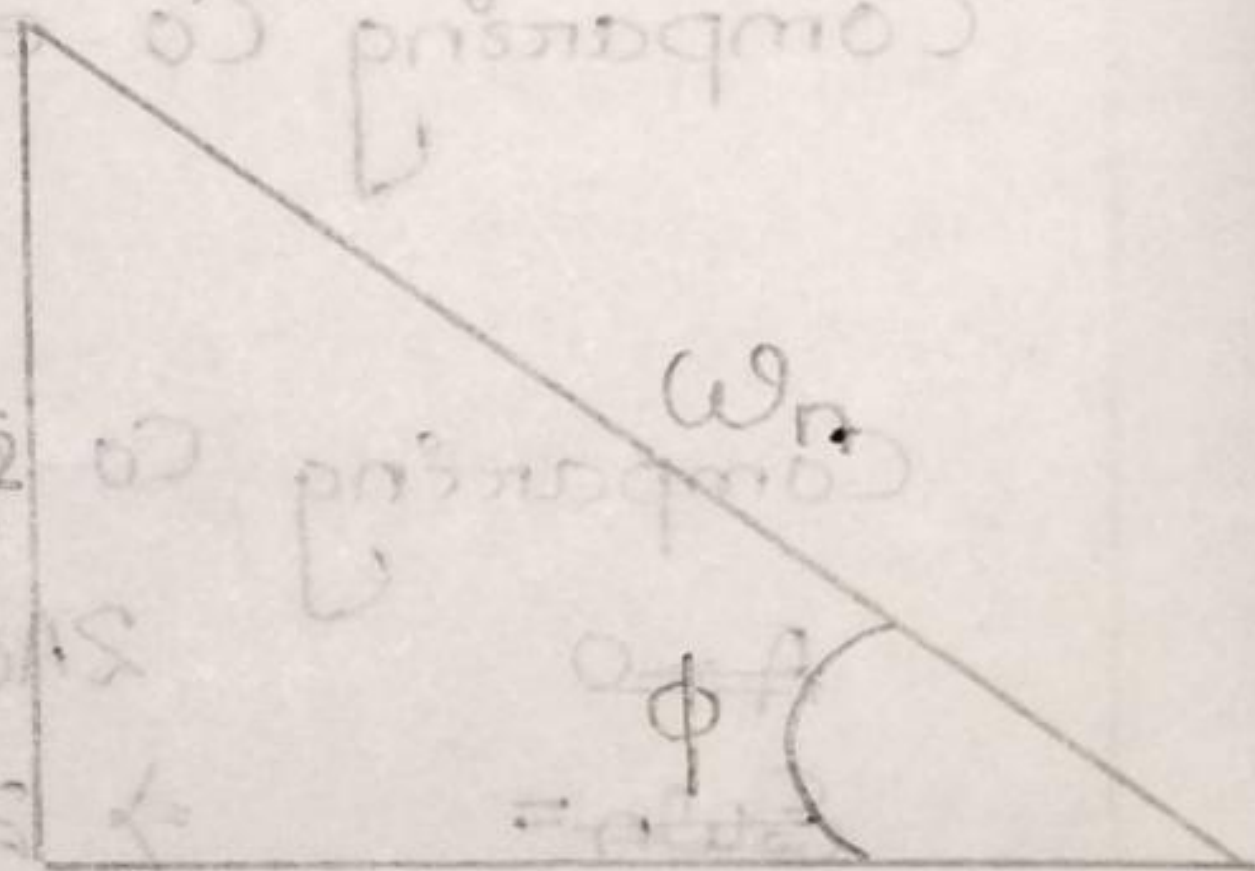
$$\tan \phi = \frac{\sqrt{1 - \xi^2}}{\xi}$$

$$= \sqrt{1 - \xi^2}$$

$$\Rightarrow \phi = \tan^{-1} \frac{\sqrt{1 - \xi^2}}{\xi}$$

$$\phi = \cos^{-1} \xi$$

$$\phi = \sin^{-1} \sqrt{1 - \xi^2}$$



$$C(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\frac{\sin\phi \cdot \cos\omega_d t + \cos\phi \cdot \sin\omega_d t}{\sqrt{1-\xi^2}} \right]$$

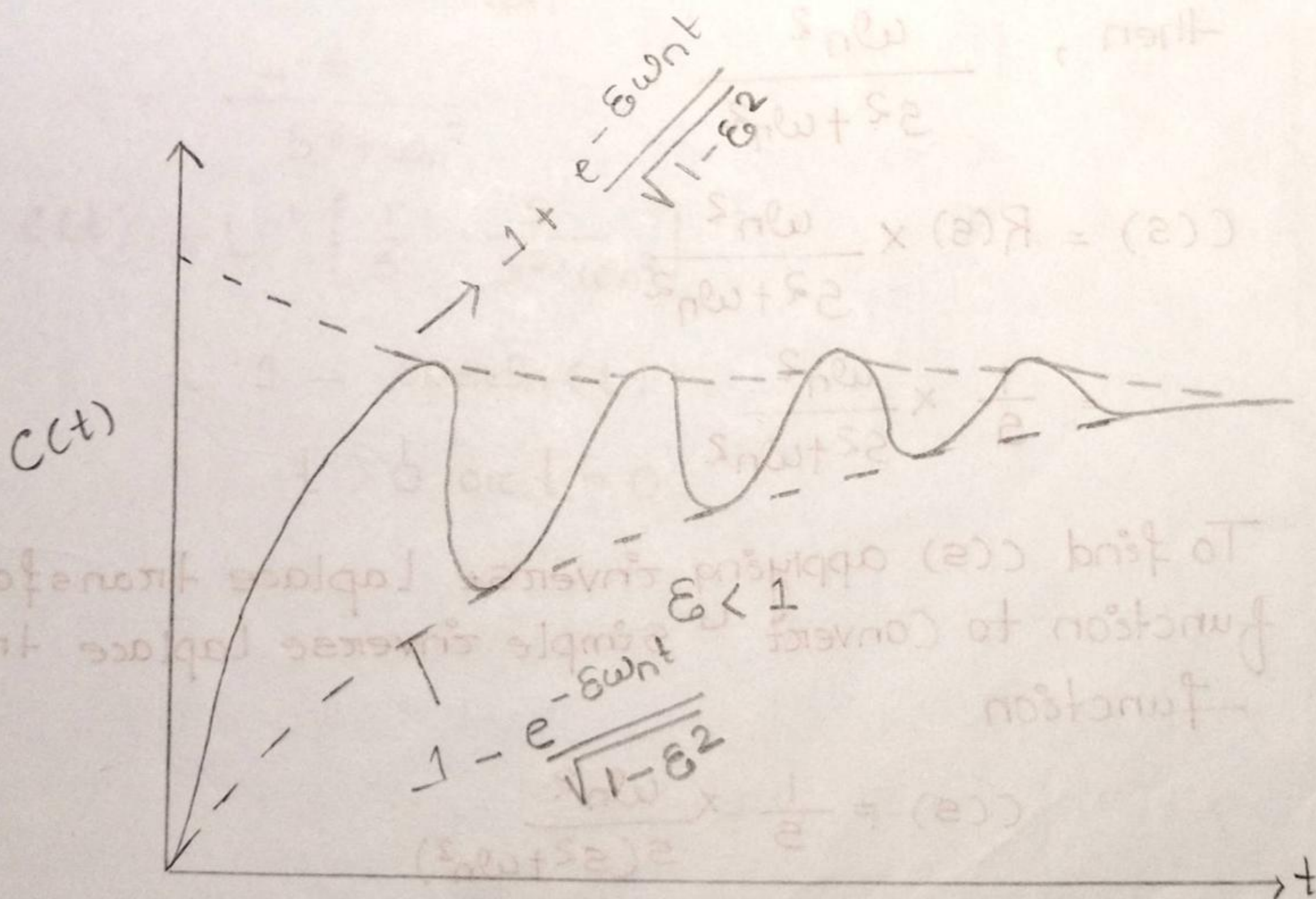
$$= 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \left[\sin(\omega_d t + \phi) \right] \text{ [Time response]}$$

Where, ξ = damping ratio of system
 ω_n = natural frequency of oscillation of the system.

ω_d = damped frequency of oscillation of the system and $\omega_d = \omega_n \sqrt{1-\xi^2}$

The steady state value of $C(t)$

$$C(t) = C_{ss}(t) = \lim_{t \rightarrow \infty} C(t) = 1$$



Hence the response of a second order system to unit step input is damped oscillating for $\zeta < 1$ the response shown in figure.

* Time response of underdamped second order system for unit step -

For unit step input $R(t) = u(t) = 1$

$$\Rightarrow L^{-1}(R\{t\}) = L^{-1}(1) = \frac{1}{s}$$

From basic definition of T.F. $= \frac{C(s)}{R(s)}$

$$= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Put $\zeta = 0$

then, $\frac{\omega_n^2}{s^2 + \omega_n^2}$

$$C(s) = R(s) \times \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$= \frac{1}{s} \times \frac{\omega_n^2}{s^2 + \omega_n^2}$$

To find $c(s)$ applying inverse Laplace transforming function to convert function

Simple inverse Laplace transform

$$C(s) = \frac{1}{s} \times \frac{\omega_n^2}{s(s^2 + \omega_n^2)}$$

By using Partial fraction -

$$\frac{A}{s} + \frac{Bs + c}{s^2 + \omega_n^2} \quad (1)$$

$$= \frac{A(s^2 + \omega_n^2) + s(Bs + c)}{s(s^2 + \omega_n^2)}$$

$$A = 1$$

$$A + B = 0$$

$$\Rightarrow B = -1$$

$$C = 0 \quad (2)$$

Put the value in equation (1)

$$\frac{1}{s} \times \frac{(-s) + 0}{s^2 + \omega_n^2}$$

$$= \frac{-s}{s^2 + \omega_n^2}$$

$$C(t) = L^{-1} \left[\frac{1}{s} - \frac{s}{s^2 + \omega_n^2} \right]$$

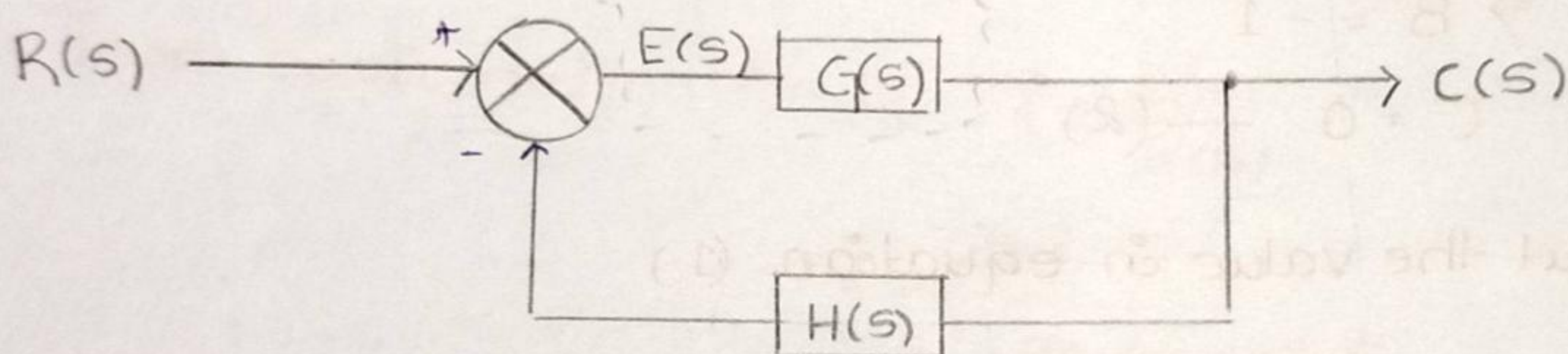
$$= 1 - \cos \omega_n t$$

$$t > 0 \text{ or } t = 0.$$

Steady State error-

The steady state error represents the accuracy of Control system, It is defined as the variation between the steady state output and i/p. If the variation is small the system is more accurate.

Hence for a good system the steady state error should be negligible. Ideal speaking the error is zero for an ideal system.



$$E(s) = R(s) - [C(s)H(s)] \quad \because C(s) = E(s)G(s)$$
$$R(s) - [E(s)G(s)H(s)]$$

$C(s)$ = Output

$R(s)$ = reference input

$E(s)$ = Error signal

$G(s)$ = Transfer function

$H(s)$ = feedback function.

$$E(s) = R(s) - [E(s)G(s)H(s)]$$

$$\Rightarrow E(s) + [E(s)G(s)H(s)] = R(s)$$

$$\Rightarrow E(s) [1 + G(s)H(s)] = R(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

To consider error signal by the time domain

$$e(t) = L^{-1} \{ E(s) \}$$

$$= L^{-1} \left\{ \frac{R(s)}{1 + G(s)H(s)} \right\}$$

E_{ss} = Steady state error by the value of $t \rightarrow \infty$

$$\Rightarrow e_{ss} = \lim_{t \rightarrow \infty} e(t)$$

$$e_{ss} = \lim_{s \rightarrow 0} s E(s)$$

Error Constant Method:—

There 3 types of Constant Steady State error method

- (i) Step input
- (ii) Ramp input
- (iii) Parabolic

Step input—

Steady state error due to step input

$$e_{ss} = \lim_{s \rightarrow 0} s E(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$R(s) = L^{-1} \{ u(t) \} \quad u(t) = 1$$

$$L^{-1} \{ 1 \} = \frac{1}{s}$$

$$e_{ss} = \lim_{s \rightarrow 0} \frac{s \times 1/s}{1 + G(s)H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)}$$

$$e_{ss} = \frac{1}{1 + \lim_{s \rightarrow 0} G(s)H(s)}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$$= \frac{1}{1 + K_p}$$

P = position error constant.

The Steady State error value when the input is unit step.

Type = 0

$$e_{ss} = \frac{1}{1 + K_p}$$

$$= \frac{1}{1 + \lim_{s \rightarrow 0} G(s)H(s)}$$

$$K_p = G(s)H(s)$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$$= \frac{K(s+z_1)(s+z_2)\dots(s+z_n)}{s^n(s+p_1)(s+p_2)\dots(s+p_n)}$$

$$\lim_{s \rightarrow 0} \frac{K(s+z_1)(s+z_2)\dots}{s^n(s+p_1)(s+p_2)\dots}$$

$$= \frac{K z_1 z_2}{p_1 p_2}$$

$$\Rightarrow e_{ss} = \frac{1}{1 + \text{Constant}}$$

$$= \text{Constant}$$

Type = 1

$$e_{ss} = \frac{1}{1 + K_p}$$

$$= \frac{1}{1 + \lim_{s \rightarrow 0} G(s)H(s)}$$

$$K_p = \frac{K (s+z_1)(s+z_2) \dots (s+z_n)}{s^n (s+p_1)(s+p_2) \dots (s+p_n)}$$

$$\Rightarrow \lim_{s \rightarrow 0} \frac{K z_1 z_2}{0}$$

$$= \infty$$

$$\therefore e_{ss} = \infty$$

So each type '1' system 1 or more order if a given unit step input the steady state error = 0

Steady State error due to unit Ramp input:-

$$e_{ss} = \lim_{s \rightarrow 0} s E(s)$$

$$E_s = \frac{R(s)}{1 + G(s)H(s)}$$

$$R(s) = \frac{1}{s^2} \text{ or } \frac{A}{s^2}$$

$$\lim_{s \rightarrow 0} \frac{s \times 1/s^2}{1 + G(s)H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{1/s}{1 + G(s)H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{1}{s [1 + G(s)H(s)]}$$

$$= \lim_{s \rightarrow 0} \frac{1}{s + sG(s)H(s)}$$

$$\lim_{s \rightarrow 0} \frac{1}{0 + \lim_{s \rightarrow 0} sG(s)H(s)}$$

$K_v = \text{Velocity error constant}$

$$= \frac{1}{0 + K_v}$$

$$= \frac{1}{K_v}$$

$$e_{ss} = \frac{1}{K_v}$$

Type = 0

$$= \frac{K (s+z_1)(s+z_2) \dots (s+z_n)}{s^n (s+p_1)(s+p_2) \dots (s+p_n)}$$

$$\lim_{s \rightarrow 0} sG(s)H(s) = \lim_{s \rightarrow 0} \frac{sK(s+z_1)(s+z_2) \dots (s+z_n)}{s^n (s+p_1)(s+p_2) \dots (s+p_n)}$$

$$e_{ss} = \frac{1}{K_v}$$

$$= \frac{1}{0}$$

$$= \infty$$

Type = 1

$$\lim_{s \rightarrow 0} s G(s) H(s)$$

$$= \lim_{s \rightarrow 0} \frac{K s (s + z_1) (s + z_2) \dots (s + z_n)}{s^n (s + p_1) (s + p_2) \dots (s + p_n)}$$

$$= \frac{K \cancel{s} (z_1 z_2)}{\cancel{s} (p_1 p_2)}$$

$$= \frac{K (z_1 z_2)}{p_1 p_2} = \text{Constant}$$

$$e_{ss} = \frac{1}{K_v} = \text{Constant}$$

Type - 2

$$\lim_{s \rightarrow 0} s G(s) H(s)$$

$$= \lim_{s \rightarrow 0} \frac{K s (s + z_1) (s + z_2) \dots (s + z_n)}{(s + p_1) (s + p_2) \dots (s + p_n)}$$

$$= \frac{K \cancel{s} (z_1 z_2)}{s^2 (p_1 p_2)}$$

$$= \frac{K (z_1 z_2)}{0}$$

$$= 0$$

$$e_{ss} = \frac{1}{K_v}$$

$$= \frac{1}{0}$$

$$= \infty$$

Steady State error due to parabolic input -

$$e_{ss} = \lim_{s \rightarrow 0} sE(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$R(s) = \frac{1}{s^3}$$

$$= \frac{1/s^3 \times s}{1 + G(s)H(s)}$$

$$= \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$\lim_{s \rightarrow 0} \frac{1}{0 + \lim_{s \rightarrow 0} s^2 G(s)H(s)}$$

$K_A =$ acceleration error constant

$$= \frac{1}{0 + K_A}$$

$$e_{ss} = \frac{1}{K_A}$$

Type-0

$$\lim_{s \rightarrow 0} s^2 G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{s^2 (s+z_1)(s+z_2) \dots (s+z_n)}{s^n (s+p_1)(s+p_2) \dots (s+p_n)}$$

$$= 0$$

$$e_{ss} = 1/K_A = 1/0 = \infty$$

Type-1

$$\lim_{s \rightarrow 0} s^2 G(s) H(s)$$

$$= \lim_{s \rightarrow 0} \frac{K s^2 (s+z_1)(s+z_2) \dots (s+z_n)}{s^n (s+p_1)(s+p_2) \dots (s+p_n)}$$

$$= \cancel{s^2} (s+z_1)(s+z_2) \dots (s+z_n) \cdot \lim_{s \rightarrow 0} \frac{s^2}{s^n (s+p_1)(s+p_2) \dots (s+p_n)}$$

$$= \frac{K (z_1 z_2)}{P_1 P_2} = \text{Constant}$$

$$e_{ss} = \frac{1}{K_A} = \text{Constant}$$

Steady State error signal -

Type	Step unit $u(t)$	Ramp $rt(t)$	Parabolic $\frac{1}{2}at^2$
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_A}$
3	0	0	0

Question-1

The openloop transfer function of a system given by $G(s) = \frac{50}{(1+0.15s)(1+2s)}$ determine a steady state for a unit step unit, unit ramp input and acceleration input $\frac{1}{2}T^2$.

$$G(s) = \frac{50}{(1+0.1s)(1+2s)}$$

$$\text{Unit Step input} = \frac{1}{1+K_p}$$

$$= \frac{1}{1 + \lim_{s \rightarrow 0} G(s)H(s)}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$$(\because H(s) = 1)$$

$$= \lim_{s \rightarrow 0} \frac{50}{(1+0.1s)(1+2s)}$$

$$= 50$$

$$E_{ss} = \frac{1}{1+50} = \frac{1}{51}$$

$$G(s) = \frac{50}{(1+0.1s)(1+2s)}$$

$$\text{Ramp input} = \frac{1}{K_v}$$

$$K_v = \frac{1}{1 + \lim_{s \rightarrow 0} sG(s)H(s)}$$

$$= \frac{1}{1 + \cancel{s} \times 50}$$

$$= \frac{1}{1 + \lim_{s \rightarrow 0} s \times \frac{50}{(1+0.1s)(1+2s)}}$$

$$= 0$$

$$E_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

$$G(s) = \frac{50}{(1+0.1s)(1+2s)}$$

$$\text{parabolic input} = \frac{1}{KA}$$

$$\lim_{s \rightarrow 0} s^2 G(s) H(s)$$

$$= \lim_{s \rightarrow 0} s^2 \times \frac{50}{(1+0.1s)(1+2s)}$$

$$= 0$$

$$E_{ss} = \frac{1}{0} = \infty$$

2) The open loop transfer function of a feedback Control System is given by $G(s)H(s) = \frac{100}{s(s+1)(s+2)}$ determine the Static error coefficient and Steady state error when $R(t) = 5 + 4t$

$$G(s)H(s) = \frac{100}{s(s+1)(s+2)}$$

$$\text{unit step input} = \frac{1}{1+K_p}$$

$$E_{ss} = \frac{1}{1+K_p} = \frac{5}{1+K_p}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{100}{s(s+1)(s+2)}$$

$$= \frac{100}{0}$$

$$= \infty$$

$$K_v = \lim_{s \rightarrow 0} s G(s)H(s)$$

$$= \lim_{s \rightarrow 0} s \times \frac{100}{s(s+1)(s+2)}$$

$$= \frac{100}{1+2}$$

$$= 100/3 = 50$$

$$E_{ss} = \frac{40}{K_v}$$

$$= \frac{40}{50}$$

$$= 0.8$$

CONTROLLER AND EFFECT :-

A Controller is a mechanism that seeks to minimize the difference between the actual value of the system and the desired value of the system.

There are 3 basic controllers.

- i)
- ii) Integral Controller
- iii) Derivative Controller.

These 3 types of controllers can be combined into a new controller.

- i) Proportional + Integral
- ii) Integral + Derivative

P-Signal:-

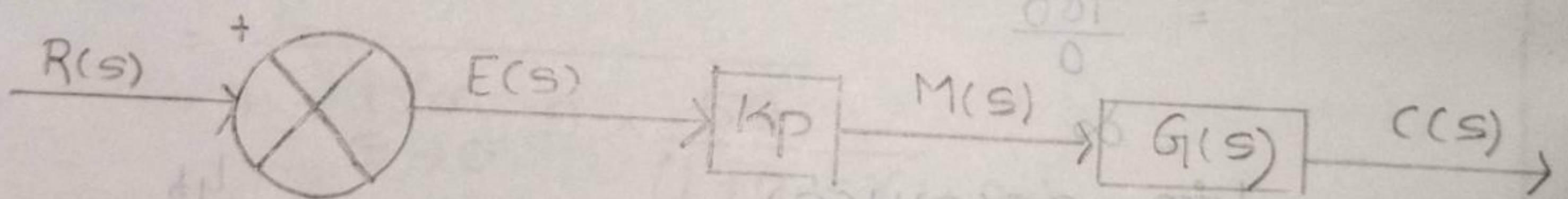
In this type of controller system the actuating signal is proportional signal.

The error signal is the difference between the reference input signal and the feedback obtained from the output.

$$M(t) = K_p e(t)$$

$$M(s) = K_p E(s)$$

$$\Rightarrow \frac{M(s)}{E(s)} = K_p$$



K_p = Proportional signal

$M(s)$ = actuating signal controller output

$E(s)$ = error signal.

Integral Controller -

In a controller with integral control action the output is proportional to integral of error signal

Mathematically -

$$M(t) = K_i \int_0^t e(t) dt$$

Where,

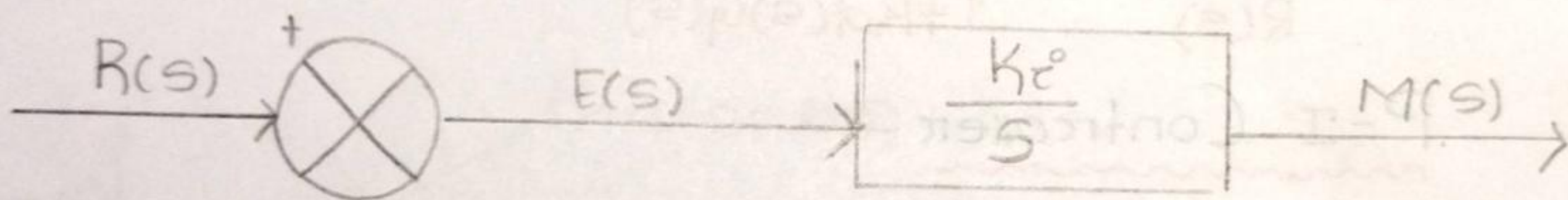
K_i = adjustable constant

The transfer function of integral control action can be

$$\frac{M(s)}{E(s)} = K_i(s)$$

It is also known as a reset control.

A controller with a integral control action is shown in figure -



Derivative Controller -

In this type of control action the actuating signal is proportional to the rate of change of the input to the controller.

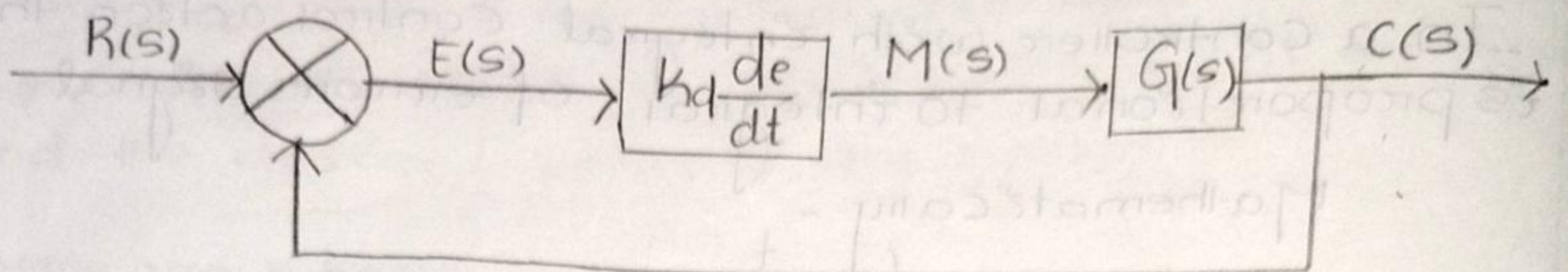
But this region the derivative controller is also known as rate controller.

Mathematically -

$$M(t) = K_d \frac{de(t)}{dt}$$

$$M(s) = K_d s E(s)$$

$$\frac{M(s)}{E(s)} = K_d s$$



$$M(s) = k_d s E(s)$$

$$\Rightarrow E(s) = R(s) - C(s)$$

$$\Rightarrow M(s) = k_d(s) [R(s) - C(s)]$$

$$\Rightarrow C(s) = M(s) G(s)$$

$$\Rightarrow \cancel{M(s) G(s)} \quad M(s) = k_d(s) [R(s) - M(s) G(s)]$$

$$\Rightarrow M(s) = k_d(s) R(s) - k_d(s) M(s) G(s)$$

$$\Rightarrow M(s) + k_d(s) M(s) G(s) = k_d(s) R(s)$$

$$\Rightarrow M(s) [1 + k_d(s) G(s)] = k_d(s) R(s)$$

$$\Rightarrow \frac{M(s)}{R(s)} = \frac{k_d(s)}{1 + k_d(s) G(s)}$$

P-I Controller

It is the combination of Proportional + Integral control action

Mathematically it can be represent as

$$M(t) = k_p e(t) + \frac{k_p}{T_i} \int_0^t e(t) dt$$

$$M(s) = k_p E(s) + \frac{k_p}{s T_i} E(s) = E(s) \left[1 + \frac{1}{s T_i} \right] k_p$$

The transfer function of this controller is

$$\frac{M(s)}{E(s)} = k_p \left[1 + \frac{1}{s T_i} \right]$$

Where

T_i = integral time

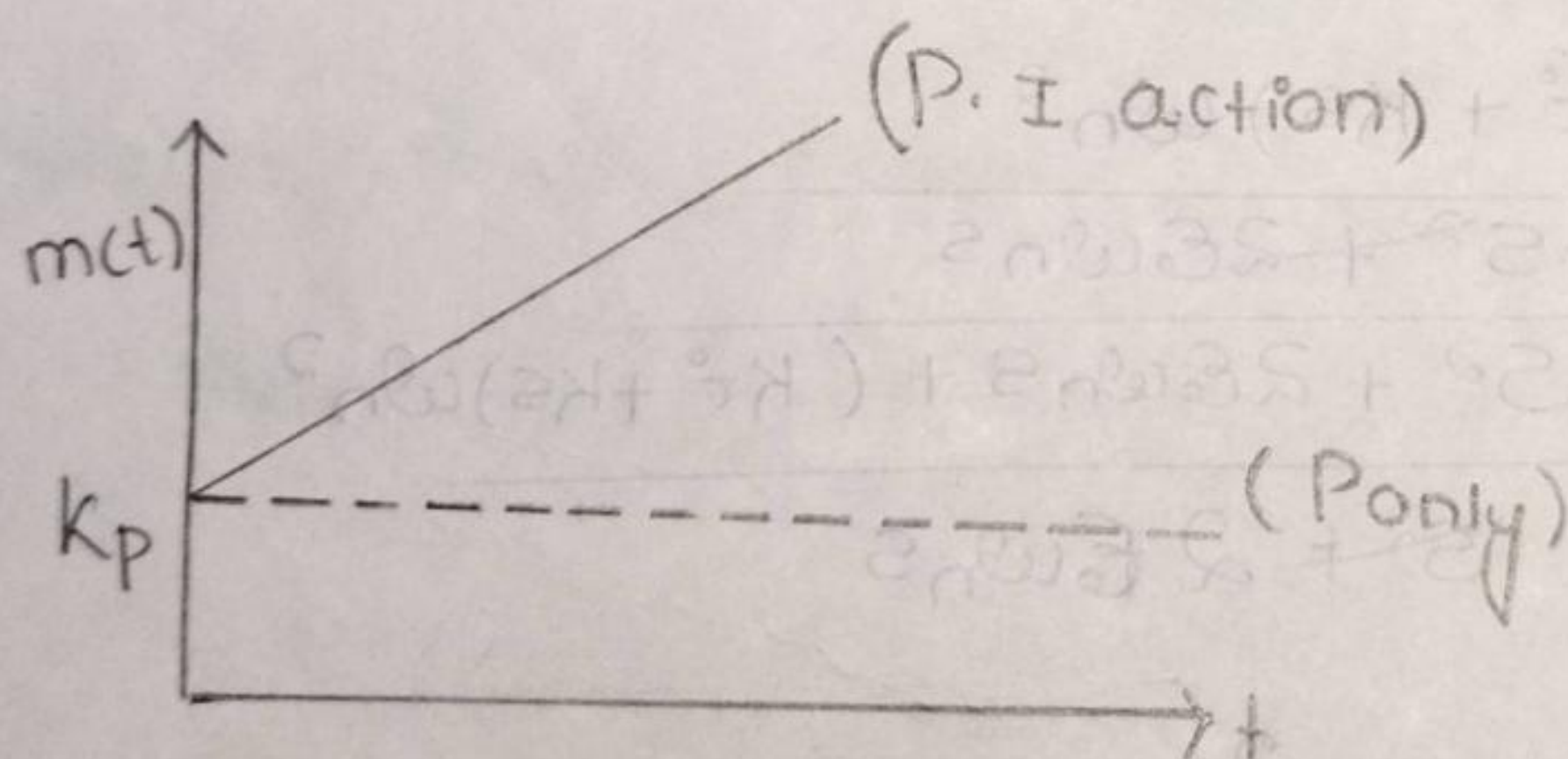
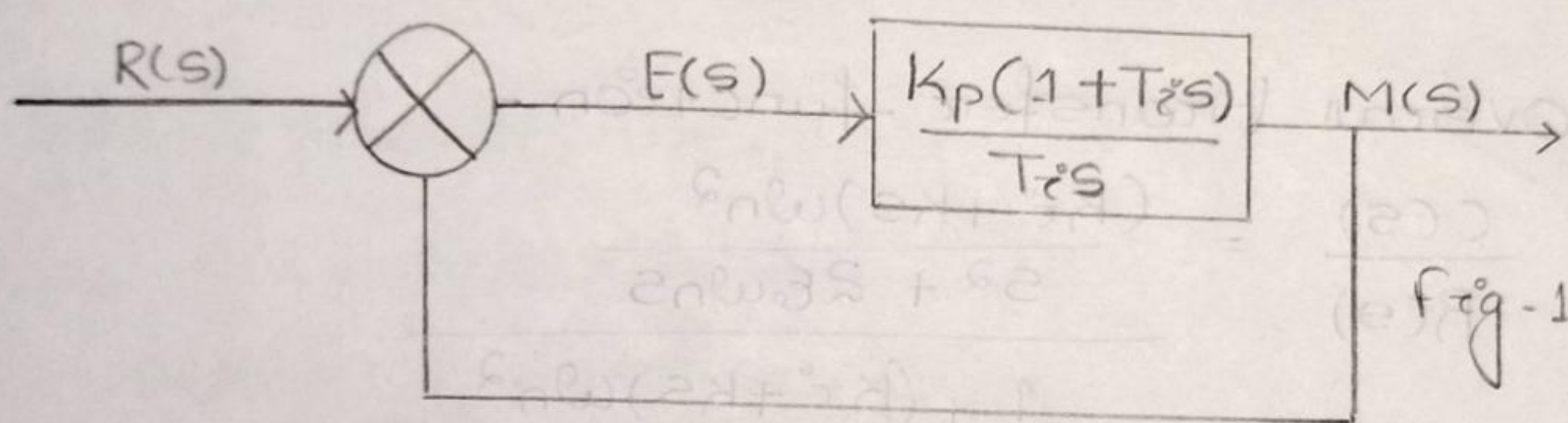
k_p = proportionality gain.

Both K_p and T_i are constant but are adjustable on the system. Any change in T_i would effect the ~~toan~~ integral control action at the controller.

Any change in K_p affects the proportional as well as integral control action.

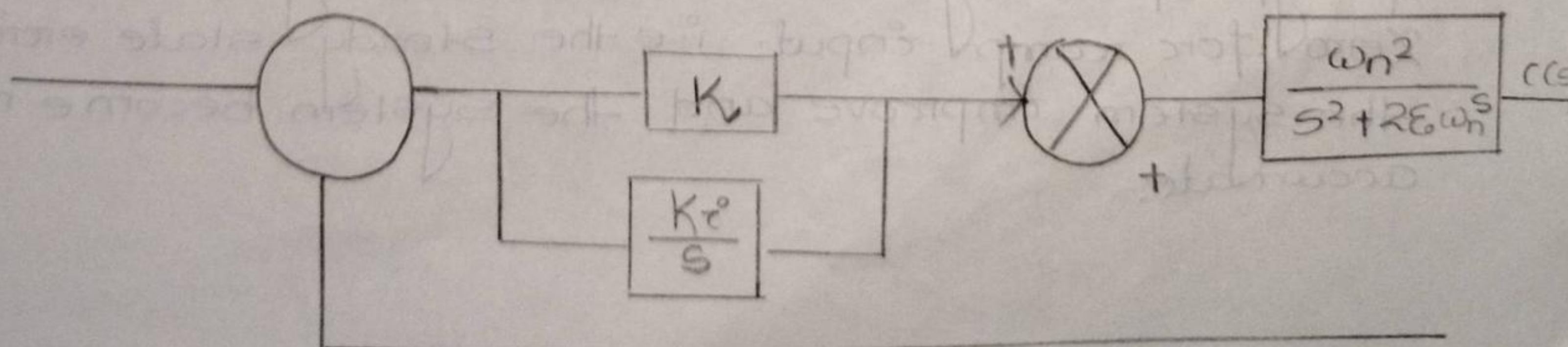
The inverse integral time T_i is known as reset rate. Reset rate can be define as the no. of times per limit that the proportional part of the response is duplicate

Therefore reset rate is also known as repeat per minute.



The PI Controller fig-1 shows as PI Controller and figure 1(a) shows unit step function.

Fig- 1(b) shows the output of such controller. per unit step input.



It shows the block diagram of typical, of P.I. Control

$$G(s) = \left(K + \frac{K\tau}{s} \right) \times \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}$$

$$= \left(\frac{Ks + K\tau}{s} \right) \times \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}$$

$$= \frac{(Ks + K\tau)\omega_n^2}{s^2 + 2\zeta\omega_n s}$$

Here,

$$Hs = 1$$

The Overall Transfer function -

$$\frac{C(s)}{R(s)} = \frac{(K\tau + Ks)\omega_n^2}{s^2 + 2\zeta\omega_n s} \times \frac{1}{1 + \frac{(K\tau + Ks)\omega_n^2}{s^2 + 2\zeta\omega_n s}}$$

$$= \frac{(K\tau + Ks)\omega_n^2}{s^2 + 2\zeta\omega_n s + \frac{(K\tau + Ks)\omega_n^2}{s^2 + 2\zeta\omega_n s}}$$

$$= \frac{(K\tau + Ks)\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 K\tau + \omega_n^2 Ks}$$

Now the order system become three and type of system is 2. the Original system is of 2 and type is 1.

As type of the system has increase by 1. error becomes zero for ramp input. i.e the steady state error of the system improve and the system become more accurate

The transfer function relating $E(s)$ to $R(s)$ is given by -

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

$$= \frac{1}{1 + (K_i + Ks)\omega_n^2}$$

$$= \frac{1}{s^3 + 2\zeta\omega_n s^2 + K_i\omega_n^2 + Ks\omega_n^2}$$

$$= \frac{s^2(s + 2\zeta\omega_n)}{s^3 + 2\zeta\omega_n s^2 + K_i\omega_n^2 + Ks\omega_n^2}$$

For unit ramp input -

$$R(s) = \frac{1}{s^2}$$

$$E(s) = \frac{1}{s^2} \times \frac{s^2(s + 2\zeta\omega_n)}{s^3 + 2\zeta\omega_n s^2 + K_i\omega_n^2 + Ks\omega_n^2}$$

Steady state error -

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = 0$$

ROOT LOCUS TECHNIQUES:-

The stability of a close loop control system is determined for the location of the root of characteristics equation

$$1 + G(s)H(s) = 0$$

For a system to be stable the root of its characteristics equation should be located in the L.H.S of s-plane

A close loop control system having forward path gain K is shown by the block diagram.

The characteristics equation of the system is given by

$$1 + G(s)H(s) = 0$$

Any root of equation satisfy following 2 condition-

$$|G(s)H(s)| = 1 \quad \text{--- (1)}$$

$$\angle G(s)H(s) = (2K + 1) = 180^\circ \quad \text{--- (2)}$$

Where $K = 0, 1, 2, 3, \dots$

The root locus method of analysis is a process of determining the points in s-plane satisfy equation (1) and equation (2) usually the forward path gain factor K is considered as an independent variable and roots of $1 + G(s)H(s) = 0$

As depending variable the roots are plotted in s-plane with K as variable parameter.

From the location of the roots in s-plane the nature of the time response and system.

General properties of Root locus-

The characteristics equation of close loop control system is $1 + G(s)H(s) = 0$

$$\Rightarrow G(s)H(s) = -1 \quad (3)$$

Hence the roots of characteristics equation occurs only for those value λ for equation (3) satisfy simultaneously.

Settling time-

It is the time required for ~~system~~ the response to reach and set within specified % of steady state value usually 2% or 5%.

Magnitude Criterion-

It is the magnitude of $G(s)H(s)$

$$|G(s) \cdot H(s)| = 1; (1 < K < \infty)$$

Using this magnitude criterion the value of the open loop gain 'K' on the root locus are drawn.

Angle Criterion-

$$\angle G(s) \cdot H(s) = \pm 180^\circ (2q + 1); (K \geq 0)$$

when q as an integer-

$$q = 0, 1, 2, 3, \dots$$

A point in the s-plane satisfying the angle criterion is the point on the root locus.

Root Locus Rules for Construction —

Rule-1

Symmetrical about real axis.

Rule-2

Roots from Openloop Poles and ends at Open loop Zeros.

Rule-3

Roots from Openloop ends at open loop Zeros.

N = no. of root locus

$N = P$ if $P > Z$

P = No. of finite poles

$N = Z$ if $Z > P$

Z = No. of finite Zeros

$N = Z$ if $P = Z$

Rule-4

Asymptotes = Branch of root locus that tends to infinite along a straight line $A = |P-Z|$

Rule-5

Angle of asymptotes (ϕ) = $\frac{(2q+1)180^\circ}{|P-Z|}$

Where $q = 0, 1, 2, 3, \dots, |P-Z|-1$.

The no. of asymptotes = $|P-Z|$

Centroid (σ_A) =

It is the point of Intersection asymptotes with real

$\sigma_A = \frac{\sum \text{real part of the openloop pole} - \sum \text{real part of the openloop Zeros}}{|P-Z|}$

$|P-Z|$

This point (σ_A) is known as the asymptotes of Centroid.

Rule-6

Angle of departure = $180^\circ -$ Sum of angle of vector drawn
vector drawn to this ~~zero~~ pole from other poles
+ sum of angle of vector

The ~~transfer function~~ forward path T.f of an unit feedback control system is given by

$$H(s)G(s) = \frac{K}{s(s+2)(s+4)}$$

No. of Zeros = 0

No. of Poles = $s = 0, -2, -4$

Root locus point

endpoint

Openloop pole, $P_1 \Rightarrow s = 0$

Open loop Zero

$P_2 \Rightarrow s = -2$

$Z = 0$

$P_3 \Rightarrow s = -4$

$Z = \infty$

$$\begin{aligned} \text{No. of root locus} &= |P - Z| \\ &= |3 - 0| \\ &= 3 \end{aligned}$$

$$\text{No. of Asymptotes} = |3 - 0| = 3$$

$$\text{Centroid of Asymptotes} = \frac{\sum \text{pole} - \sum \text{Zero}}{|P - Z|}$$

$$= \frac{(0 - 2 - 4) - 0}{3}$$

$$= \frac{-6 - 0}{3}$$

$$= \frac{-6}{3}$$

$$= -2$$

$$\text{Angle of asymptotes } \phi_A = \frac{180(2q+1)}{|P-z|}$$

$$q = 0, 1, 2, \dots, |P-z| - 1$$

$$= |3-0| - 1$$

$$= 2$$

$$q = 0 = \frac{(2q+1)180^\circ}{|P-z|} \quad q = 1 = \frac{(2q+1)180^\circ}{|P-z|}$$

$$= \frac{(2 \times 0 + 1)180^\circ}{3}$$

$$= \frac{180^\circ}{3}$$

$$= 60^\circ$$

$$= \frac{180^\circ(2 \times 1 + 1)}{3}$$

$$= \frac{180^\circ \times 3}{3}$$

$$= 180^\circ$$

$$q = 2 = \frac{(2q+1)180^\circ}{|P-z|}$$

$$= \frac{(2 \times 2 + 1)180^\circ}{3}$$

$$= \frac{4 \times 180^\circ}{3}$$

$$= 240^\circ$$

Breakaway Point =

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\Rightarrow \frac{s(s+2)(s+4) + K}{s(s+2)(s+4)} = 0$$

$$\Rightarrow s(s+2)(s+4)+K=0$$

$$\Rightarrow (s^2+2s)(s+4)+K=0$$

$$\Rightarrow Ks^3+4s^2K+2s^2K+8sK=0$$

$$\Rightarrow K(s^3+4s^2+2s^2+8s)=0$$

$$\Rightarrow (s^3+6s^2+8s)+K=0$$

$$\Rightarrow s^3+6s^2+8s+K=0$$

$$\Rightarrow K+s^3+6s^2+8s=0$$

$$\Rightarrow K=-s^3-6s^2-8s$$

$$\frac{dK}{ds} = \frac{d(-s^3-6s^2-8s)}{ds}$$

$$= -3s^2 - 12s - 8$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 3(-8)}}{2(-3)}$$

$$= \frac{-12 \pm \sqrt{144 + 96}}{-6}$$

$$= -3.15, -0.84$$

Interaction of root Locus - $K(1+s)(s+2)$

$$s^3 \quad 1 \quad 8 \quad 0 = (K+1)(2s+2) \leftarrow$$

$$s^2 \quad 6 \quad K \quad 0 = K(2s+2) \leftarrow$$

$$s^1 \quad \frac{48-K}{6} = 0 \Rightarrow 48 \leftarrow$$

$$s^0 \quad 6s^2 + K = 0$$

$$\Rightarrow 6s^2 + 48 = 0$$

$$\Rightarrow 6s^2 = -48$$

$$\Rightarrow s^2 = -48/6$$

$$\Rightarrow s = \sqrt{-48/6}$$

$$= \pm 2j \cdot 828$$

The characteristic equation of a feedback Control system is $s^3 + 2s^2 + (K+1)s + 5K = 0$. Sketch the root locus of this equation for $0 < K < \infty$ show all the regional point On the Locus.

Characteristics equation =

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + s^3 + 2s^2 + (K+1)s + 5K = 0$$

$$\Rightarrow 1 + s^3 + 2s^2 + sK + 5 + 5K = 0$$

$$\Rightarrow s^3 + 2s^2 + sK + 5K + 5 = 0$$

$$\Rightarrow \frac{K(s+5)}{s^3+2s^2+2s} = \frac{K(s+5)}{s(s^2+2s+1)}$$

~~No. of Zeros = 0~~
~~0~~

~~No. of Poles = 3~~
~~3~~

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4 \times 1 \times 1}}{2 \times (-1)}$$

$$= \frac{-2}{-2}$$

$$= 1$$

No. of Poles = 0, -1, -1

No. of Zeros = -5

No. of root locus = $|P - Z| = |3 - 1| = 2$

No. of asymptotes = $|P - Z| = |3 - 1| = 2$

Centroid of Asymptotes = $\frac{\sum \text{pole} - \sum \text{zero}}{|P - Z|}$

$$= \frac{(0, -1, -1) - (-5)}{2}$$

$$= \frac{-3 - 5}{2} = -\frac{8}{2} = -4$$

$$= -4$$

Angle of asymptotes = $\frac{(2q+1)180^\circ}{|P - Z|}$

$$q = |P - Z| - 1 = 2 - 1 = 1$$

$$q = 0$$

$$\frac{(2q+1)180^\circ}{|P - Z|} = 180^\circ$$

$$q = 1$$

$$= \frac{(2q+1)180^\circ}{|P - Z|}$$

$$= 3 \times 180^\circ$$

$$= 540^\circ$$

$$\text{Breakout point} = \frac{K(s+5)}{s^3+2s^2+s}$$

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K(s+5)}{s^3+2s^2+s} = 0$$

$$\Rightarrow s^3+2s^2+s+Ks+5K=0$$

$$\frac{s}{-s}$$

No. of poles = 0, -1, -2
No. of zeros = -2

No. of root locus = 12-2 = 10
No. of asymptotes = 12-2 = 10

$$\frac{\sum \text{poles} - \sum \text{zeros}}{12-2}$$

$$\frac{(0-1-2)}{10}$$

$$\sigma = \frac{-1-2}{10} = -0.3$$

$$\frac{\angle(180^\circ)}{12-2}$$

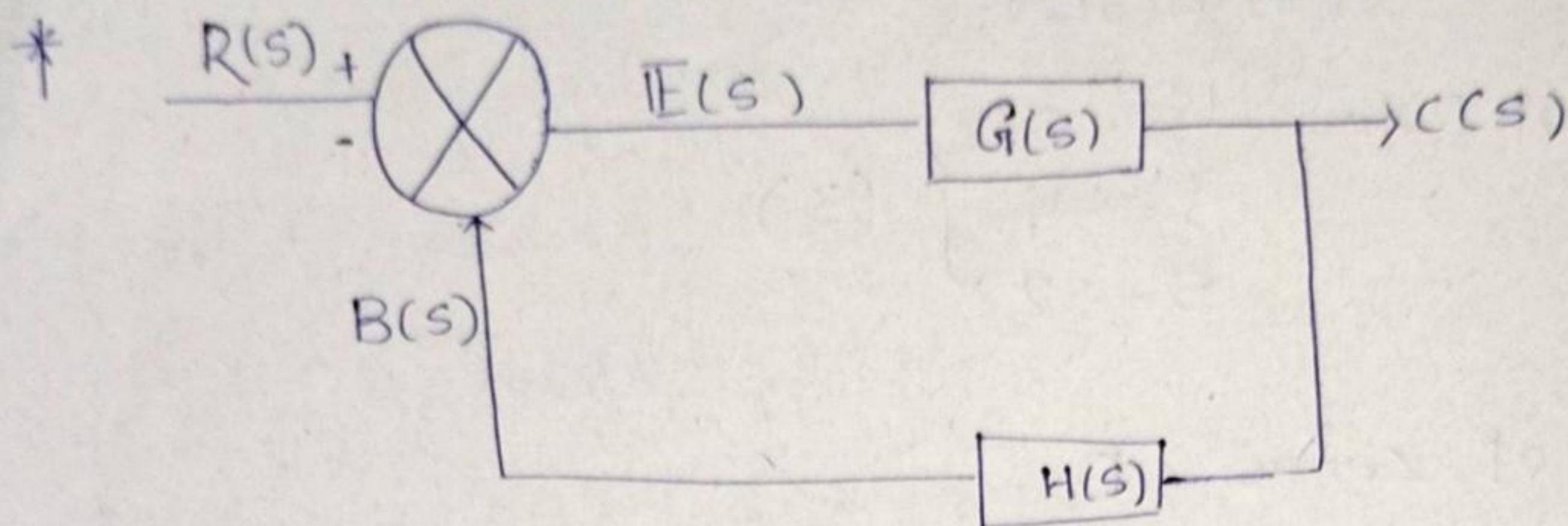
$$\angle = \frac{180^\circ}{10} = 18^\circ$$

$$\frac{\angle(180^\circ)}{12-2}$$

$$\angle = \frac{180^\circ}{10} = 18^\circ$$

closed loop :-

(1)



$R(s)$ = reference input

$C(s)$ = output

$H(s)$ = feedback path

$B(s)$ = feedback signal

$E(s)$ = Error signal

$$T.f = \frac{C(s)}{R(s)}$$

$$C(s) = E(s) \cdot G(s)$$

$$E(s) = R(s) - B(s)$$

$$B(s) = C(s) \cdot H(s)$$

$$C(s) = [R(s) - B(s)] \cdot G(s)$$

$$= [R(s) - C(s) \cdot H(s)] G(s)$$

$$= R(s)G(s) - C(s)H(s)G(s)$$

$$\Rightarrow C(s) + C(s)H(s)G(s) = R(s)G(s)$$

$$\Rightarrow C(s) [1 + G(s)H(s)] = R(s)G(s)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$* G(s) = \frac{k}{s(s+4)(s+2)}$$

$$\left. \begin{array}{l} \text{No. of Poles} = s = 0 \\ s = -4 \\ s = -2 \end{array} \right\} (3)$$

$$\text{No. of Zeros} = 0$$

$$\text{No. of root locus} = |P - Z| = |3 - 0| = 3$$

$$\text{No. of asymptotes} = 3$$

$$\begin{aligned} \text{Centroid of asymptotes} &= \frac{\sum P - \sum Z}{|P - Z|} \\ &= \frac{(0 + 4 + 2) - 0}{|3 - 0|} = \frac{6}{3} = 2 \end{aligned}$$

$$\begin{aligned} \text{angle of asymptotes} &= |P - Z| - 1 \\ &= 3 - 1 = 2 \end{aligned}$$

$$q = 0$$

$$\frac{(2q + 1) 180^\circ}{|P - Z|}$$

$$= \frac{180}{3}$$

$$= 60^\circ$$

$$q = 1$$

$$\frac{(2q \times 1 + 1) 180}{3}$$

$$= 180^\circ$$

$$q = 2$$

$$\frac{(2 \times 2 + 1) 180}{3}$$

$$= \frac{5 \times 180}{3}$$

$$= 300$$

Breakaway Point -

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{k}{s(s+4)(s+2)} = 0$$

$$\Rightarrow \frac{s(s+4)(s+2) + K}{s(s+4)(s+2)} = 0$$

$$\Rightarrow s[s^2 + 2s + 4s + 8] + K = 0$$

$$\Rightarrow s^3 + 6s^2 + 8s + K = 0$$

$$\Rightarrow K = -(s^3 + 6s^2 + 8s)$$

$$\Rightarrow \frac{dK}{ds} = -(3s^2 + 12s + 8)$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12 \pm \sqrt{144 - 96}}{6}$$

$$= \frac{-12 \pm \sqrt{48}}{6}$$

$$= -2 + \frac{2}{3}\sqrt{3}, \quad -2 - \frac{2}{3}\sqrt{3}$$

Point of intersection -

$$s^3 + 6s^2 + 8s + K$$

$$s^3 \quad 1 \quad 8$$

$$s^2 \quad 6 \quad K$$

$$s^1 \quad \frac{48 - K}{6} \quad 0$$

$$s^0 \quad K$$

$$\text{For } K = 0 \quad 48$$

$$6s^2 + 48 = 0$$

$$\Rightarrow s^2 = -8$$

Control System -

(2)

Transfer function -

It is defined as the ratio between Laplace Output to Laplace input ignoring all the Condition

Let Consider a Laplace input $R(s)$

Output $= C(s)$

$$\text{Then T.f} = \frac{C(s)}{R(s)}$$

Root Locus -

It is defined as the graphical representation in a Complex S-plane in a possible location for its close loop poles varying in value of a certain system parameter.

- Control system.
- Transducer.
- Openloop, close loop.
- Comparison between Openloop and close loop. C.S.
- system.
- Feedback system and type.
- Automatic Control system.
- Test signal.
- step, unit, Ramp, Parabolic & Impulse Signal.

→ Properties of transfer function.

* Independent of the magnitude and nature of the input.

* It doesnot Provide any information about the physical structure of the system.

* Transfer function of a physical difference can be identical.

* The system poles and zeroes can be found out from transfer system.

* Stability can be found from characteristic equation.

→ Advantages and disadvantages of T.F.

→ Problems of transfer function network -

① Applying KVL in mesh - 1

$$V_i(t) = R(t) + \int \frac{e}{C} dt$$

Applying KVL mesh - 2

$$V_o(t) = \int \frac{e}{C} dt$$

Now Laplace transform of both side -

$$V_i(s) = R(s) + \frac{1}{C(s)} I(s)$$

$$V_o(s) = \frac{1}{C(s)} I(s).$$

- Block diagram reduction rule.
- Mason's gain formula $\left(\sum_{K=1}^n \frac{P_K \Delta_K}{\Delta} \right)$
- Properties of SFG.
- Procedure for Construction of a signal flow graph.

→ Breakaway Point.

→ Angle of departure.

Due to presence of Complex poles the net angle Contribution ~~is at~~ these poles ~~in other~~ with other Open loop poles and zeros is called angle of departure

$$\phi_d = 180^\circ (2q + 1)$$

→ Polar plot

→ Routh Hurwitz Criterion.

→ Gain margin - $20 \log \left[\frac{1}{G(j\omega)_c \cdot H(j\omega)_c} \right] \text{ dB}$

gain margin is defined as the ~~gain needed to~~ amount of change in Open loop gain needed to make a close loop system unstable.

→ Phase margin -

It defined as the difference between phase lag and -180° .

Or

It is defined as the amount of change ~~needed to~~ in a Open loop phase needed to make a close loop system unstable.

$$* G(s) = \frac{10}{(s+1)(s+3)}, \quad H(s) = 1$$

characteristics equation-

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{10}{(s+1)(s+3)} = 0$$

$$\Rightarrow (s+1)(s+3) + 10 = 0$$

$$\Rightarrow s^2 + 3s + s + 3 + 10 = 0$$

$$\Rightarrow s^2 + 4s + 13 = 0 \quad \text{---(1)}$$

(ii) Damping ratio -

Comparing equation (1) with 2nd Order System -

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$2\zeta\omega_n = 4$$

$$\omega_n^2 = 13$$

$$\Rightarrow \zeta\omega_n = 2$$

$$\Rightarrow \omega_n = 3.6$$

$$\Rightarrow \zeta = \frac{2}{3.6}$$

$$= 0.55$$

(iv) time Period of Oscillation = $\frac{2\pi}{\omega_n}$

(iii) Time at which first Overshoot will Occure.

$$\frac{1}{\omega_n \sqrt{1 - \zeta^2}} = \frac{1}{3.6 \sqrt{1 - (0.55)^2}} =$$

$$* G(s) = \frac{1}{s(s-1)} \quad (\text{Polar plot})$$

(3)

Put,

$$s = j\omega$$

$$G(j\omega) = \frac{1}{j\omega(j\omega-1)}$$

$$\text{Magnitude} = \frac{1}{\omega(\omega^2+1)}$$

$$\text{Phase angle} = -90^\circ + \tan^{-1} \omega$$

$$\omega = 0$$

$$\begin{aligned} \phi &= -90^\circ + \tan^{-1} \omega \\ &= -90^\circ \end{aligned}$$

$$\omega = \infty$$

$$\begin{aligned} \phi &= -90^\circ + \tan^{-1} \omega \\ &= -90^\circ + 90^\circ \\ &= 0. \end{aligned}$$

Now,

$$\begin{aligned} & \frac{1}{j\omega(j\omega-1)} \\ &= \frac{1}{-\omega^2 - j\omega} \\ &= \frac{-\omega^2 + j\omega}{(-\omega^2 - j\omega)(-\omega^2 + j\omega)} \\ &= \frac{-\omega^2 + j\omega}{\omega^4 + \omega^2} \\ &= \frac{-\omega^2}{\omega^4 + \omega^2} + j \frac{\omega}{\omega^4 + \omega^2} \end{aligned}$$

Now

$$\Rightarrow \frac{-\omega^2}{\omega^4 + \omega^2} = 0$$

$$\Rightarrow -\omega^2 = 0$$

$$\Rightarrow \omega = 0 \text{ \& \& } \infty$$

$$j \frac{\omega}{\omega^4 + \omega^2} = 0$$

$$\Rightarrow j\omega = 0$$

$$\Rightarrow \omega = 0 \text{ \& \& } \infty$$

Routh Hurwitz Criterion -

* $f(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15 = 0$

s^5	1	2	3
s^4	1	2	15
s^3	0	-12	0
s^2	∞		
s^1			
s^0			

Routh array failed
now we take $0 = \epsilon$

s^5	1	2	3
s^4	1	2	15
s^3	ϵ	-12	0

$$s^2 \quad \begin{array}{cc} \textcircled{x} & \\ \frac{(2\epsilon + 12)}{\epsilon} & \frac{15\epsilon}{\epsilon} \end{array} \quad 0$$

$$s^1 \quad \begin{array}{cc} \textcircled{y} & \\ \frac{12 \left(\frac{2\epsilon + 12}{\epsilon} \right) - 15\epsilon}{\frac{2\epsilon + 12}{\epsilon}} & 0 \end{array}$$

s^0

$$\frac{2\epsilon + 12}{\epsilon} \times (-12) - 15\epsilon = y$$

$$\frac{2\epsilon + 12}{\epsilon} = x$$

	s^5	1	2	3
	s^4	1	2	15
Cr	s^3	ϵ	-12	0
	s^2	x	15	0
	s^1	y	15	
	0	15		

Now $\lim_{\epsilon \rightarrow 0}$

$$\lim_{\epsilon \rightarrow 0} \frac{2\epsilon + 12}{\epsilon}$$

$$= \lim_{\epsilon \rightarrow 0} \frac{2\epsilon}{\epsilon} + \lim_{\epsilon \rightarrow 0} \frac{12}{\epsilon}$$

$$= \infty$$

$$\lim_{\epsilon \rightarrow 0} \left(\frac{2\epsilon + 12}{\epsilon} \right) \times (-12) - 15\epsilon$$

$$\frac{2\epsilon + 12}{\epsilon}$$

$$= \lim_{\epsilon \rightarrow 0} \frac{-12 - 15\epsilon}{\epsilon}$$

$$= -12$$

$$= -ve$$

$\therefore$ System is unstable.

→ Nichol's chart.

* Nichol's chart is very useful technique for determining stability and the close loop frequency of a feedback system.

* Stability is determined from a plot of the open loop gain vs phase characteristics.

~~* Close loop frequency is determined from~~

The M and N loci are repeated every 360° .

$$\frac{C(j\omega)}{R(j\omega)} = M(\omega) e^{j\alpha(\omega)}$$

where -

$M(\omega)$ = amplitude Component

$\alpha(\omega)$ = phase Component

The radian frequency at which $\frac{C(j\omega)}{R(j\omega)}$ occurs is

Called resonance frequency of the system.

It can be used to determine -

Complete frequency response value of resonance Peak.

M_{rc} and ω_{rc} for Closesloop C.S.

(4)

* Nyquist Stability Criterion -

It states that the no. of encirclement about the critical point $(-1 + j0)$ must be equal to the poles of the characteristic equation.

that means the poles of the open loop transfer function in the right half the s -plane.

From other angle it also reviewed as per Principle of argument, there will be 'P' poles and 'Z' zeros enclosed by the s -plane close path. so we can write the no. of encirclement as $N = P - Z$

enclosed
If the s -plane contains only poles then the direction of the encirclement in $G(s)H(s)$ plane will be opposite to the direction of the enclosed close path in s -plane.

If there is only zero, then $G(s)H(s)$ ~~will be~~ direction will be same in the direction of the enclosed close path in s -plane.

If $P=0$ then $N=0$ (stable)

No open loop pole in the right half of s -plane.

*

* M-circle -

Let us Consider

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Put $s = j\omega$.

For unity feedback system $H(s) = 1$.

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1 + G(j\omega)}$$

Put, $G(j\omega) = u(\omega) + jv(\omega) = x + jy$

$$\text{Magnitude, } \left| \frac{C(j\omega)}{R(j\omega)} \right| = \left| \frac{x + jy}{1 + [u(\omega) + jv(\omega)]} \right|$$

$$M = \frac{x + jy}{1 +}$$

$$\text{Magnitude, } \left| \frac{C(j\omega)}{R(j\omega)} \right| = \left| \frac{x + jy}{1 + x + jy} \right|$$

$$M = \frac{\sqrt{x^2 + y^2}}{\sqrt{(1+x)^2 + y^2}}$$

$$\Rightarrow M^2 = \frac{x^2 + y^2}{(1+x)^2 + y^2}$$

$$\Rightarrow M^2 = \frac{x^2 + y^2}{(1 + x^2 + 2x) + y^2}$$

$$\Rightarrow M^2(1 + x^2 + 2x) + y^2 = x^2 + y^2$$

$$\Rightarrow M^2 + M^2x^2 + M^22x + y^2 = x^2 + y^2$$

$$\Rightarrow M^2 = x^2 + y^2 - M^2x^2 - M^22x - M^2y^2$$

$$\Rightarrow M^2 = x^2 - M^2x^2 + y^2 - M^2y^2 - 2M^2x$$

$$\Rightarrow M^2 = x^2(1 - M^2) + y^2(1 - M^2) - 2M^2x$$

dividing both the sides by $1 - M^2$

$$\Rightarrow \frac{M^2}{1 - M^2} = x^2 + y^2 - \frac{2M^2x}{1 - M^2}$$

Adding $\left(\frac{M^2}{1 - M^2}\right)^2$

$$\Rightarrow \frac{M^2}{1 - M^2} + \left(\frac{M^2}{1 - M^2}\right)^2 = x^2 + y^2 - \frac{2M^2x}{1 - M^2} + \left(\frac{M^2}{1 - M^2}\right)^2$$

$$\Rightarrow \left[x^2 - \frac{2M^2x}{1 - M^2} + \left(\frac{M^2}{1 - M^2}\right)^2 \right] + y^2$$

$$= \frac{M^2}{1 - M^2} + \left(\frac{M^2}{1 - M^2}\right)^2$$

$$\Rightarrow \left(x - \frac{M^2}{1 - M^2} \right)^2 + y^2 = \frac{M^2}{(1 - M^2)^2}$$

This is the equation of a circle having Origin

$$\left(\frac{M^2}{1 - M^2} \right) = 0.$$

N-circle-

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

For unit feedback system $H(s) = 1$

$$\text{Put } s = j\omega$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1 + G(j\omega)}$$

$$j\omega = x + yj$$

$$\tan \alpha = \tan^{-1} \left(\frac{y}{x} \right) - \tan^{-1} \left(\frac{y}{1+x} \right)$$

$$\Rightarrow \tan \alpha = N$$

$$\Rightarrow N = \tan \left[\tan^{-1} \left(\frac{y}{x} \right) - \tan^{-1} \left(\frac{y}{1+x} \right) \right]$$

$$= \tan \left[\tan^{-1} \left(\frac{y}{x} \right) \right] - \tan \left[\tan^{-1} \left(\frac{y}{1+x} \right) \right]$$

$$= \tan \left[\frac{\tan^{-1} \left(\frac{y}{x} \right) - \tan^{-1} \left(\frac{y}{1+x} \right)}{1 + \tan^{-1} \left(\frac{y}{x} \right) \cdot \tan^{-1} \left(\frac{y}{1+x} \right)} \right]$$

$$= \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y}{x} \cdot \frac{y}{1+x}}$$

$$= \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y^2}{x(1+x)}}$$

$$= \frac{y(1+x) - yx}{x(1+x) + y^2}$$

$$= \frac{y + yx - yx}{x + x^2 + y^2}$$

$$N = \frac{y}{x + x^2 + y^2}$$

$$\Rightarrow N(x + x^2) + y^2 = y$$

$$\Rightarrow N(1 + Nx(1+x)) + y^2 = y$$

$$\Rightarrow 1 + x(1+x) + y^2 = \frac{y}{N}$$

$$\Rightarrow \text{Adding both the side } \left[\frac{1}{4} + \frac{1}{(2N)^2} \right]$$

$$\Rightarrow x^2 + x^2 + y^2 + \frac{1}{4} + \frac{1}{(2N)^2} - \frac{y}{N} = 0$$

$$\Rightarrow x^2 + x + \frac{1}{4} + y^2 - \frac{y}{N} + \frac{1}{(2N)^2} = 0$$

$$\Rightarrow \left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = 0$$

This equation represent the equation of a circle with centre $(-\frac{1}{2})$ and $\frac{y}{(2N)^2}$ and radius $\sqrt{(\frac{1}{4})^2 + (\frac{y}{2N})^2}$

→ The frequency at which the Nyquist diagram (1) crosses the unit circle is known as ~~Cross over~~ gain cross-over frequency.

→ All pass minimum phase system.

* In a Control theory and signal processing a system is said to be minimum phase if the system and its inverse are causal or stable.

Ex - A discrete-time system with rational T.F. $H(z)$ can only satisfy causality and stability requirement if all of the poles of the system are inside the unit circle then we are free to choose whether the zero is inside or outside of the unit circle.

When we impose the causality and stability then the ~~system~~ inverse system is unique and the system (H) and its inverse (H_{inv}) are called minimum phase.

In all pass frequency analysis an all pass filter is considered and it is a signal processing filter that passes all frequency equally, but changes the phase relationship among various frequency.

→

$$* G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$

put $s = j\omega$

$$G(s) = \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

Magnitude -

$$\left| \frac{K}{j\omega(1+j\omega T_1)(1+j\omega T_2)} \right|$$

$$= \frac{K}{\omega \sqrt{1+\omega^2 T_1^2} \sqrt{1+\omega^2 T_2^2}}$$

$$\text{Phase} = -90^\circ - \tan^{-1} \omega T_1 - \tan^{-1} \omega T_2$$

Now,

$$\lim_{\omega \rightarrow 0} -90^\circ - \tan^{-1} \omega T_1 - \tan^{-1} \omega T_2 = 0$$

lim

$$\omega \rightarrow \infty -90^\circ - \tan^{-1} \infty - 90^\circ - 90^\circ = -270^\circ.$$

?

$$* G(s) = \frac{10(s+4)}{s(s-2)}$$

Put

$$s = j\omega$$

$$= \frac{10(j\omega + 4)}{j\omega(j\omega - 2)}$$

$$= \frac{40 + j10\omega}{-\omega^2 - j2\omega}$$

$$= \frac{40}{-\omega^2 - j2\omega} + j \frac{10\omega}{-\omega^2 - j2\omega}$$

$$\text{Phase} = \tan^{-1} \frac{\omega}{4} - \tan^{-1} 90^\circ - \tan^{-1} \left(\frac{\omega}{2} \right)$$

$$\lim_{\omega \rightarrow 0} 0 - 90^\circ - 0 = -90^\circ$$

$$\lim_{\omega \rightarrow \infty} \tan^{-1} \infty - 90^\circ - \tan^{-1} \infty = 90^\circ$$

Putting

$$j \frac{10\omega}{-\omega^2 - j2\omega} = 0 \Rightarrow 0$$

→ Time response of underdamped second order system. (8)

→ Time response of second order system.

→ Time response of first order system with unit impulse ~~in~~ input.

→ Standard test signal

→ Root locus (Q) -

$$G(s) = \frac{K_1}{s(s+6)(s^2+4s+13)}$$

No. of poles = 4 $s=0$

No. of zeros = 0.

$$s+6=0$$

$$\Rightarrow s = -6$$

$$\begin{aligned} & \frac{s^2+4s+13}{2} \\ &= \frac{-4 \pm \sqrt{16-52}}{2} \end{aligned}$$

$$= \frac{-4 \pm \sqrt{-36}}{2}$$

$$= -2 \pm 3i$$

No. of asymptotes root locus = $4-0=4$

No. of asymptotes = $|P-Z| = |4-0|=4$

Centroid of asymptotes = $\frac{\sum P - \sum Z}{|P-Z|}$

$$= \frac{0+6+2+3i+2-3i}{4} = \frac{10}{4} = 2.5$$

$$\text{Angle of asymptotes} = \frac{(2q+1)180^\circ}{|P-Z|}$$

$$q = |P-Z| - 1 = 3$$

$q=0$	$q=1$	$q=2$	$q=3$
$\Rightarrow \frac{(2q+1)180^\circ}{4}$	$= \frac{3 \times 180}{4}$	$= \frac{5 \times 180^\circ}{4}$	$= \frac{7 \times 180^\circ}{4}$
$= 45^\circ$	$= 135^\circ$	$= 225^\circ$	$= 315^\circ$

Breakaway Point -

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+6)(s^2+4s+13)} = 0$$

$$\Rightarrow s(s+6)(s^2+4s+13) + K = 0$$

$$\Rightarrow (s^2+6s)(s^2+4s+13) + K = 0$$

$$\Rightarrow s^4 + 4s^3 + 13s^2 + 6s^3 + 24s^2 + 78s + K = 0$$

$$\Rightarrow s^4 + 6s^3 + 41s^2 + 78s + K = 0$$

$$\frac{dK}{ds} = -(4s^3 + 18s^2 + 82s + 78) = 0$$

$$\Rightarrow s^4 + 6s^3 + 41s^2 + 78s + K = 0$$

$$\Rightarrow K = -(s^4 + 10s^3 + 37s^2 + 78s)$$

$$\frac{dk}{ds} = -(4s^3 + 30s^2 + 74s + 78)$$

Point of intersection -

$$s^4 \quad 1 \quad 3.7 \quad K$$

$$s^3 \quad 10 \quad 78$$

$$s^2 \quad 29.2 \quad K$$

$$s^1$$

$$s^0$$

$$\frac{29.2 \times 78 - 10K}{29.2 \times 78} = 0$$

$$\Rightarrow -10K = -29.2 \times 78$$

$$\Rightarrow K = 227.26$$

From auxiliary equation -

$$29.2s^2 + K = 0$$

$$\Rightarrow 29.2s^2 + 227.26 = 0$$

$$\Rightarrow s^2 = \frac{-227.26}{29.2}$$

→ Rise time and peak time.

$$* G(s) = \frac{K}{(1+sT_1)(1+sT_2)}$$

Put
 $s = j\omega$

$$= \frac{K}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$\text{Magnitude} = \frac{K}{\sqrt{1+\omega^2 T_1^2} \sqrt{1+\omega^2 T_2^2}}$$

$$\text{Phase} = -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$$

Taking the limit for magnitude-

$$\lim_{\omega \rightarrow 0} \frac{K}{\sqrt{1+\omega^2 T_1^2} \sqrt{1+\omega^2 T_2^2}} = K_v$$

$$\lim_{\omega \rightarrow \infty} \frac{K}{\sqrt{1+\omega^2 T_1^2} \sqrt{1+\omega^2 T_2^2}} = \infty$$

Taking the limit for Phase angle.

$$\lim_{\omega \rightarrow 0} -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = 0$$

$$\begin{aligned} \lim_{\omega \rightarrow \infty} -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 \\ = -90^\circ - 90^\circ \\ = \infty \end{aligned}$$

→ Nyquist Contour -

It is a Contour s -plane which enclose the entire right half s -plane. This contour having the encirclement in clockwise direction and radius approaches infinity.

→ Steady state error -

It is defined to be the error between the actual output and desire input as t tends to ∞

i.e

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} [SE(s)]$$

$$= \lim_{t \rightarrow \infty} [r(t) - c(t)]$$

→ Transient response specification of 2nd Order system -

- (i) Delay time (t_d)
- (ii) Rise time (t_r)
- (iii) Peak time (t_p)
- (iv) Peak overshoot (M_p)
- (v) Setting time (t_s)
- (vi) Steady state error (e_{ss})

Delay time - (t_d) =

It is the time required for the ~~reach~~ response to reach from 0% to 50% the the steady state value.

→ Time response of the first order system with unit step input:

$$R(t) = u(t) = 1$$

$$L[R(t)] = \frac{1}{s}$$

$$\frac{C(s)}{R(s)} = \frac{1}{1 + sT}$$

$$\Rightarrow C(s) = R(s) \left[\frac{1}{1 + sT} \right]$$

$$= \frac{1}{s} \times \frac{1}{1 + sT}$$

$$= \frac{1}{s} - \frac{1}{1 + sT}$$

$$= \frac{1}{s} - \frac{1}{s + 1/T}$$

Taking the inverse Laplace.

$$L^{-1}[C(s)] = L^{-1} \left[\frac{1}{s} - \frac{1}{s + 1/T} \right]$$

$$\Rightarrow c(t) = 1 - e^{-t/T}$$

$$e(t) = r(t) - c(t)$$

$$= 1 - (1 - e^{-t/T})$$

$$= e^{-t/T}$$

The steady state error is $\lim_{t \rightarrow \infty} e^{-t/T}$

$$= 0.$$

Separating the real and imaginary Part. (9)

$$= \frac{K}{(1 + j\omega T_1)(1 + j\omega T_2)}$$

$$= \frac{K (1 - j\omega T_1)(1 - j\omega T_2)}{(1 + j\omega T_1)(1 + j\omega T_2)(1 + j\omega T_1)(1 - j\omega T_2)}$$

$$= \frac{K (1 - \omega^2 T_1 T_2)}{(1 + \omega^2 T_1^2)(1 + \omega^2 T_2^2)}$$

$$= \frac{K (1 - \omega^2 T_1 T_2)}{1 + \omega^2 T_2^2 + \omega^2 T_1^2 + \omega^4 T_1^2 T_2^2}$$

equating real part with 0

$$\Rightarrow \frac{K (1 - \omega^2 T_1 T_2)}{1 + \omega^2 T_2^2 + \omega^2 T_1^2 + \omega^4 T_1^2 T_2^2} = 0$$

$$\Rightarrow K (1 - \omega^2 T_1 T_2) = 0$$

$$\Rightarrow 1 - \omega^2 T_1 T_2 = 0$$

$$\Rightarrow \omega^2 T_1 T_2 = 1$$

$$\Rightarrow \omega = \frac{1}{\sqrt{T_1 T_2}}$$

→ How type and Order of a Control system is determined?

The Openloop transfer function is explained as -

$$T.f = G(s)H(s) \\ = \frac{K(1+T_a s)(1+T_b s)(1+T_c s) + \dots (1+T_m s)}{s^N(1+T_1 s)(1+T_2 s)(1+T_3 s) + \dots (1+T_n s)}$$

where ,

$$N = \text{typeno}$$

i.e $N = 0, 1, 2, \dots$

$N = 0$, Type - 0 system

$N = 1$, Type - 1 system

$N = 2$, Type - 2 system.

Again the Order of a system found out to be equations involves in N^{π} & D^{π} i.e

the system is 1st, 2nd and a 3rd order.

Let,

$$\frac{C(s)}{R(s)} = \frac{1}{1 + sT}$$

∴ First Order system.

Rise time (t_r) -

It is defined as the time ^{taken for the} ~~required~~ response to go from 0 - 100% of the Steady state value.

For critically and overdamped system ~~the~~ the response go from 10% to 90% of the Steady state value.

Peak time (t_p) -

It is the time required for the response to reach the first peak overshoot.

Peak overshoot (M_p) -

It is the time deviation of instantaneous output over the Steady state value.

Setting time (T_s) -

It is the time required for the response to reach and stay within Specified ~~per~~ % of the Steady state value.

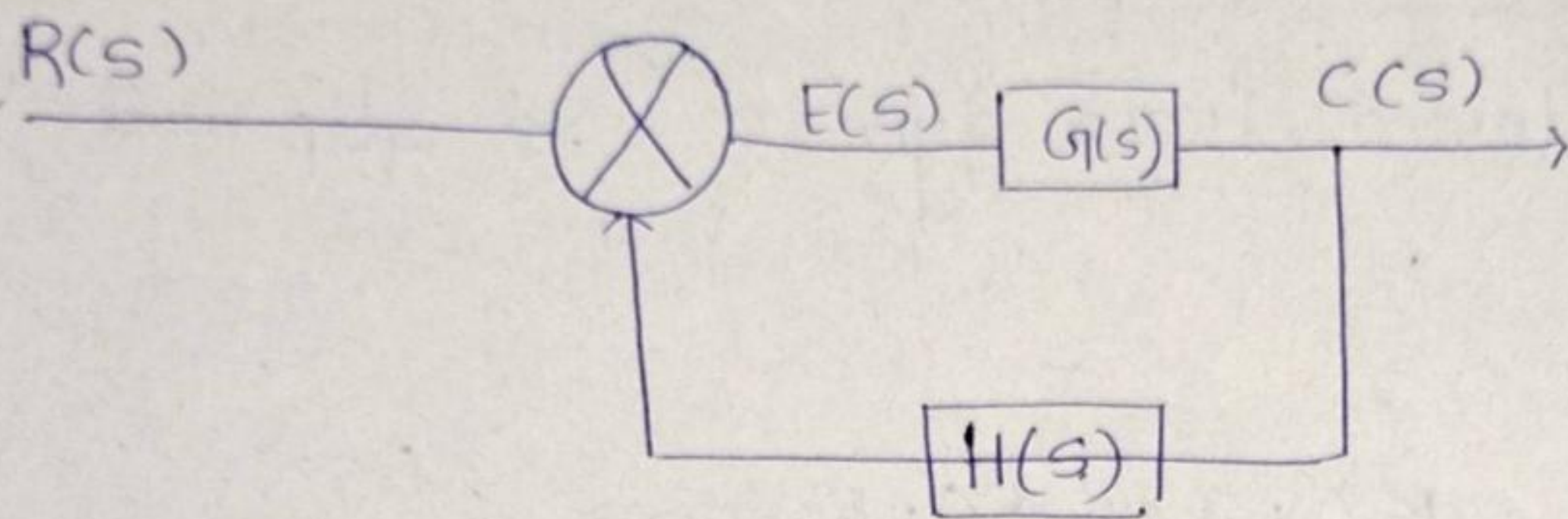
Steady state error (e_{ss}) -

It is the ~~actual~~ error between the actual output and desired output at 't' tends to ∞ .

$$e_{ss} = \lim_{t \rightarrow \infty} S[E(s)]$$

$$= \lim_{t \rightarrow \infty} [r(t) - c(t)]$$

→ Steady state error and error constant 11



$$E(s) = R(s) - C(s)H(s)$$

We know ,

$$C(s) = E(s)G(s)$$

$$\Rightarrow E(s) = R(s) - [E(s)G(s)H(s)]$$

$$\Rightarrow E(s) + E(s)G(s)H(s) = R(s)$$

$$\Rightarrow E(s)[1 + G(s)H(s)] = R(s)$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

To Consider error signal by time domain

$$L^{-1}[E(s)] = L^{-1}[E(s)]$$

$$= L^{-1}\left[\frac{R(s)}{1 + G(s)H(s)}\right]$$

e_{ss} = ~~at~~ steady state error by the value tending to $t \rightarrow \infty$

$$e_{ss} = \lim_{t \rightarrow \infty}$$

$$e_{ss} = \lim_{t \rightarrow \infty} e(t)$$

Error Constant -

Position Error Constant (K_p) -

This can be obtained from Type-0 unit step unit r/p system -

$$R(s) = \frac{1}{s}$$

$$G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\dots}{K(1+sT_a)(1+sT_b)\dots}$$

$$\lim_{s \rightarrow 0} G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{K(1+sT_1)(1+sT_2)\dots}{s(1+sT_a)(1+sT_b)\dots}$$

$$= K \times 1$$

$$= K$$

Velocity Error Constant (K_v)

This can be obtained from type-0 unit ramp r/p.

$$R(s) = \frac{1}{s^2}$$

$$K_v = \lim_{s \rightarrow 0} s G(s)H(s)$$

$$= \lim_{s \rightarrow 0} s \times \frac{K(1+sT_1)(1+sT_2)\dots}{(1+sT_a)(1+sT_b)\dots}$$

$$= K$$

Acceleration error Constant -

This can be obtained from the type-0 system with unit parabolic system

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s)$$

$$= \lim_{s \rightarrow 0} s^2 \times \frac{K(1+sT_1)(1+sT_2) \dots}{s^2(1+sT_a)(1+sT_b) \dots}$$

$$= K$$

* Rules for Construction of Root Locus -

* symmetrical about χ -axis

* find no. of Poles and Zeros.

* find no. of root locus by using

$$N = |P - Z|$$

N = no. of root locus

P = " Poles

Z = " Zeros

* Find no. of asymptotes -

* ~~The~~ find Centroid of asymptotes using formula.

$$\sigma = \frac{\sum P - \sum Z}{|P - Z|}$$

P = no. of Poles, Z = no. of Zeros

σ = Centroid of asymptotes.

* Find angle of asymptotes by using $\frac{(2q+1)180^\circ}{|P-Z|}$

where $q = |P - Z| - 1$

* Find breakaway point ~~an~~ from characteristics equation

* find point of intersection.

50ⁿ

500¹

50²

* If there is a Complex poles or zeros then angle of arrival and departure is to be found out.

→ Controller.

→ P, I and D

P+I, I+D, ~~P+D~~

→ $P = M(t) = K_p(t)$

$$\Rightarrow M(s) = K_p E(s)$$

$$\Rightarrow \frac{M(s)}{E(s)} = K_p$$

P - Controller.

K_p = Proportional signal

$M(s)$ = actuating

$E(s)$ = error signal

→ Integral signal -

$$M(t) = K_i \int_0^t e(t) dt$$

where

K_i = adjustable constant

→ Derivative Controller -

$$M(t) = K_d \frac{de(t)}{dt}$$

$$M(s) = K_d E(s)$$

$$\Rightarrow \frac{M(s)}{E(s)} = K_d$$

→ Polar Plot:-

* It is a frequency domain plot in which a target is located on the map by means of polar co-ordinates.

* In this plot the graph is drawn between the real frequency and imaginary frequency.

* Both the phase and magnitudes are determined at $\omega = 0$ and $\omega = \infty$

→ Feedback system Component -

Reference ~~output~~ input, Processing being controller
Output, Sensing element, Controller.

→ The Two electrical analogous for mechanical system

- i) Forced voltage analogy
- ii) Forced Current analogy.

→ Order of system -

It is defined as the no. of independent energy storage elements in the system.

→ Transient Response

Transient state is essentially the time between the beginning of the event and the ~~steady~~ state.

Steady state response

Steady state establishes after a specific time in a system.

→ Damping ratio -

It is defined as the number of oscillation in a system that can decay ~~on~~ after an interruption.

→ Nature of Second Order system with different type of damping -

* If $\zeta > 1$ then both poles are -ve and real.

* If $\zeta = 1$ then both poles are equal, -ve and real.

→ Advantages of generalized error - co-efficient

- (i) It is used to find closed loop frequency response from open loop frequency response.
- (ii) The frequency domains specification can be determined from Nichol's chart.

→ Why derivative Controller is not used in control system?

It is only suitable for the system which is having very less time sampling time or time constant. For those system which is having large time constant derivative Controller is not used.

→ Damped frequency of Oscillation -

It is frequency at which the system maintains its equilibrium and the amplitude of the vibration can be completely reduced.

→ Frequency Response -

It describes the steady-state response of a system to sinusoidal inputs of varying frequencies.

→ Bandwidth -

It is the frequency at which the magnitude of the closed loop system equal to 0.707.

Bandwidth indicates the frequency range for which satisfactory operation occurs.

→ Bode plot :-

Bode plot is a graph of the frequency response of a system.

It is the combination of bode magnitude and bode phase plot.

Advantages-

It provides relative stability in terms of gain margin and phase margin.

It covers from low frequency and high frequency.

→ Gain Margin

The frequency at which the magnitude = 1.

Gain Crossover = ω_{gc}

Phase Margin

The frequency at which angle is -180°

Phase Crossover = ω_{pc}

→ Corner frequency-

It is a boundary in a system's frequency response at which energy flowing through the system begins to reduce.

→ Phase lag and phase lead-

Phase lag refers to the wave that occurs behind another wave of the same frequency.

Phase Lead refers to the wave that occurs ahead of another wave of the same frequency.

→ Nichol's chart -

It is very useful technique for determining stability and the close loop frequency of a feedback system.

→ Two Contours of Nichol's chart -

Constant closed-loop magnitude
Phase angle

→ Polar plot -

It is the plot of the magnitude vs the phase angle on the polar co-ordinates.

For $G(s)H(s) = \frac{K}{s(s+2)}$

Adding a pole to the function $G(s)H(s)$ half of plane have the effect of pushing original root locus toward right half of s -plane.

Corresponding root locus is showing in the figure.

Add pole $G(s)H(s) = \frac{K}{s(s+2)(s+4)}$ and

Now the pole at $s = -4$ is added to $G(s)H(s)$ so for any value of K before addition of pole in left half system is totally stable.

But after addition of pole in let have 2 branches of root locus after some value of K move in right half s -plane so system to this value of ' K ' is stable

After this value K system become unstable so stability of system get restricted

At $G(s)H(s) = \frac{K}{s(s+2)(s+4)(s+6)}$ it now one more pole

add $s = -6$ is added to the system breakaway point in section $s=0$ and $s=-2$ get shifted towards right.

As compare to previous case system stability further get restricted.

Effect of addition Openloop pole can be summarized -

Root locus shifted towards imaginary axis.
System stability relatively decreases.

System becomes more oscillatory in the nature.
Range of operating value of 'K' for stability of the system decreases.

$$G(s)H(s) = \frac{K(s+4)}{s(s+2)}$$

Let ~~as introduce~~ add introduce a zero add
 $s = -4$

It can be seen that root locus shift towards left i.e. towards zero which is added

So as roots move towards left half of s-plane.
Relatively stability increases as compare to previous case.

Add $G(s)H(s) = \frac{K(s+4)(s+6)}{s(s+2)}$

Let's introduce one more zero $s = -6$
As we gone adding zero to left resultant root locus

In such effect of addition of zero