LECTURE NOTE ON

Structural Mechanics

Prepared by:

Shriman Kumar Mishra

Lecture in

CIVIL ENGINEERING

DHABALESWAR INSTITUTE OF POLYTECHNIC,

RADHADAMODARPUR

ATHAGARH, CUTTACK-29

C. TOPIC WISE DISTRIBUTION

Chapter	Name of topics	Periods
1	Review of Basic Concepts	04
2	Simple and Complex Stress, Strain	15
3	Stresses in Beams	10
4	Columns and Struts	04
5	Shear Force and Bending Moment	12
6	Slope and Deflection	10
7	Indeterminate Beams	10
8	Trusses and Frames	10

D. Course Contents:

1 Review Of Basic Concepts

1.1 Basic Principle of Mechanics: Force, Moment, support conditions, Conditions of equilibrium, C.G & MI, Free body diagram

1.2 Review of CG and MI of different sections

2 Simple And Complex Stress, Strain

2.1 Simple Stresses and Strains

Introduction to stresses and strains: Mechanical properties of materials – Rigidity, Elasticity, Plasticity, Compressibility, Hardness, Toughness, Stiffness, Brittleness, Ductility, Malleability, Creep, Fatigue, Tenacity, Durability, Types of stresses -Tensile, Compressive and Shear stresses, Types of strains - Tensile, Compressive and Shear strains, Complimentary shear stress - Diagonal tensile / compressive Stresses due to shear, Elongation and Contraction, Longitudinal and Lateral strains, Poisson's Ratio, Volumetric strain, computation of stress, strain, Poisson's ratio, change in dimensions and volume etc, Hooke's law - Elastic Constants, Derivation of relationship between the elastic constants.

2.2 Application of simple stress and strain in engineering field:

Behaviour of ductile and brittle materials under direct loads, Stress Strain curve of a ductile material, Limit of proportionality, Elastic limit, Yield stress, Ultimate stress, Breaking stress, Percentage elongation, Percentage reduction in area, Significance of percentage elongation and reduction in area of cross section, Deformation of prismatic bars due to uniaxial load, Deformation of prismatic bars due to its self weight.

2.3 Complex stress and strain

Principal stresses and strains: Occurrence of normal and tangential stresses, Concept of Principal stress and Principal Planes, major and minor principal stresses and their orientations, Mohr's Circle and its application to solve problems of complex stresses

3 **Stresses In Beams and Shafts**

3.1 Stresses in beams due to bending: Bending stress in beams – Theory of simple bending – Assumptions – Moment of resistance – Equation for Flexure– Flexural stress distribution – Curvature of beam – Position of N.A. and Centroidal Axis – Flexural rigidity – Significance of Section modulus

3.2 Shear stresses in beams: Shear stress distribution in beams of rectangular, circular and standard sections symmetrical about vertical axis.

3.3 Stresses in shafts due to torsion: Concept of torsion, basic assumptions of pure torsion, torsion of solid and hollow circular sections, polar moment of inertia, torsional shearing stresses, angle of twist, torsional rigidity, equation of torsion

3.4 Combined bending and direct stresses: Combination of stresses, Combined direct and bending stresses, Maximum and Minimum stresses in Sections, Conditions for no tension, Limit of eccentricity, Middle third/fourth rule, Core or Kern for square, rectangular and circular sections, chimneys, dams and retaining walls

4 **Columns and Struts**

4.1 Columns and Struts, Definition, Short and Long columns, End conditions, Equivalent length / Effective length, Slenderness ratio, Axially loaded short and long column, Euler's theory of long columns, Critical load for Columns with different end conditions

5 **Shear Force and Bending Moment**

5.1 Types of loads and beams:

Types of Loads: Concentrated (or) Point load, Uniformly Distributed load (UDL), Types of Supports: Simple support, Roller support, Hinged support, Fixed support, Types of Reactions: Vertical reaction, Horizontal reaction, Moment reaction, Types of Beams based on support conditions: Calculation of support reactions using equations of static equilibrium.

5.2 Shear force and bending moment in beams:

Shear Force and Bending Moment: Signs Convention for S.F. and B.M, S.F and B.M of general cases of determinate beams with concentrated loads and udl only, S.F and B.M diagrams for Cantilevers, Simply supported beams and Over hanging beams, Position of maximum BM, Point of contra flexure, Relation between intensity of load, S.F and B.M.

6 **Slope and Deflection**

6.1 Introduction: Shape and nature of elastic curve (deflection curve); Relationship between slope, deflection and curvature (No derivation), Importance of slope and deflection.

6.2 Slope and deflection of cantilever and simply supported beams under concentrated and uniformly distributed load (by Double Integration method, Macaulay's method).

7 **Indeterminate Beams**

7.1 Indeterminacy in beams, Principle of consistent deformation/compatibility, Analysis of propped cantilever, fixed and two span continuous beams by principle of superposition, SF and BM diagrams (point load and udl covering full span)

8 **Trusses**

8.1 Introduction: Types of trusses, statically determinate and indeterminate trusses, degree of indeterminacy, stable and unstable trusses, advantages of trusses.

STRUCTURAL MECHANICS

INTRODUCTION:

Structural mechanics or Mechanics of structures is the computation of deformations, deflections, and internal forces or stresses (stress equivalents) within structures, either for design or for performance evaluation of existing structures. It is one subset of structural analysis.

What is structure?

The term 'structure' refers to anything that is constructed or built from different interrelated parts with a fixed location on the ground.

BASIC PRINCIPLE OF MECHANICS

Force:

- *Force is an external agent capable of changing the state of rest or motion of a particular body*
- *It has a magnitude and a direction*
- *The Force can be measured using a spring balance.*
- *The SI unit of force is Newton(N)*

  $F = m \times a$

  Where ,

  M = *mass*

  a = *acceleration*

Common symbols:	$\vec{F}$, F
SI unit:	Newton
In SI base units:	$kg{\cdot}m/s^2$
Other units:	dyne, poundal, pound-force, kip, kilo pond
Derivations from other quantities:	F = m a
Dimension:	LMT^{-2}

MOMENT:

It is the turning effect produced by a force, on the body, on which it acts. The moment is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of force.

i.e. M = F x l

In vector form, $\vec{M} = \vec{r} \times \vec{F}$

While the moment M (vector) of a force about a point depends upon the magnitude, the line of action, and the *sense of the force, it does not depend upon the actual position of the point of application of the force along its line of action.*

Types: i. Clockwise moment "+ve"

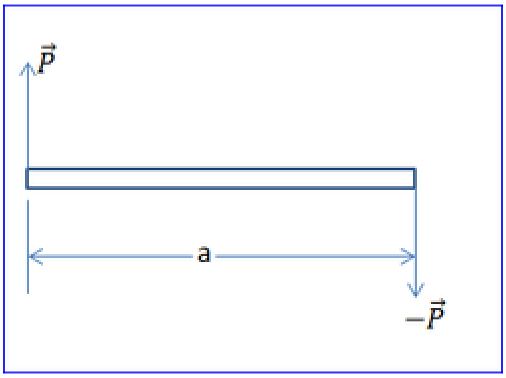
ii. Anticlockwise moment "-ve

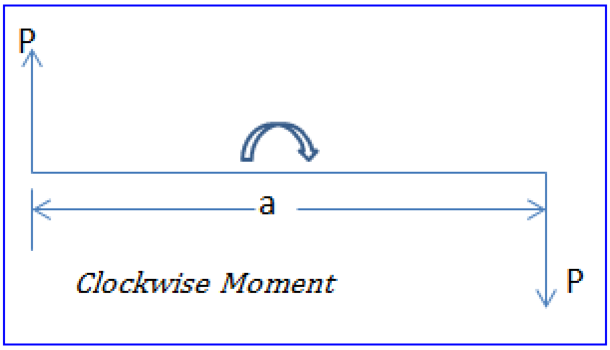
Law of Moments

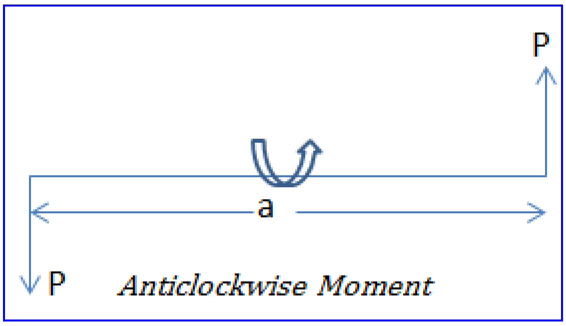
It states that "If a number of forces, all being in one plane, are acting at a point in equilibrium, the sum of clockwise moments must be equal to the sum of anticlockwise moments taken about any point in the plane of forces.

Couple

- *Couple is defined as combination of two equal and opposite forces separated by a certain distance.*
- *Couple is produced due to equal but unlike forces.*
- *Couple is unable to produce any translatory motion (i.e. motion in a straight line). It produces only rotation in a body.*
- *Couple Moment = M = P x a*
- *Two forces P (vector) and -P (vector) having same magnitude, parallel line of action and opposite senses are said to form a couple.*







SUPPORT CONDITION:

There is three types of supports in structure.

- Roller support
- Hinged support
- Fix support

Roller support:

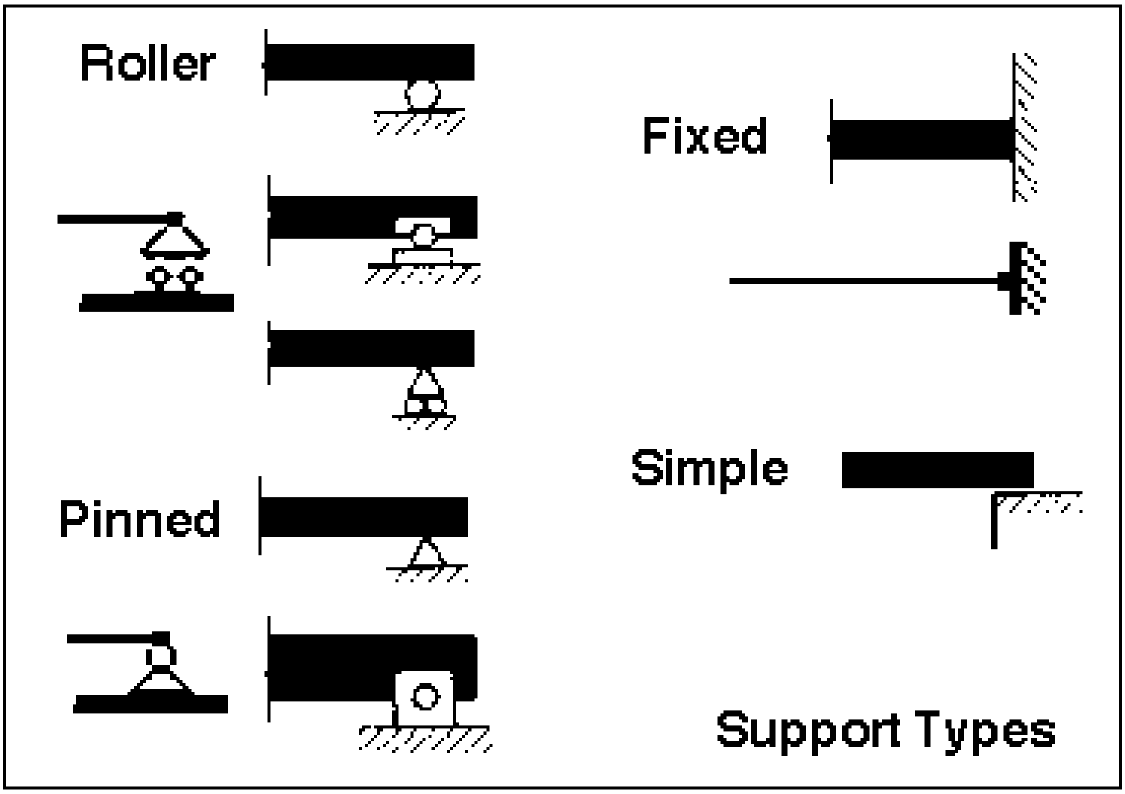
Roller supports are free to rotate and translate along the surface upon which the roller rests. The surface can be horizontal, vertical, or sloped *at any angle. ... Roller supports can also take the form of rubber bearings, rockers, or a set of gears which are designed to allow a limited amount of lateral movement.*

Hinged support:

The hinge support is capable of resisting forces acting in any direction of the plane. This support does not provide any resistance to rotation. The horizontal and vertical component of reaction can be determined using equation of equilibrium.

Fixed support:

It exerts forces acting in any direction and prevents all translational movements (horizontal and vertical) as well as all rotational movement of a member. These supports' reaction forces are horizontal and vertical components of a linear resultant; a moment. It is a rigid type of support or connection.


Roller
Fixed
Simple
Pinned
Support Types

What is equilibrium?

A body is said to be in equilibrium if no net force acts on it. Newton's first law of motion tells us that a body continues its state of rest or of uniform motion in a straight line if no resultant or net force acts on it.

For example, a book lying on a table or a picture hanging on a wall is at rest. The weight of the book acting downward is balanced by the upward reaction of the table

Condition of equilibrium:

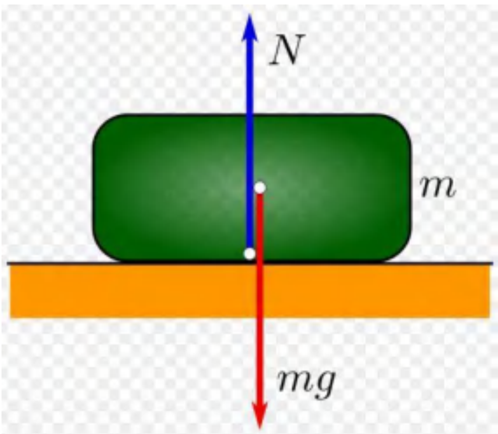
In the above examples, we see that a body at rest or in uniform motion is in equilibrium if the resultant force acting on it is zero. For a body in equilibrium, it must satisfy certain conditions. There are two conditions for a body to be in equilibrium.

First condition of equilibrium

A body is said to satisfy the first condition for equilibrium if the resultant of all the forces acting on it is zero. Let n number of forces F_1, F_2, F_3,......, F_n is acting on a body such that:

$$F_1 + F_2 + F_3 + \ldots\ldots + F_n = 0$$

or $\sum F = 0$ ......(1)



The symbol ∑ is a Greek letter called sigma used for summation. Equation (1) is called the first condition of equilibrium.

The first condition for equilibrium can also be stated in terms of X and Y components of the forces acting on the body as:

$$F_{1x} + F_{2x} + F_{3x} + \ldots\ldots + F_{nx} = 0$$

And

$$F1y + F2y + F3y + \ldots\ldots + F_{ny} = 0$$

or

$$\sum Fx = 0$$

$$\sum Fy = 0$$

First condition of equilibrium examples

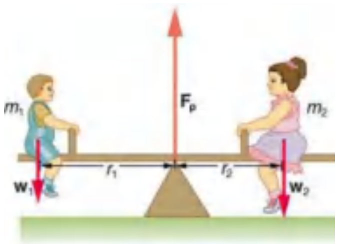
- *A book lying on a table.*
- *A picture hanging on a wall, are at rest and thus satisfies the first condition for equilibrium.*

Second condition of equilibrium

If the body is not in equilibrium although the first condition for equilibrium is still satisfied. It is because the body has the tendency to rotate. This situation demands another condition in addition to the first condition for equilibrium. According to this, a body satisfies the second condition for equilibrium when the resultant torque acting on it is zero.

Mathematically:

$$\sum \tau = 0$$



Second condition of equilibrium examples

- *The force applying on the steering of the car*
- *Couple*
- *Children playing on the sea saw*

NOTE:

***torque**: A rotational or twisting effect of a force; (SI unit newton-meter or Nm; imperial unit foot-pound or ft-lb.*

Subject: Structural Mechanics
Class No: - 03

Centre of Gravity

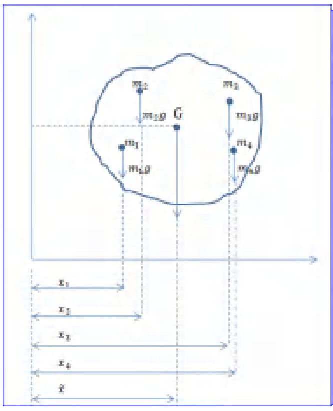
Centre of Gravity (C.G.) is that point through which the resultant of a system of parallel forces formed by the weights of all particles of the body passes.

In other words, the point through which the whole weight of the body acts is known as centre of gravity.
Every body has one and only one c.g.

Centroid

- *The plane figures (like triangle, quadrilateral, circle, trapezoid, etc.) have only areas but no mass. The centre of area of such figure is known as centroid.*
- *It is also called the geometrical centre or the centre of gravity*

Derivation for Centre of Gravity



Let us consider a lamina with definite area. Its plane consists of a number of particles with masses m_1, m_2, m_3.......... and hence, weights m_1g, m_2g, m_3g .... with co-ordinates (x_1, y_1), (x_2, y_2), (x_3, y_3)........

Let 'G' be the centre of gravity with co-ordinates $(\bar{x}, \bar{y})$ where resultant weight 'Mg' acts.

Since the sum of the moments of a system of coplanar forces equals the moment of resultant:

Taking moments about O, with the reference axis OX,

$$m_1gx_1 + m_2gx_2 + m_3gx_3 + \cdots \ldots\ldots = Mg\bar{x}$$

$$Hence, \bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots \ldots\ldots}{m_1 + m_2 + m_3 + \cdots \ldots\ldots}$$

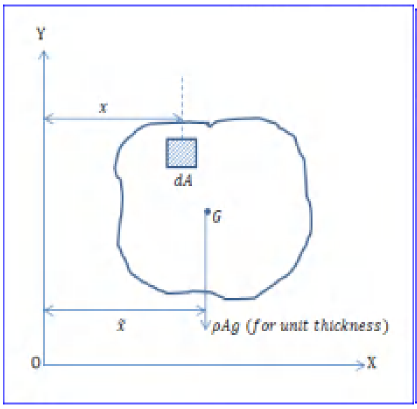
$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

Similarly, taking moments about O, with the reference axis OY,

$$\bar{y} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots \ldots\ldots}{m_1 + m_2 + m_3 + \cdots \ldots\ldots}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$

Derivation for the Centroid of Area



Let, $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centroid w.r.t. some axis of reference.

Consider an elemental are 'dA' of the lamina at a distance 'x' from the axis OY. The weight of the elemental path is $\rho.dA.g$. The moment of this force about the axis OY = $\rho.dA.g.x$

Total moment of the weight of lamina = $\rho.A.g.\bar{x}$

Now, $\rho.A.g.\bar{x} = \sum \rho.dA.g.\bar{x}$

$$Hence, \bar{x} = \frac{\Sigma dA.x}{A}, \bar{y} = \frac{\Sigma dA.y}{A}$$

(For Discrete Areas)

$$\bar{x} = \frac{\int dA.x}{\int dA}, \bar{y} = \frac{\int dA.y}{\int dA}$$

(For Continuous Areas)

Axis of Symmetry

Axis of Symmetry is a line or axis which divides the given line, are or volume into two equal and identical parts.

Centroid always lies along the line of axis of symmetry.

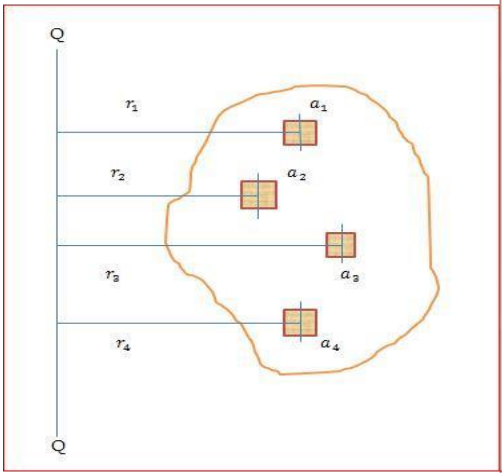
How to calculate the position of C.G.?

1. *Choose the reference axis (if not given) by considering the axis of symmetry.*
2. *Divide the given area into a number of elements with defined geometry.*
3. *Calculate the area and position of c.g. of each area from the reference axis*

Moment of Inertia

- *The moment of inertia about any point or axis is the product of the area and the perpendicular distance between the point or axis to the centre of gravity of the area. This is called the first moment of area.*
- *If this first moment of area is again multiplied by the perpendicular distance between them, the product is known as second moment of area.*
- *In case of a rigid body, its mass is considered and it is called second moment of mass.*
- *Second moment of area = Moment of Inertia*
- *Second moment of mass = Mass Moment of Inertia*
- *Denoted by MI or MOI*

MOI of Plane Area



Consider a_1, a_2, a_3 and a_4 are the elementary areas and r_1, r_2, r_3 and r_4 are the distances from the reference axis QQ.

MOI of area about the reference line QQ,

$$I_{QQ} = a_1r_1^2 + a_2r_2^2 + a_3r_3^2 + a_4r_4^2 + \dots\dots + a_nr_n^2$$

$$I_{QQ} = \sum_{i=1}^{n} a_i(r_i)^2$$

Mass Moment of Inertia about reference axis or line QQ is

$$I_{QQ} = \sum_{i=1}^{n} m_i(r_i)^2$$

MOI of plane by integration:

$$I_X = y_1^2 \; x \; dA_1 + y_2^2 \; x \; dA_2 + \cdots \dots\dots + y_n^2 \; x \; dA_n$$

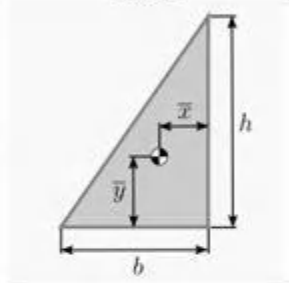
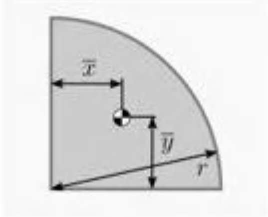
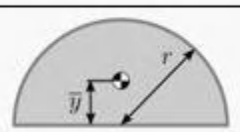
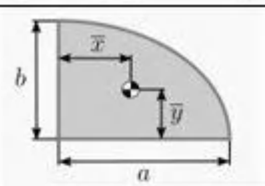
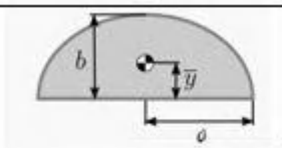
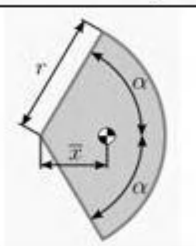
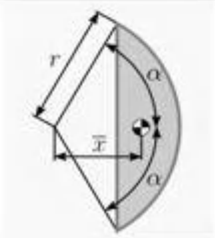
$$Hence, I_{XX} = \int y^2 dA$$

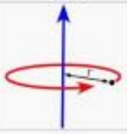
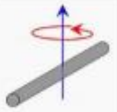
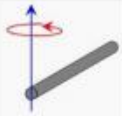
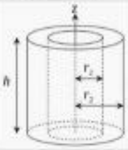
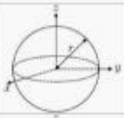
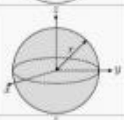
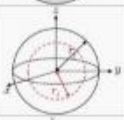
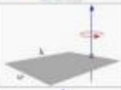
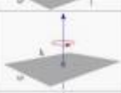
$$And, I_{YY} = \int x^2 dA$$

Probable question:

1. *What is Moment of inertia ?*
2. *How to calculate position of C.G ?*
3. *What centre of gravity ?*
4. *What is centroid ?*

Subject: Structural Mechanics
Class No: - 04
CG & MI in different section

Shape	Figure	X	Y	Area
Triangle		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter Circle		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semi-Circle		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter Ellipse		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semi-Ellipse		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Circular Sector		$\frac{2r\sin\alpha}{3\alpha}$	0	ar^2
Circular Segment		$\frac{4r(\sin\alpha)^3}{3(2\alpha-\sin 2\alpha)}$	0	$\frac{r^2}{2}(2\alpha - \sin 2\alpha$

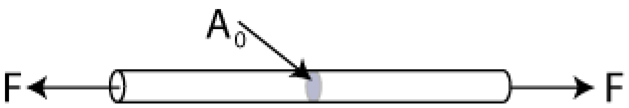
Description	Figure	Moment(s) of inertia
Point mass m at a distance r from the axis of rotation. A point mass does not have a moment of inertia around its own axis, but using the parallel axis theorem a moment of inertia around a distant axis of rotation is achieved.		$I = mr^2$
Two point masses, M and m, with reduced mass μ and separated by a distance, x about an axis passing through the center of mass of the system and perpendicular to line joining the two particles.		$I = \frac{Mm}{M+m}x^2 = \mu x^2$
Rod of length L and mass m, rotating about its center. This expression assumes that the rod is an infinitely thin (but rigid) wire. This is a special case of the thin rectangular plate with axis of rotation at the center of the plate, with $w = L$ and $h = 0$.		$I_{center} = \frac{mL^2}{12}$ [1]
Rod of length L and mass m, rotating about one end. This expression assumes that the rod is an infinitely thin (but rigid) wire. This is also a special case of the thin rectangular plate with axis of rotation at the end of the plate, with $h = L$ and $w = 0$.		$I_{end} = \frac{mL^2}{3}$ [1]
Thin circular hoop of radius r and mass m. This is a special case of a torus for $b = 0$ (see below), as well as of a thick-walled cylindrical tube with open ends, with $r_1 = r_2$ and $h = 0$.		$I_z = mr^2$ $I_x = I_y = \frac{mr^2}{2}$
Thin, solid disk of radius r and mass m. This is a special case of the solid cylinder, with $h = 0$. That $I_x = I_y = \frac{I_z}{2}$ is a consequence of the perpendicular axis theorem.		$I_z = \frac{mr^2}{2}$ $I_x = I_y = \frac{mr^2}{4}$
Thin cylindrical shell with open ends, of radius r and mass m. This expression assumes that the shell thickness is negligible. It is a special case of the thick-walled cylindrical tube for $r_1 = r_2$. Also, a point mass m at the end of a rod of length r has this same moment of inertia and the value r is called the radius of gyration.		$I = mr^2$ [1]
Solid cylinder of radius r, height h and mass m. This is a special case of the thick-walled cylindrical tube, with $r_1 = 0$. (Note: X-Y axis should be swapped for a standard right handed frame).		$I_z = \frac{mr^2}{2}$ [1] $I_x = I_y = \frac{m}{12}(3r^2 + h^2)$
Thick-walled cylindrical tube with open ends, of inner radius r_1, outer radius r_2, length h and mass m.		$I_z = \frac{m}{2}(r_1^2 + r_2^2) = mr_2^2\left(1 - t + \frac{t^2}{2}\right)$ [1] [2] where $t = (r_2 - r_1)/r_2$ is a normalized thickness ratio; $I_x = I_y = \frac{m}{12}(3(r_2^2 + r_1^2) + h^2)$
With a density of ρ and the same geometry		$I_z = \frac{\pi\rho h}{2}(r_2^4 - r_1^4)$ $I_x = I_y = \frac{\pi\rho h}{12}(3(r_2^4 - r_1^4) + h^2(r_2^2 - r_1^2))$
Regular tetrahedron of side s and mass m		$I_{solid} = \frac{ms^2}{20}$ $I_{hollow} = \frac{ms^2}{12}$ [3]
Hollow sphere of radius r and mass m. A hollow sphere can be taken to be made up of two stacks of infinitesimally thin, circular hoops, where the radius differs from 0 to r (or a single stack, where the radius differs from $-r$ to r).		$I = \frac{2mr^2}{3}$ [1]
Solid sphere (ball) of radius r and mass m. A sphere can be taken to be made up of two stacks of infinitesimally thin, solid discs, where the radius differs from 0 to r (or a single stack, where the radius differs from $-r$ to r).		$I = \frac{2mr^2}{5}$ [1]
Sphere (shell) of radius r_2, with centered spherical cavity of radius r_1 and mass m. When the cavity radius $r_1 = 0$, the object is a solid ball (above). When $r_1 = r_2$, $\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right) = \frac{5}{3}r_2^2$, and the object is a hollow sphere.		$I = \frac{2m}{5}\left(\frac{r_2^5 - r_1^5}{r_2^3 - r_1^3}\right)$ [1]
Right circular cone with radius r, height h and mass m		$I_z = \frac{3mr^2}{10}$ [4] $I_x = I_y = \frac{3m}{20}(r^2 + 4h^2)$ [4]
Torus of tube radius a, cross-sectional radius b and mass m.		About an axis passing through the center and perpendicular to the diameter: $\frac{m}{4}(4a^2 + 3b^2)$ [5] About a diameter: $\frac{m}{8}(4a^2 + 5b^2)$ [5]
Thin rectangular plate of height h, width w and mass m (Axis of rotation at the end of the plate)		$I_e = \frac{m}{12}(4h^2 + w^2)$
Thin rectangular plate of height h, width w and mass m (Axis of rotation at the center)		$I_c = \frac{m}{12}(h^2 + w^2)$ [1]
Solid cuboid of height h, width w, and depth d, and mass m. For a similarly oriented cube with sides of length s, $I_{CM} = \frac{ms^2}{6}$		$I_h = \frac{m}{12}(w^2 + d^2)$ $I_w = \frac{m}{12}(d^2 + h^2)$ $I_d = \frac{m}{12}(w^2 + h^2)$

Chapter no: 02

Simple stress & strains: -

What is stress?

The term stress (s) is used to express the loading in terms of force applied to a certain cross-sectional area of an object. From the perspective of loading, stress is the applied force or system of forces that tends to deform a body.



$$\text{Stress, } \sigma = \frac{\text{Force}}{\text{Cross-Sectional Area}} = \frac{F}{A_0}$$

It is measured in N/m2 and this unit is specifically called Pascal (Pa). A bigger unit of stress is the mega Pascal (MPa).

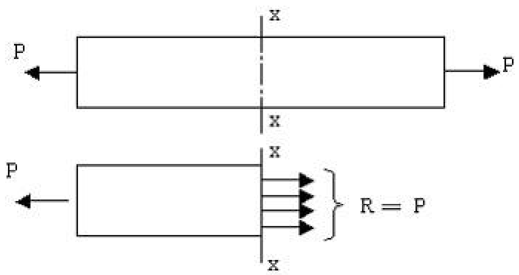
1 Pa = 1N/m2,
1MPa = 106 N/m2 =1N/mm2.

There are 3 basic type of stresses identify,

1. *Compressive stress*
2. *Shear stress*
3. *Tensile stress*

Compressive stress tends to squeeze a body, tensile stress to stretch (extend) it, and shear stress to cut it.

Tensile stress:



Consider a uniform bar of cross-sectional area A subjected to an axial tensile force P.

The stress at any section x-x normal to the line of action of the tensile force P is specifically called tensile stress pt. Since internal resistance R at x-x is equal to the applied force P, we have,

pt. = (internal resistance at x-x)/ (resisting area at x-x)

=R/A

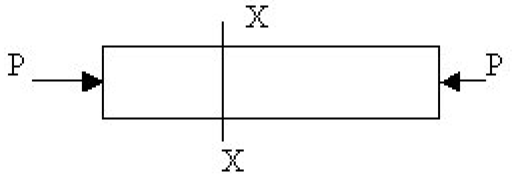
=P/A.

Under tensile stress the bar suffers stretching or elongation.

Compressive stress:

If the bar is subjected to axial compression instead of axial tension, the stress developed at x-x is specifically called compressive stress pc.

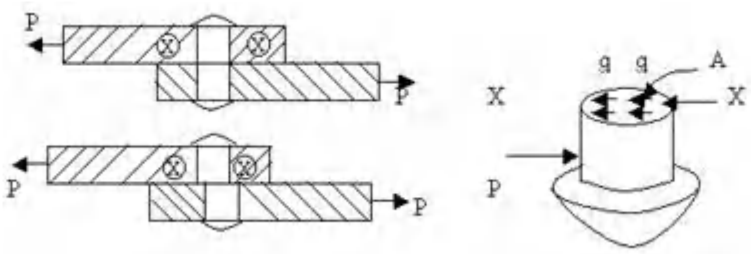
pc =R/A= P/A



Under compressive stress the bar suffers shortening

Shear stress:

Consider the section x-x of the rivet forming joint between two plates subjected to a tensile force P as shown in figure



The stresses set up at the section x-x acts along the surface of the section, that is, along a direction tangential to the section. It is specifically called shear or tangential stress at the section and is denoted by q.

q =R/A=P/A

Normal or Direct Stresses

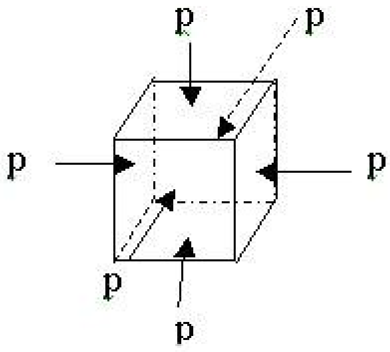
When the stress acts at a section or normal to the plane of the section, it is called a normal stress or a direct stress. It is a term used to mean both the tensile stress and the compressive stress.

Simple and Pure Stresses

The three basic types of stresses are tensile, compressive and shear stresses. The stress developed in a body is said to be simple tension, simple compression and simple shear when the stress induced in the body is (a) single and (b) uniform. If the condition (a) alone is satisfied, the stress is called pure tension or pure compression or pure shear, as the case may be.

Volumetric Stress

Three mutually perpendicular like direct stresses of same intensity produced in a body constitute a volumetric stress. For example, consider a body in the shape of a cube subjected *equal normal pushes on all its six faces. It is now subjected to equal compressive stresses p in all the three mutually perpendicular directions. The body is now said to be subjected to a volumetric compressive stress p.*



Volumetric stress produces a change in volume of the body without producing any distortion to the shape of the body.

<u>Assignment</u>

1. *What is MI of rod rotating about its centre ?*
2. *What is the MI of thick hollow sphere?*
3. *What is normal stress ?*
4. *What is stress ? also define 3 basic type of stress.*

Subject: Structural Mechanics
Class No: - 05

Strain:

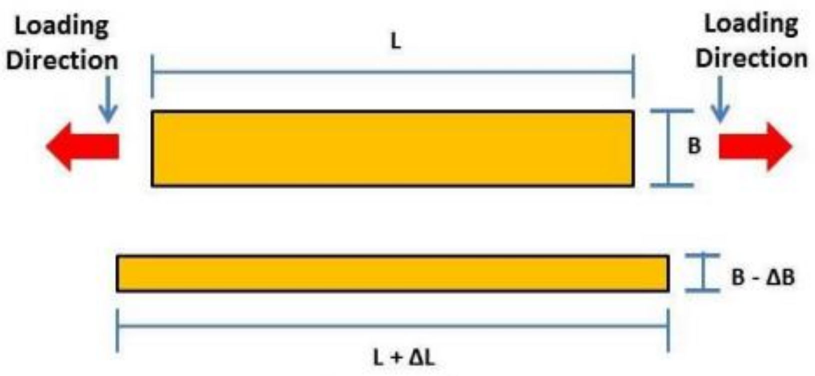
Strain is a measurement quantity which is ratio of change in length to original length under loading condition. But when we stretch a body, its dimensions changes in all directions. According to it strain can be divided into two types.

Types of strain:

Strain can be categorized into following aspects.

According to the direction of loading:

Let's try to learn with example. Take a rubber piece of L length, and B width. Apply a pull load on its two parallel faces along its length and observe it. The length of the rubber piece slightly increases, suppose it is ΔL. We also observed some little change in the width and height as they both decrease slightly. Suppose change in width is ΔB and change in height is ΔW. So we have observed three value of change in length in all three direction. According to it strain can be divided into two types.



Rubber Wire

Lateral Strain:

The ratio of change in length to its original length in the perpendicular direction of the loading is called lateral strain. In the above example it can be represented by

$$\varepsilon_{\text{lateral}} = \frac{\Delta B}{B} \text{ or } \frac{\Delta W}{W}$$

Longitudinal Strain:

The ratio of change in length to its original length in the direction of loading is known as longitudinal strain. In the above example longitudinal strain can be represent by.

$$\varepsilon_{\text{longitudinal}} = \frac{\Delta L}{L}$$

According to the loading:

According to the types of loading strain can be divided into three types

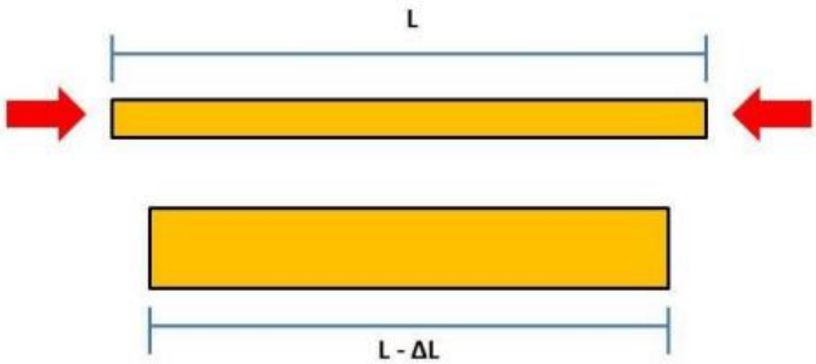
- *Normal strain*

- *Shear strain*
- *Volumetric strain*

Normal strain:
It can further divide into two types.

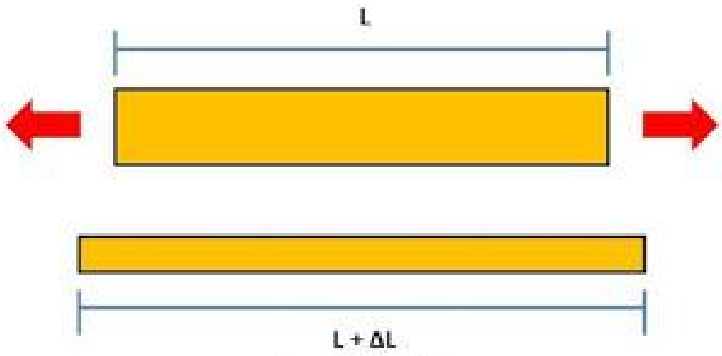
Compressive strain

Strain measure under compressive loading is known as compressive strain. It tends to increase the cross-section area and decrease the length of the body.



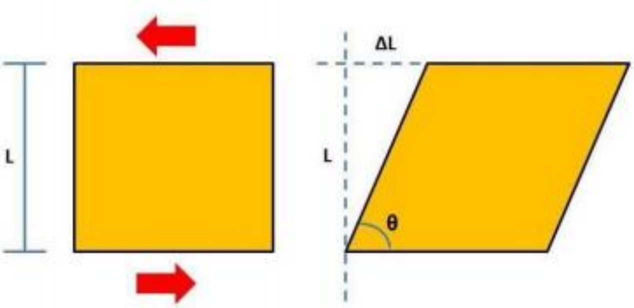
Compressive Strain

Tensile Strain:
Strain measure under tensile loading is known as tensile strain. It tends to decrease the cross section and increase the length of the body.



Tensile Strain

Shear Strain:
When the shear stress is applied on a body, it tends to deform the shape of the body as shown in the figure. The change in tangential angle in the direction of the loading is called shear strain.



Shear Strain

$$\varepsilon_{shear} = \frac{\Delta L}{L} \text{ or } \tan\theta$$

Volumetric strain:

The ratio of change in volume to original volume under normal loading condition is known as volumetric strain.

Mechanical of material:

Rigidity

Rigidity, also called stiffness, is a measure of elasticity, and represents a material's resistance to permanent deformation. Rigidity is closely related to strength, but differs in that brittle materials can be rigid, but not strong, and softer malleable metals, such as lead, can be strong, but not rigid. Rigidity is a material's resistance to bending, whereas strength is a material's resistance to breakage.

Rigidity is measured by finding the Young's modulus of a particular material. The Young's modulus is measured by dividing the stress acting upon a material by the strain which it undergoes.

Elasticity

Elasticity is a physical property of a material whereby the material returns to its original shape after having been stretched out or altered by force. Substances that display a high degree of elasticity are termed "elastic." The SI unit applied to elasticity is the pascal (Pa), which is used to measure the modulus of deformation and elastic limit.

The causes of elasticity vary depending on the type of material. Polymers, including rubber, may exhibit elasticity as polymer chains are stretched and then subsequently return to their original form when the force is removed. Metals may display elasticity as atomic lattices change shape and size, again, returning to their original form once energy is removed.

Plasticity

The plasticity of a material is its ability to undergo some degree of permanent deformation without rupture or failure. This property is important in forming, shaping, extruding and many other hot and cold working processes. Materials such as clay, lead etc., are plastic at room temperature. Steel is plastic when at bright red heat. Generally, plasticity increases with increase in temperature.

Compressibility

Compressibility is the measure of a liquid's relative volume change when the pressure acting on it changes. Compressibility is related to thermodynamics and fluid mechanics. It is denoted by beta "B". The Compressibility of a fluid depends on adiabatic or isothermal process. It can be represented in the formula below.

$$\beta = -\frac{1}{V}\frac{\partial V}{\partial p}$$

Where

B = compressibility

V = Volume

P = Pressure

Hardness

It is the ability of a material to resist to permanent shape change due to external stress. There is various measure of hardness – Scratch Hardness, Indentation Hardness and Rebound Hardness.

1. *Scratch Hardness*
   Scratch Hardness is the ability of materials to the oppose the scratches to outer surface layer due to external force.
2. *Indentation Hardness*
   It is the ability of materials to oppose the dent due to punch of external hard and sharp objects.
3. *Rebound Hardness*
   Rebound hardness is also called as dynamic hardness. It is determined by the height of "bounce" of a diamond tipped hammer dropped from a fixed height on the material.

Toughness

It is the ability of a material to absorb the energy and gets plastically deformed without fracturing. Its numerical value is determined by the amount of energy per unit volume. Its unit is Joule/ m^3. Value of toughness of a material can be determined by stress-strain characteristics of a material. For good toughness, materials should have good strength as well as ductility.
For example: brittle materials, having good strength but limited ductility is not tough enough. Conversely, materials having good ductility but low strength are also not tough enough. Therefore, to be tough, a material should be capable to withstand both high stress and strain.

Stiffness

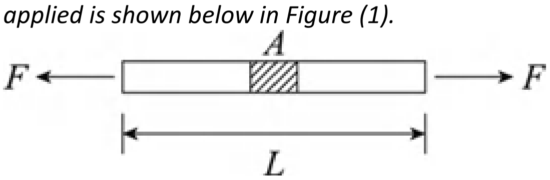
It is defined as the property of a material which is rigid and difficult to bend. The example of stiffness is rubber band. If single rubber band is stretch by two fingers the stiffness is less and the flexibility is more. Similarly, if we use the set of rubber band and stretched it by two fingers, the stiffness will be more, rigid and flexibility is less.

The expression of stiffness for an elastic body is as below.

Here, the stiffness is k, applied force is F, and deflection is δ.

The unit of stiffness is Newtons per meter.

Stiffness is applied to tension or compression. The length of a bar with cross-sectional area and tensile force applied is shown below in Figure (1).



The formula for axial stiffness is expressed as,

$$k = \frac{F}{\frac{FL}{AE}}$$

$$k = \frac{FAE}{FL}$$

$$k = \frac{AE}{L}$$

Here, cross-sectional area of an object is *A*, length is *L*, applied force is *F*, and elastic modulus is *E*.

Subject: Structural Mechanics
Class No: - 06

Brittleness

Brittleness of a material indicates that how easily it gets fractured when it is subjected to a force or load. When a brittle material is subjected to a stress it observes very less energy and gets fractures without significant strain. Brittleness is converse to ductility of material. Brittleness of material is temperature dependent. Some metals which are ductile at normal temperature become brittle at low temperature.

Ductility

Ductility is a property of a solid material which indicates that how easily a material gets deformed under tensile stress. Ductility is often categorized by the ability of material to get stretched into a wire by pulling or drawing. This mechanical property is also an aspect of plasticity of material and is temperature dependent. With rise in temperature, the ductility of material increases.

Malleability

Malleability is a property of solid materials which indicates that how easily a material gets deformed under compressive stress. Malleability is often categorized by the ability of material to be formed in the form of a thin sheet by hammering or rolling. This mechanical property is an aspect of plasticity of material. Malleability of material is temperature dependent. With rise in temperature, the malleability of material increases.

Creep and Slip

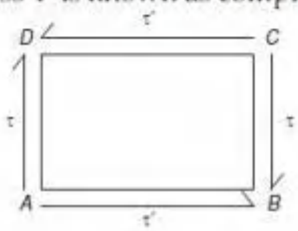
Creep is the property of a material which indicates the tendency of material to move slowly and deform permanently under the influence of external mechanical stress. It results due to long time exposure to large external mechanical stress with in limit of yielding. Creep is more severe in material that are subjected to heat for long time. Slip in material is a plane with high density of atoms.

Fatigue

Fatigue is the weakening of material caused by the repeated loading of the material. When a material is subjected to cyclic loading, and loading greater than certain threshold value but much below the strength of material (ultimate tensile strength limit or yield stress limit), microscopic cracks begin to form at grain boundaries and interfaces. Eventually the crack reaches to a critical size. This crack propagates suddenly and the structure gets fractured. The shape of structure affects the fatigue very much. Square holes and sharp corners lead to elevated stresses where the fatigue crack initiates.

Complementary shear stress

Complementary shear Stress: When ever a shear stress τ is applied on parallel surface of body then to keep the body in equilibrium a shear stress 'τ'' is induced on remaining surface of body. These stresses form a couple. The couple form due to shear stress τ produces clockwise moment. For equilibrium this couple is balanced by couple developed by τ'. This resisting shear stress τ' is known as complementary shear stress.



Couple produced by τ

$$(\tau\, BC) \times AB$$

Couple produced by τ'

$$\tau'(CD) \times BC$$

For equilibrium

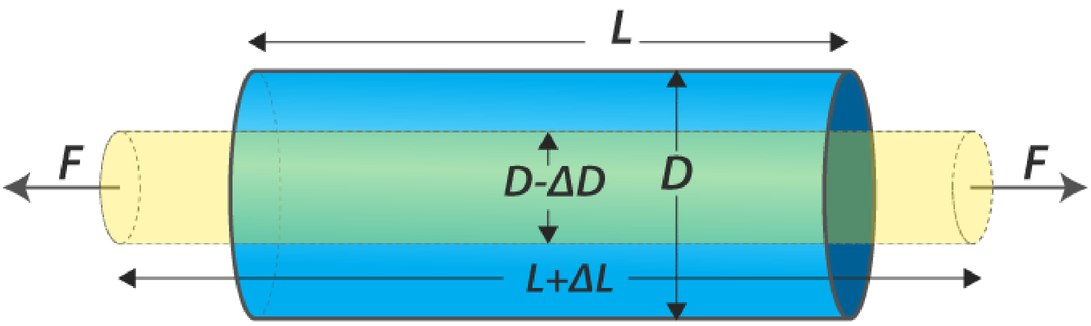
$$(\tau\, BC) \cdot AB = (\tau'\, CD) \cdot BC$$

$$\Rightarrow \quad \tau = \tau' \;\{AB = CD\}$$

What is a Longitudinal Strain?

A longitudinal strain is defined as

Change in the length to the original length of an object
It is caused due to longitudinal stress and is denoted by the Greek letter epsilon ε.



Longitudinal Strain Formula

Consider a cylinder. When that longitudinal stress acts on it, there will be a change in the length of the cylinder. Then the longitudinal strain can be mathematically expressed as follows:

Longitudinal Strain=Change in the length/ Original Length $\varepsilon=\Delta L/L$

Where,

- L is the original length
- ΔL change in length

Longitudinal Strain Unit

It is expressed as $\varepsilon=\Delta L L$. Here the fundamental unit of length is the meter. Substituting it in the formula we get:
SI Unit of Longitudinal Stress=m/m

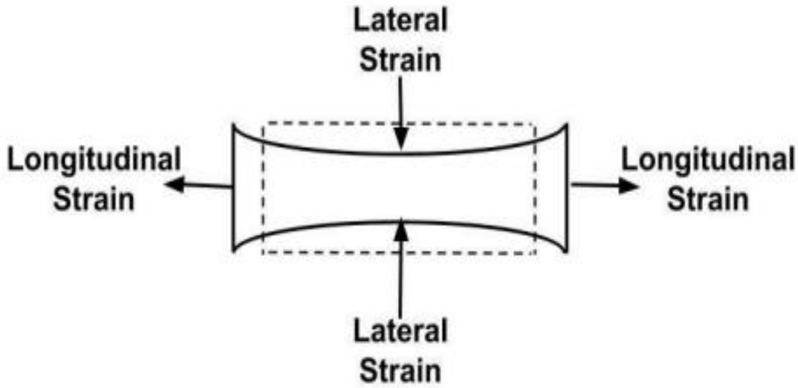
They cancel each other, making it unit less or dimensionless quantity

Lateral strain

Lateral strain, also known as transverse strain, is defined as the ratio of the change in diameter of a circular bar of a material to its diameter due to deformation in the longitudinal direction. It occurs when under the action of a longitudinal stress, a body will extend in the direction of the stress and contract in the transverse or lateral direction (in the case of tensile stress). When put under compression, the body will contract in the direction of the stress and extend in the transverse or lateral direction. It is a dimensionless quantity, as it is a ratio between two quantities of the same dimension.

What is Poisson's Ratio

Poisson’s Ratio is the ratio of Lateral Strain and Longitudinal Strain within Elastic Limits.



$$\text{Poisson's Ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

Symbol	Greek letter ‘nu’, v
Formula	Poisson’s ratio = – Lateral strain / Longitudinal strain
Range	-1.0 to +0.5
Units	Unitless quantity
Scalar / Vector	Scalar quantity

Subject: Structural Mechanics
Class No: - 07

Volumetric Strain

It is defined as the *ratio between change in volume and original volume of the body,* and is denoted by e_v.

$$\therefore \quad e_v = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\delta V}{V} \quad \text{...(1.3)}$$

The strains which disappear with the removal of load are termed as *elastic strains* and the body which regains its original position on the removal of force is called an *elastic body.* The body is said to be *plastic* if the *strains exist even after the removal of external force.* There is always a limiting value of load up to which the strain totally disappears on the removal of load—the stress corresponding to this load is called *elastic limit.*

Robert Hooke discovered experimentally that within elastic limit, stress varies *directly* as strain

i.e., $\text{Stress} \propto \text{Strain}$

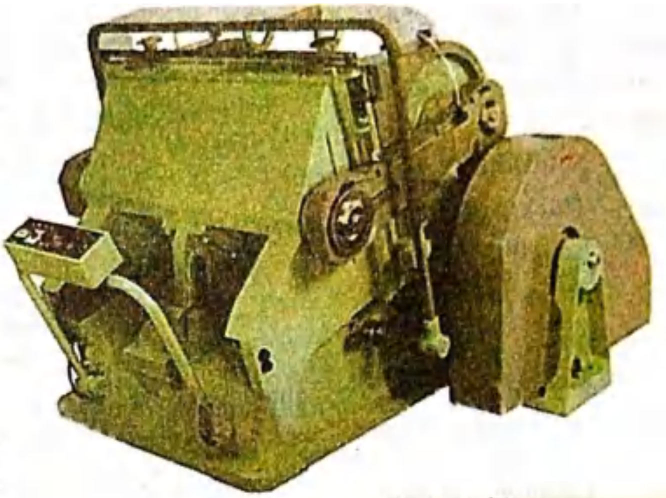
or, $\dfrac{\text{Stress}}{\text{Strain}} = \text{a constant}$

This constant is termed as *Modulus of elasticity.*

(i) Young's modulus:

It is the *ratio between tensile stress and tensile strain or compressive stress and compressive strain.* It is denoted by *E.* It is the same as modulus of elasticity.

$$\text{or,} \quad E = \frac{\sigma}{e}\left[= \frac{\sigma_t}{e_t} \text{ or } \frac{\sigma_c}{e_c}\right] \quad \text{...(1.4)}$$



Heavy duty punching machine.

(ii) Modulus of rigidity:

It is defined as the *ratio of shear stress τ (tau) to shear strain* and is denoted by *C, N* or *G* It is also called *shear modulus of elasticity.*

$$\text{or,} \quad \frac{\tau}{e_s} = C, N \text{ or } G \quad \text{...(1.5}$$

(iii) Bulk or volume modulus of elasticity:

It may be defined as the *ratio of normal stress* (on each face of a solid cube) *to volumetri strain* and is denoted by the letter *K.*

$$\text{or,} \quad \frac{\sigma_n}{e_v} = K \quad \text{...(1.6}$$

Some important problems:

Example 1.1. *A square steel rod 20 mm × 20 mm in section is to carry an axial load (compressive) of 100 kN. Calculate the shortening in a length of 50 mm. E = 2.14 × 10^8 kN/m^2.*

Solution.

Area, $A = 0.02 \times 0.02 = 0.0004\ m^2$

Length, $l = 50$ mm or 0.05 m

Load, $P = 100$ kN

$E = 2.14 \times 10^8\ kN/m^2$

Shortening of the rod δl:

Stress, $\sigma = \frac{P}{A}$

∴ $\sigma = \frac{100}{0.0004} = 250000\ kN/m^2$

Also, $E = \frac{\text{Stress}}{\text{Strain}}$

or, $\text{Strain} = \frac{\text{Stress}}{E} = \frac{250000}{2.14 \times 10^8}$

or, $\frac{\delta l}{l} = \frac{250000}{2.14 \times 10^8}$

∴ $\delta l = \frac{250000}{2.14 \times 10^8} \times l = \frac{250000}{2.14 \times 10^8} \times 0.05$

$= 0.0000584$ m or 0.0584 mm

Hence, the shortening of the rod = **0.0584 mm** (Ans.)

Example 1.2. *A hollow cast-iron cylinder 4 m long, 300 mm outer diameter, and thickness of metal 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 kN/m². Assume Young's modulus for cast-iron as 1.5 × 10⁸ kN/m² and find (i) magnitude of the load, (ii) longitudinal strain produced, and (iii) total decrease in length.*

Solution. Outer diameter, $D = 300 \text{ mm} = 0.3 \text{ m}$

Thickness, $t = 50 \text{ mm} = 0.05 \text{ m}$

Length, $l = 4 \text{ m}$

Stress produced, $\sigma = 75000 \text{ kN/m}^2$

$E = 1.5 \times 10^8 \text{ kN/m}^2$

Inner diameter of the cylinder, $d = D - 2t = 0.3 - 2 \times 0.05 = 0.02 \text{ m}$

(*i*) Magnitude of the load *P* :

Using the relation, $\sigma = \dfrac{P}{A}$

or, $P = \sigma \times A = 75000 \times \dfrac{\pi}{4}(D^2 - d^2) = 75000 \times \dfrac{\pi}{4}(0.3^2 - 0.2^2)$

or, $P = \mathbf{2945.2 \text{ kN}}$ **(Ans.)**

(*ii*) Longitudinal strain produced, *e* :

Using the relation,

Strain, $e = \dfrac{\text{Stress}}{E} = \dfrac{75000}{1.5 \times 10^8}$

$= \mathbf{0.0005}$ **(Ans.)**

(*iii*) Total decrease in length, δ*l* :

Using the relation,

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

$$0.0005 = \frac{\delta l}{4}$$

$$\delta l = 0.0005 \times 4 \text{ m} = 0.002 \text{ m} = 2 \text{ mm}$$

Hence, decrease in length = **2 mm (Ans.)**

Example 1.3. *The following observations were made during a tensile test on a mild steel specimen 40 mm in diameter and 200 mm long.*

Elongation with 40 kN load (within limit of proportionality),

$$\delta l = 0.0304 \text{ mm}$$

Yield load = *161 kN*

Maximum load = *242 kN*

Length of specimen at fracture

= *249 mm*

Determine :

(*i*) *Young's modulus of elasticity,* (*ii*) *Yield point stress,*

(*iii*) *Ultimate stress, and* (*iv*) *Percentage elongation.*

Solution.

(*i*) Young's modulus of elasticity *E* :

Stress, $$\sigma = \frac{P}{A} = \frac{40}{\frac{\pi}{4} \times (0.04)^2} = 3.18 \times 10^4 \text{ kN/m}^2$$

Strain, $$e = \frac{\delta l}{l} = \frac{0.0304}{200} = 0.000152$$

$\therefore$ $$E = \frac{\text{Stress}}{\text{Strain}} = \frac{3.18 \times 10^4}{0.000152}$$

$$= \mathbf{2.09 \times 10^8 \text{ kN/m}^2} \quad \textbf{(Ans.)}$$

(*ii*) Yield point stress :

Yield point stress $$= \frac{\text{Yield point load}}{\text{Area}}$$

$$= \frac{161}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{12.8 \times 10^4 \text{ kN/m}^2} \quad \textbf{(Ans.)}$$

(*iii*) Ultimate stress :

Ultimate stress $$= \frac{\text{Maximum load}}{\text{Area}}$$

$$= \frac{242}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{19.2 \times 10^4 \text{ kN/m}^2} \quad \textbf{(Ans.)}$$

(*iv*) Percentage elongation :

Percentage elongation $$= \frac{\text{Length of specimen at fracture} - \text{original length}}{\text{Original length}}$$

$$= \frac{249 - 200}{200} = 0.245 = \mathbf{24.5\%} \quad \textbf{(Ans.)}$$

Example 1.9. *A member LMNP is subjected to point loads as shown in Fig. 1.14.*

Calculate :

(i) Force P necessary for equilibrium.

(ii) Total elongation of the bar.

Take $E = 210\ GN/m^2$

Solution. Refer to Fig. 1.14

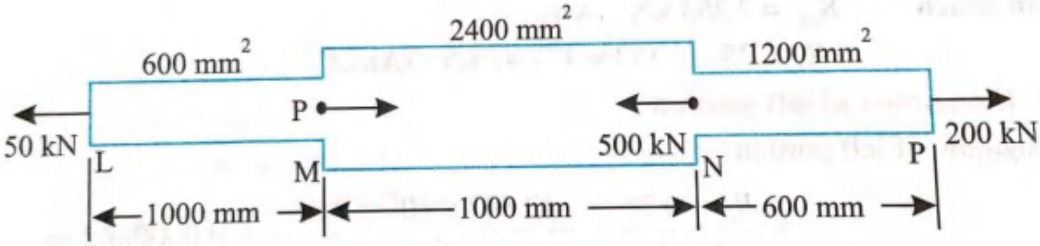


Fig. 1.14

(i) Force *P* necessary for equilibrium :

Resolving the forces on the rod along its axis, we get

$$50 + 500 = P + 200$$

$$\mathbf{P = 350\ kN} \quad \textbf{(Ans.)}$$

(ii) Total elongation of the bar :

Let δl_1, δl_2 and δl_3 be the changes in lengths *LM*, *MN* and *NP* respectively.

Then,

$$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{50 \times 1000 \times 1}{600 \times 10^{-6} \times 210 \times 10^9} = 3.97 \times 10^{-4} \text{ m ....increase (+)}$$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{300 \times 1000 \times 1}{2400 \times 10^{-6} \times 210 \times 10^9} = 5.95 \times 10^{-4} \text{ m ....decrease (-)}$$

$$\delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{200 \times 1000 \times 0.6}{1200 \times 10^{-6} \times 210 \times 10^9} = 4.76 \times 10^{-4} \text{ m ....increase (+)}$$

$\therefore$ Total elongation, $\delta l = \delta l_1 - \delta l_2 + \delta l_3 = 3.97 \times 10^{-4} - 5.95 \times 10^{-4} + 4.76 \times 10^{-4}$

$= 10^{-4}\,(3.97 - 5.95 + 4.76) = 2.78 \times 10^{-4}$ m, or, 0.278 mm

Hence, total elongation of the bar = **0.278 mm** **(Ans.)**

Example 1.10. *In Fig. 1.15 is shown a steel bar of cross-sectional area 250 mm² held firmly by the end supports and loaded by an axial force of 25 kN.*

Determine :

(*i*) *Reactions at L and M.*

(*ii*) *Extension of the left portion.*

$E = 200\ GN/m^2$.

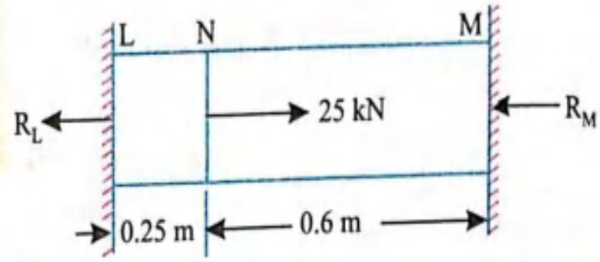


Fig. 1.15

Solution. Refer to Fig. 1.15

(*i*) Reactions at *L* and *M* :

As the bar is in equilibrium.

$$\therefore \quad R_L + R_M = 25 \text{ kN} \qquad ...(i)$$

Also, since total length of the bar remains unchanged,

∴ Extension in *LN* = contraction in MN

$$\frac{R_L \times 0.25}{A \times E} = \frac{R_M \times 0.6}{A \times E}$$

$$R_L \times 0.25 = R_M \times 0.6$$

$$R_L = \frac{R_M \times 0.6}{0.25} = 2.4\ R_M$$

Substituting the value of R_L in (*i*), we get

$$2.4\ R_M + R_M = 25$$

From which $\quad$ $\mathbf{R_M = 7.353\ kN}$ **(Ans.)**

$\therefore \quad R_L = 25 - 7.353 = \mathbf{17.647\ kN}$ **(Ans.)**

(*ii*) Elongation of left portion :

Elongation of left portion

$$= \frac{R_L \times 0.25}{A \times E} = \frac{17.647 \times 10^3 \times 0.25}{250 \times 10^{-6} \times 200 \times 10^9} = 0.0000882 \text{ m}$$

= 0.0882 mm (Ans.)

Example 1.20. *A concrete cylinder of diameter 150 mm and length 300 mm when subjected to an axial compressive load of 240 kN resulted in an increase of diameter by 0.127 mm and a decrease in length of 0.28 mm. Compute the value of Poisson's ratio* $\mu\left(=\frac{1}{m}\right)$ *and modulus of elasticity E.*

Solution. Diameter of the cylinder, $d = 150$ mm

Length of the cylinder, $l = 300$ mm

Increase in diameter, $\delta d = 0.127$ mm (+)

Decrease in length $l = 0.28$ mm (–)

Axial compressive load, $P = 240$ kN

Poisson's ratio, μ :

We know that,

Linear strain $= \frac{\delta l}{l} = \frac{0.28}{300} = 0.000933$

and, lateral strain $= \frac{\delta d}{d} = \frac{0.127}{150} = 0.000846$

∴ Poisson's ratio, $\mu = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{0.000846}{0.000933} = 0.907$

Modulus of elasticity, *E* :

Using the relation, $E = \frac{\text{Stress}}{\text{Strain (linear)}} = \frac{P/A}{\delta l/l}$

$$E = \frac{240/\left(\frac{\pi}{4} \times 0.15^2\right)}{(0.00028/0.3)} = \frac{240 \times 4 \times 0.3}{\pi \times 0.15^2 \times 0.00028}$$

$$= 14.55 \times 10^6 \text{ kN/m}^2 = 14.55 \text{ GN/m}^2$$

∴ Young's modulus, $\mathbf{E = 14.55 \text{ GN/m}^2}$ **(Ans.)**

Subject: Structural Mechanics
Topic: -Application of simple stress and strain in engineering field
Class No: - 08

Ductile and Brittle Materials

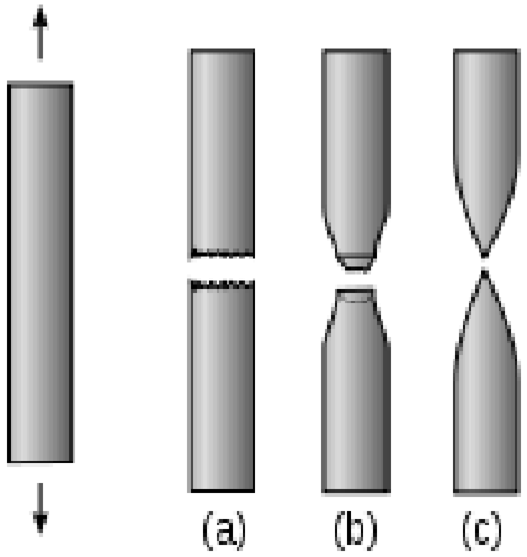
Materials ability to undergo significant plastic deformation under tensile stress before rapture called ductile properties of materials. In other words, if materials ductile, materials stretch under tensile load. The ductile materials are Steel, Aluminium, copper etc.

Brittle materials break without significant plastic deformation under tensile stress. Also called sudden failure. Brittle material absorbs little energy prior to rapture. The brittle material is glass, Plain concrete, cast iron, etc.

In this figure you can observe materials (a) break without losing its cross-section area means materials is brittle. This type of materials fails suddenly without any notice.

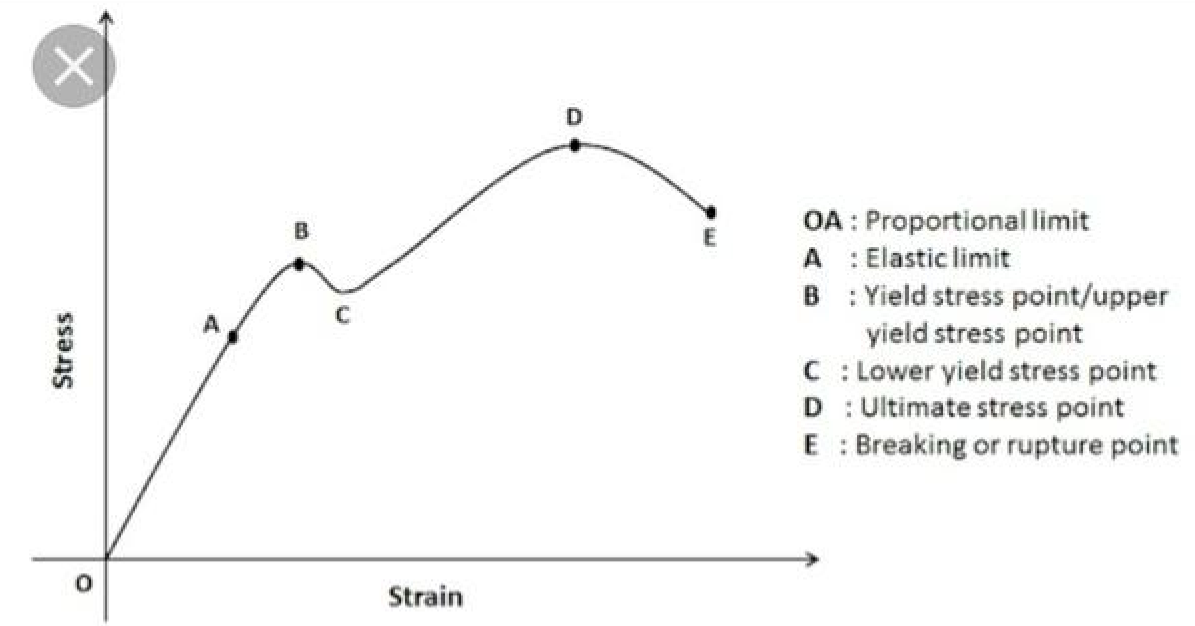
Failure of material (b) shows, it's a ductile material. Ductile materials reduced its cross-section before rapture.

Material (c) it's called complete ductile materials.



This ability to deform helps structure to resist seismic force. When the seismic load acted on building ductile structure perform well compare to other types of structure.

STRESS STRAIN CURVE OF DUCTILE MATERIAL



The significance for the points on the graph are given below:

Proportional Limit: This limit is represented by point A on the graph. Up to this limit, the stress and the strain induced in the specimen are directly proportional to each other, i.e. the specimen obeys Hooke's law. Beyond this point, the stress is not proportional to the strain.

Elastic Limit: This limit is represented by point B on the graph. Up to this limit, the material is said to be elastic. This implies that the specimen regains its original shape and dimensions after the removal of the external load. There are no residual deformations seen in the specimen, on removal of the load. After this point, the material is said to become plastic.

Yield Point: Contrary to what the name suggests, this is a region rather than a point. It is limited by the upper yield point 'C' and the lower yield point 'D'. The stress – strain curve in this part of the graph is almost horizontal, which implies that there is an appreciable increase in strain for a negligible increase in stress. Yielding starts at 'C' and ends at 'D'. After the point 'D', the material, due to strain hardening again starts taking load and the curve rises, as seen in the figure. The material now is said to be plastic and the deformation is of nearly permanent nature.

Ultimate Stress: This is shown by the point 'E' on the graph. It represents the maximum stress that a material can take before it fails. The specimen however does not fail at this point. After this point, the curve starts dropping.

Breaking Point: This is the point at which the specimen fails. After the ultimate stress point, necking of the specimen takes place, which causes a loss in the load carrying capacity of the specimen and ultimately causes it to fail. This point is represented on the curve, by point 'F'.

PERCENTAGE OF ELONGATION:

Percent elongation is a measurement that captures the amount a material will plastically and elastically deform up to fracture. Percent elongation is one way to measure and quantify the ductility of a material.

The material's final length is compared with its original length to determine the percent elongation and the material's ductility.

PERCENTAGE REDUCTION IN AREA:

Reduction of area is a comparison between the original cross-sectional area of a sample and the minimum cross-sectional area of the same sample after complete fracture failure. It is used as an indicator to show to what extent a material will deform when subjected to a tensile load. Reduction of area is normally displayed as a percentage.

Reduction of area is determined by applying a tensile load to a test sample. The test sample is normally circular in cross section, but other profiles may also be used. The test sample consists of a single piece of material (no chemical or mechanical joints) and has enlarged ends which allow it to be clamped into a tensile testing machine.

The tensile testing machine applies a tensile force along the longitudinal axis of the test sample, gradually pulling its ends further apart. The sample elongates along the longitudinal axis causing it to thin, or 'neck'. Necking continues until failure occurs and the sample fractures, breaking into two separate pieces. A tensile test can be performed on any type of material.

SIGNIFICANCE OF PERCENTAGE ELONGATION AND REDUCTION IN AREA OF CROSS SECTION

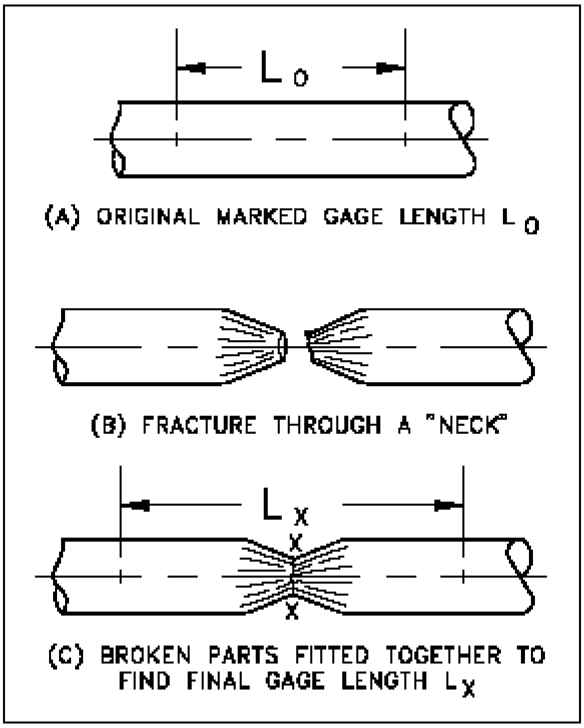


Figure 6 Measuring Elongation After Fracture

$$\text{Percent elongation} = \frac{\text{final gage length} - \text{initial gage length}}{\text{initial gage length}}$$

$$= \frac{L_x - L_o}{L_o} = \text{inches per inch x 100}$$

Percent reduction of area (RA) =

$$\frac{\text{area of original cross section} - \text{minimum final area}}{\text{area of original cross section}}$$

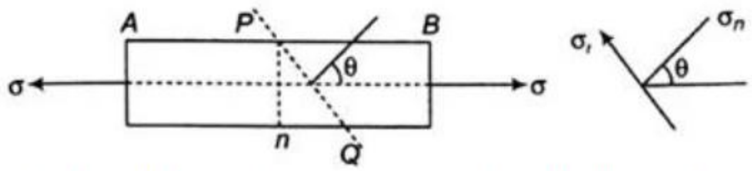
$$= \frac{A_o - A_{min}}{A_o} = \frac{\text{decrease in area}}{\text{original area}} = \frac{\text{square inches}}{\text{square inches}} \text{ x 100}$$

PROBABLE QUESTION:

1. *What is ductile material?*
2. *What is yield point?*
3. *Draw the stress strain curve of ductile material?*
4. *How to calculate %elongation and %reduction of area?*
5. *What is breaking point?*

Subject: Structural Mechanics
Topic: -complex stress and strain
Class No: - 09
Principal stresses and strain:

Consider a rectangular beam and we have to calculate the stress on an inclined section as shown in the figure.



Induced stress is divided into two components which are given as-

Normal stress

Normal Stress on an inclined section.

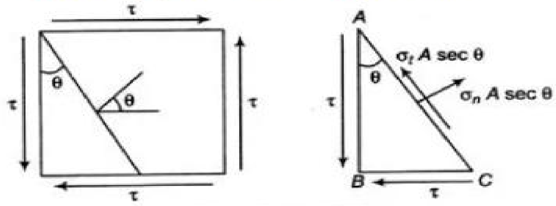
$$\sigma_n = \sigma \cos^2 \theta$$

Tangential stress

Shear Stress on an inclined section.

$$\sigma_t = -\frac{\sigma}{2} \sin 2\theta$$

Stress on Inclined Section PQ due to Shear Stress



Stress simple shear

Induced stress is divided into two components which are given as -

Normal stress

Normal Stress on an inclined section.

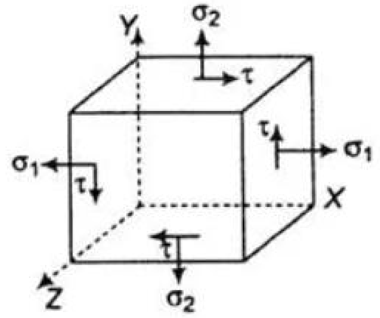
$$\sigma_n = \tau \sin 2\theta$$

Tangential Stress

Shear Stress on an inclined section.

$$\sigma_t = \tau \cos 2\theta$$

Stress on Inclined Section PQ due to combination of Axial Stress and Shear Stress



Induced stress body diagram

Induced stress is divided into two components which are given as -

Normal stress

Normal Stress on an inclined section.

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta + \tau \sin 2\theta$$

Tangential stress

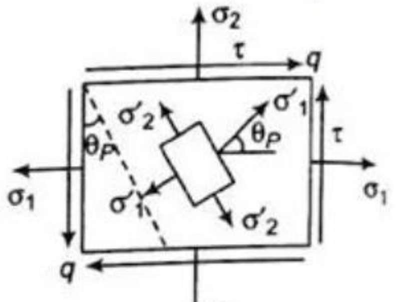
Shear Stress on an inclined section.

$$\sigma_1 = -\left(\frac{\sigma_1 + \sigma_2}{2}\right)\sin 2\theta + \tau \cos 2\theta$$

Principal Stresses and Principal Planes

The plane carrying the maximum normal stress is called the major principal plane and normal stress is called major principal stress.

The plane carrying the minimum normal stress is known as minor principal plane and normal stress is called minor principal stress.



Principal stress and planes

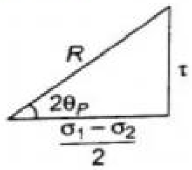
Major principal stress

$$\sigma_1' = \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{2} + \tau^2}$$

Minor principal stress

$$\sigma_2' = \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{2} + \tau^2}$$

Major & Minor principal plane angle



2 θ_P triangle

$$\tan 2\theta_p = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$\sigma_1' + \sigma_2' = \sigma_1 + \sigma_2$$

$$when\ 2\theta_p = 0$$

$$\Rightarrow \sigma_1' = \sigma_1\ and\ \sigma_2' = \sigma_2$$

Note: Across maximum normal stresses acting in plane shear stresses are zero.

Computation of Principal Stress from Principal Strain

The three stresses normal to shear principal planes are called principal stress, while a plane at which shear strain is zero is called principal strain.

For two-dimensional stress system, $\sigma_3 = 0$

$$\sigma_1 = \frac{E_1(\varepsilon_1 + \mu\varepsilon_2)}{1-\mu^2},\ \sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2}$$

Maximum Shear Stress

The maximum shear stress is equal of one half the difference between the largest and smallest principal stresses and acts on the plane that bisects the angle between the directions of the largest and smallest principal stress, i.e. the plane of the maximum shear stress is oriented 45° from the principal stress planes.

$$\tau' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2}{2} + \tau^2}$$

$$\tau' = \frac{\sigma_1' - \sigma_2'}{2}$$

$$\tan 2\theta_s = -\frac{\sigma_1 - \sigma_2}{2\tau} = -\frac{1}{\tan 2\theta_p}$$

$$2\theta_s = 2\theta_p \pm 90^0, \theta_s = \theta_p \pm 45^0$$

Principal Strain

For two-dimensional strain system,

$$\varepsilon_I = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$\varepsilon_{II} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{1}{2}\sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

Where, ε_x = Strain in x-direction

ε_y = Strain in y-direction

γ_{xy} = Shearing strain

Maximum Shear Strain

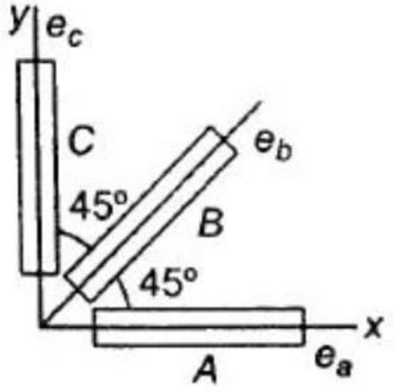
The maximum shear strain also contains normal strain which is given as

$$\gamma_{max} = \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2}$$

$$\left(\frac{\gamma}{2}\right)_{max} = \frac{\varepsilon_I - \varepsilon_{II}}{2}$$

Strain Measuring Method - 45° Strain Rosette or Rectangular Strain Rosette

Rectangular strains Rosette is inclined 45° to each other



45° strain Rosette

$$e_a = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) + \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\cos 2\theta$$

$$e_b = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) - \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\sin 2\theta$$

$$e_c = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) - \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\cos 2\theta$$

Principal strain can be calculated from above equation

Subject: Structural Mechanics
Topic: -complex stress and strain
Class No: - 10

MOHR'S CIRCLE AND ITS APPLICATION

2.6. GRAPHICAL METHODS

2.6.1. Mohr's Circle

A German scientist Otto Mohr devised a graphical method for finding out the normal and shear stresses on any interface of an element when it is subjected to two perpendicular stresses. This method is explained as follows:

2.6.1.1. Mohr's circle construction for "like stresses"

Refer to Fig. 2.7. *Steps of construction*:

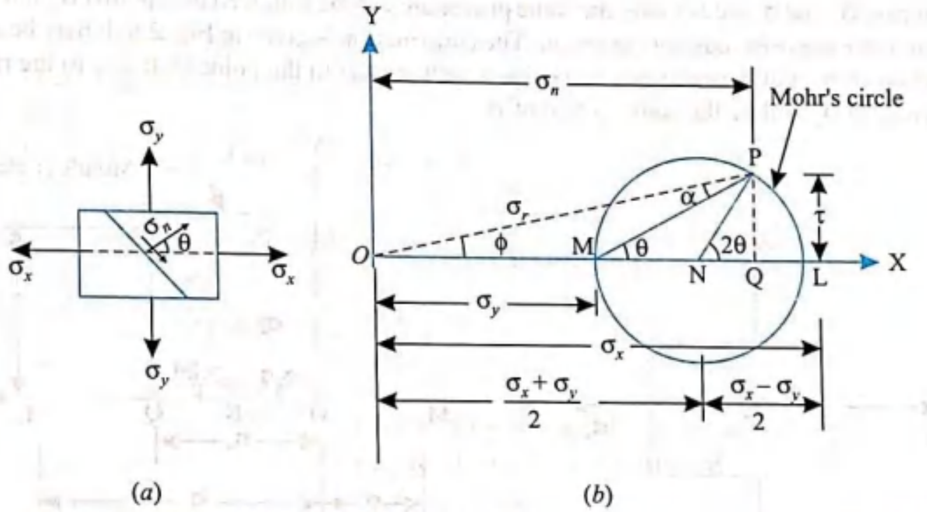


Fig. 2.7

1. Using some suitable scale, measure *OL* and *OM* equal to σ_x and σ_y respectively on the axis *OX*.
2. Bisect *LM* at *N*.
3. With *N* as centre and *NL* or *NM* radius, draw a circle.

4. At the centre N draw a line NP at an angle 2θ, in the same direction as the normal to the plane makes with the direction of σ_x. In Fig. 2.7 (*a*) which represents the stress system, the normal to the plane makes an angle θ with the direction of σ_x in the anticlockwise direction. The line NP therefore, is drawn in the anticlockwise direction.
5. From P, drop a perpendicular PQ on the axis OX. PQ will represent τ and OQ σ_n.

Now, from stress diagram

$$NP = NL = \frac{\sigma_x - \sigma_y}{2}$$

$$PQ = NP \sin 2\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta = \tau \qquad \text{(Eqn. 2.7)}$$

Similarly,
$$OQ = ON + NQ = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta = \sigma_n \qquad \text{(Eqn. 2.6)}$$

Also, from stress circle, τ is maximum when

$$2\theta = 90°, \text{ or } \theta = 45°$$

and,
$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} \qquad \text{(Eqn. 2.7 a)}$$

Sign conventions used:

(*i*) In order to mark τ in stress system, we will take the *clockwise shear as positive* and *anticlockwise shear as negative.*

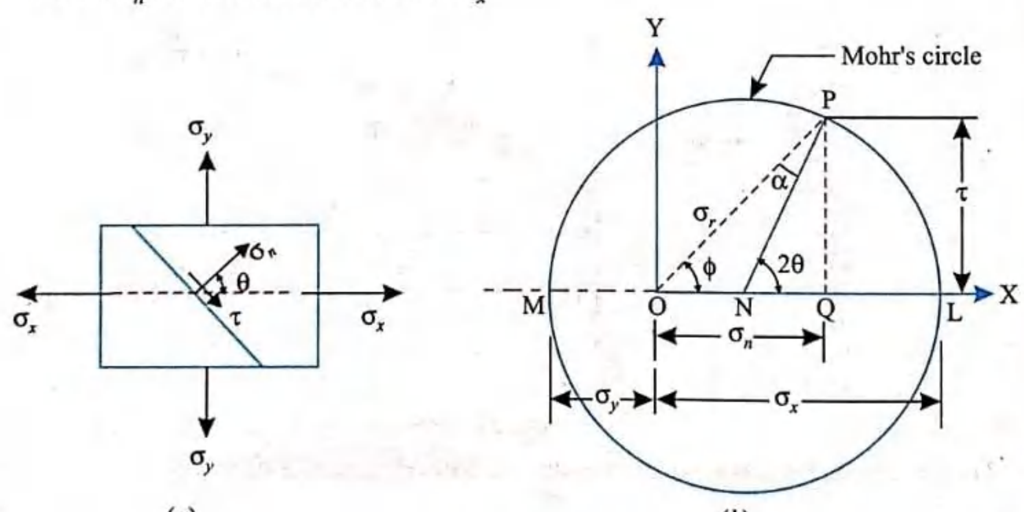
(*ii*) *Positive* values of τ will be *above* the axis and *negative* values *below* the axis.

(*iii*) If θ is in the anticlockwise direction, the radius vector will be above the axis and θ will be reckoned positive. If θ is in the clockwise direction, it will be negative and the radius vector will be below the axis.

(*iv*) *Tensile stress will be reckoned positive and will be plotted to the right of the origin O. Compressive stress will be reckoned negative and will be plotted to the left of the origin O.*

2.6.1.2. Mohr's circle construction for "unlike stresses"

In case σ_x and σ_y are *not like,* the same procedure will be followed except that σ_x and σ_y *will be measured to the opposite sides of the origin.* The construction is given in Fig. 2.8. It may be noted that the direction of σ_n will depend upon its position with respect to the point O. If it is to the right of O, the direction of σ_n will be the same as that of σ_x.



(*a*) (*b*)

Fig. 2.8

2.6.1.3. Mohr's circle construction for two perpendicular direct stresses with state of simple shear

Refer to Fig. 2.9. Following steps of construction are followed if the material is subjected to direct stresses σ_x and σ_y along with a state of simple shear:

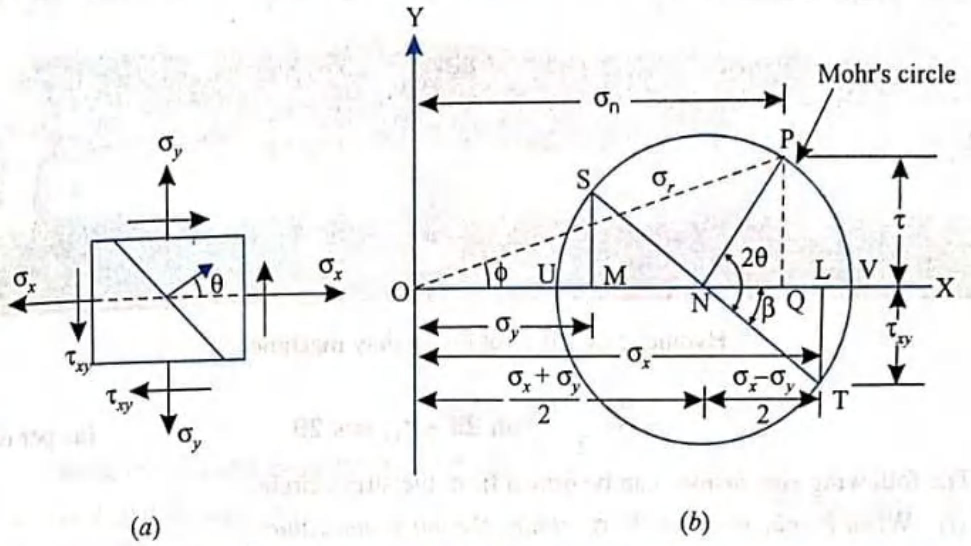


Fig. 2.9

1. Using some suitable scale, measure $OL = \sigma_x$ and $OM = \sigma_y$ along the axis OX.
2. At L draw LT perpendicular to OX and equal to τ_{xy}. LT has been drawn downward (as per sign conventions adopted) because τ_{xy} is acting up with respect to the plane across which σ_x is acting, tending to rotate it in the *anticlockwise direction* and is *negative*.
3. Similarly, make MS perpendicular to OX and equal to τ_{xy}, but above OX.
4. Join ST to cut the axis in N.
5. With N as centre and NS or NT as radius, draw a circle.
6. At N make NP at angle 2θ with NT in the anticlockwise direction.
7. Draw PQ perpendicular to the axis. PQ will give τ while OQ will give σ_n and OP will give σ_r.

Proof. Let the radius of the stress circle be R.

Then,
$$R = \sqrt{NL^2 + LT^2} = \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2}$$

Also,
$$R\cos\beta = NL = \frac{\sigma_x - \sigma_y}{2}$$
$$R\sin\beta = LT = \tau_{xy}$$

Now,
$$OQ = ON + NQ = ON + R\cos(2\theta - \beta)$$
$$= ON + R\cos 2\theta\cos\beta + R\sin 2\theta\sin\beta$$
$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{(as per eqn. 2.8)}$$
$$= \sigma_n$$

Similarly,
$$PQ = R\sin(2\theta - \beta) = R\sin 2\theta\cos\beta - R\cos 2\theta\sin\beta$$

Hydraulic cylinders of heavy duty machines.

$$= \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \qquad \text{(as per eqn. 2.9)}$$

The following *conclusions* can be drawn from the stress circle:

(i) When *P coincides with V,* σ_n *attains the maximum value.*

$$\sigma_{n(max)} = OV = ON + NV$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \qquad \left[\because NV = NT \text{ and, } NT^2 = NL^2 + LT^2\right]$$

$\sigma_{n\,(max)}$ (or σ_1) is known as *major principal stress.*

$$\tau = 0;\ \sigma_{r\,(max)} = \sigma_{n\,(max)}$$

$$\tan 2\theta = \tan \beta = \frac{\tau_{xy}}{\left[\frac{\sigma_x - \sigma_y}{2}\right]} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \text{(as per eqn. 2.10)}$$

(ii) When *P coincides with U,* σ_n *attains the minimum value,*

$$\sigma_{n\,(min)} = OU = ON - NU$$

$$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} \qquad \left[\because NU = NS = NT \text{ and } NU^2 = NT^2 = NL^2 + LT^2\right]$$

$\sigma_{n(min)}$ (or σ_2) is known as *minor principal stress.*

$$\tau = 0;\ \sigma_{r\,(min)} = \sigma_{n\,(min)};\ \theta = 90° + \beta/2$$

(iii) When $2\theta = \beta + 90°$, τ attains the maximum value,

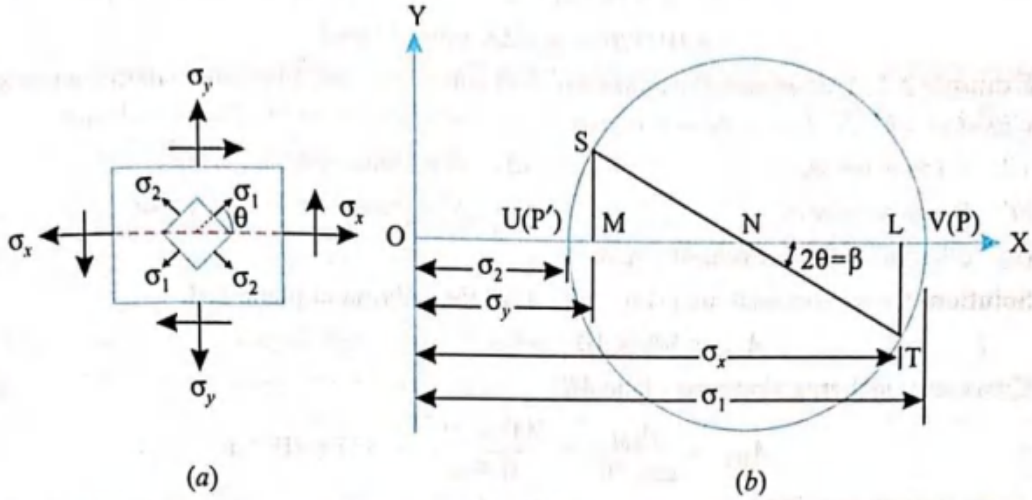
$$\tau_{max} = \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

When, $\quad 2\theta = \beta + 270°$

$$\tau_{max} = -\sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2}$$

2.6.1.4. Mohr's circle construction for principal stresses

Refer to Fig. 2.10. The following are the *steps* of construction :



(*a*) (*b*)

Fig. 2.10

1. Mark *OL* and *OM* proportional to σ_x and σ_y.
2. At *L* and *M*, erect perpendiculars *LT* = *MS* proportional to τ_{xy} in appropriate directions.
3. Join *ST*, intersecting the axis in *N*.

Since $\tau = 0$, *NV* represents the major principal plane, *P* coinciding with *V*. Similarly *NP′* represents minor principal plane, *P′* coinciding with *U*.

$$OV = ON + NV = \frac{\sigma_x + \sigma_y}{2} + R, \quad \text{where } R \text{ is the radius of the circle.}$$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_1$$

Similarly,
$$OU = ON - NU$$

$$= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_2$$

$$\tan \beta = \frac{LT}{LN} = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$$= \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \tan 2\theta \qquad \text{(where, } \beta = 2\theta\text{)}$$

The notch in the sample affects the results of the impact test, thus it is necessary for the notch to be of regular dimensions and geometry. The size of the sample can also affect results, since the dimensions determine whether or not the material is in plane strain. This difference can greatly affect the conclusions made.

CHAPTER-2 SIMPLE STRESS AND STRAIN BEHAVIOUR OF MATERIALS

1. Introduction

When a force is applied on a body it suffers a change in shape, that is, it deforms. A force to resist the deformation is also set up simultaneously within the body and it increases as the deformation continues. The process of deformation stops when the internal resisting force equals the externally applied force. If the body is unable to put up full resistance to external action, the process of deformation continues until failure takes place. The deformation of a body under external action and accompanying resistance to deform are referred to by the terms strain and stress respectively.

2. Stresses

Stress is defined as the internal resistance set up by a body when it is deformed. It is measured in N/m^2 and this unit is specifically called Pascal (Pa). A bigger unit of stress is the mega Pascal (MPa).

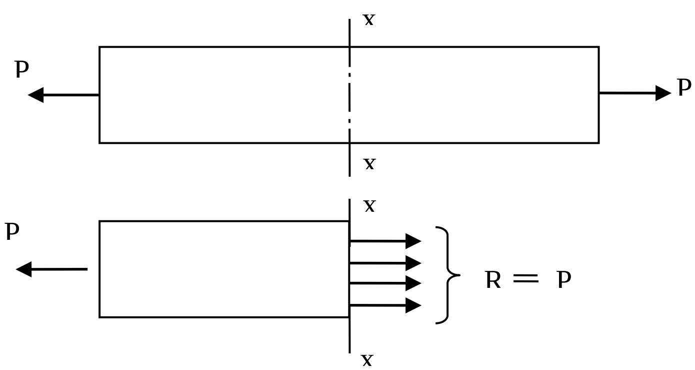
1 Pa = $1N/m^2$,

1MPa = 10^6 N/m^2 =$1N/mm^2$.

2.1. Three Basic Types of Stresses

Basically three different types of stresses can be identified. These are related to the nature of the deforming force applied on the body. That is, whether they are tensile, compressive or shearing.

2.1.1. Tensile Stress



Consider a uniform bar of cross sectional area A subjected to an axial tensile force P. The stress at any section x-x normal to the line of action of the tensile force P is specifically called tensile stress p_t . Since internal resistance R at x-x is equal to the applied force P, we have,

p_t = (internal resistance at x-x)/(resisting area at x-x)

=R/A

=P/A.

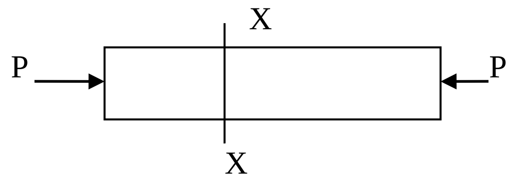
Under tensile stress the bar suffers stretching or elongation.

2.1.2. Compressive Stress

If the bar is subjected to axial compression instead of axial tension, the stress developed at x-x is specifically called compressive stress p_c.

p_c =R/A

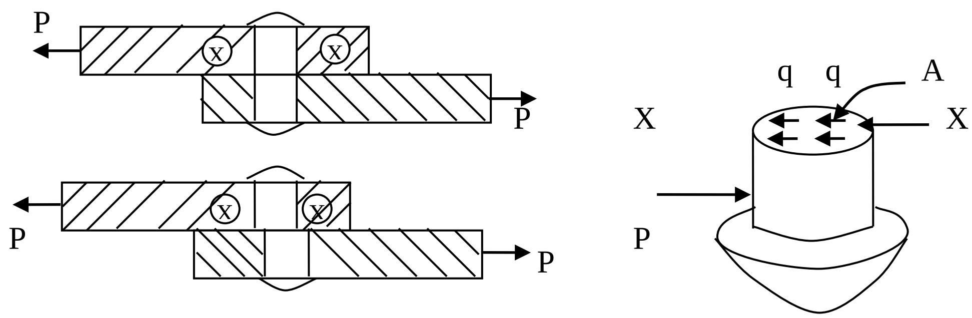
= P/A.



Under compressive stress the bar suffers shortening.

2.1.3. Shear Stress

Consider the section x-x of the rivet forming joint between two plates subjected to a tensile force P as shown in figure.



The stresses set up at the section x-x acts along the surface of the section, that is, along a direction tangential to the section. It is specifically called shear or tangential stress at the section and is denoted by q.

$q = R/A$

$= P/A.$

2.2. Normal or Direct Stresses

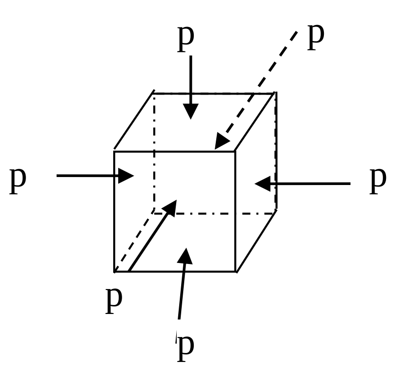
When the stress acts at a section or normal to the plane of the section, it is called a normal stress or a direct stress. It is a term used to mean both the tensile stress and the compressive stress.

2.3. Simple and Pure Stresses

The three basic types of stresses are tensile, compressive and shear stresses. The stress developed in a body is said to be simple tension, simple compression and simple shear when the stress induced in the body is (a) single and (b) uniform. If the condition (a) alone is satisfied, the stress is called pure tension or pure compression or pure shear, as the case may be.

2.4. Volumetric Stress

Three mutually perpendicular like direct stresses of same intensity produced in a body constitute a volumetric stress. For example consider a body in the shape of a cube subjected equal normal pushes on all its six faces. It is now subjected to equal compressive stresses p in all the three mutually perpendicular directions. The body is now said to be subjected to a volumetric compressive stress p.



Volumetric stress produces a change in volume of the body without producing any distortion to the shape of the body.

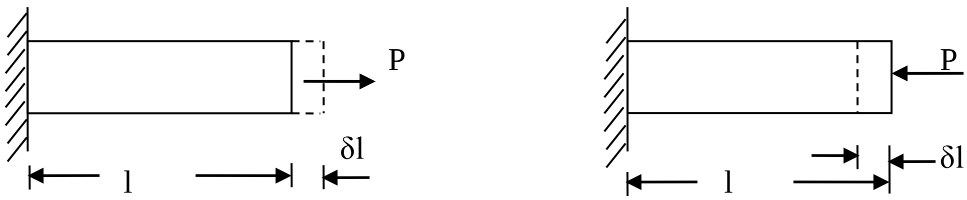
3. Strains

Strain is defined a the ratio of change in dimension to original dimension of a body when it is deformed. It is a dimensionless quantity as it is a ratio between two quantities of same dimension.

3.1. Linear Strain

Linear strain of a deformed body is defined as the ratio of the change in length of the body due to the deformation to its original length in the direction of the force. If l is the original length and δl the change in length occurred due to the deformation, the linear strain e induced is given by

$$e = \delta l / l.$$



Linear strain may be a tensile strain , $\mathbf{e_t}$ or a compressive strain $\mathbf{e_c}$ according as δl refers to an increase in length or a decrease in length of the body. If we consider one of these as +ve then the other should be considered as –ve, as these are opposite in nature.

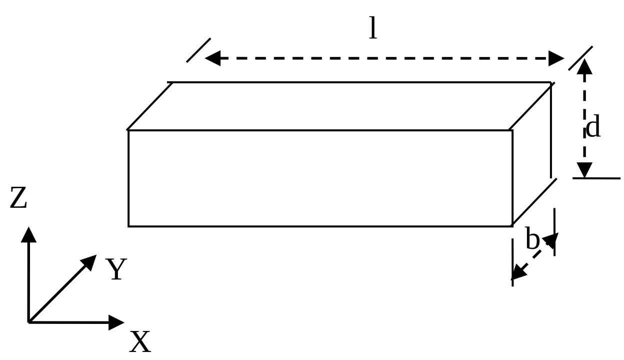
3.2.Lateral Strain

Lateral strain of a deformed body is defined as the ratio of the change in length (breadth of a rectangular bar or diameter of a circular bar) of the body due to the deformation to its original length (breadth of a rectangular bar or diameter of a circular bar) in the direction perpendicular to the force.

3.3. Volumetric Strain

Volumetric strain of a deformed body is defined as the ratio of the change in volume of the body to the deformation to its original volume. If V is the original volume and δV the change in volume occurred due to the deformation, the volumetric strain e_v induced is given by $\mathbf{e_v = \delta V/V}$

Consider a uniform rectangular bar of length l, breadth b and depth d as shown in figure. Its volume V is given by,



$V = lbd$

$\delta V = \delta l\, bd + \delta b\, ld + \delta d\, lb$

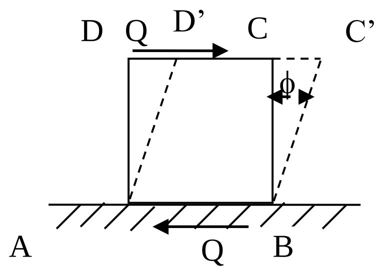
$\delta V / V = (\delta l / l) + (\delta b / b) + (\delta d / d)$

$e_v = e_x + e_y + e_z$

This means that volumetric strain of a deformed body is the sum of the linear strains in three mutually perpendicular directions.

3.4. Shear Strain

Shear strain is defined as the strain accompanying a shearing action. It is the angle in radian measure through which the body gets distorted when subjected to an external shearing action. It is denoted by Φ.



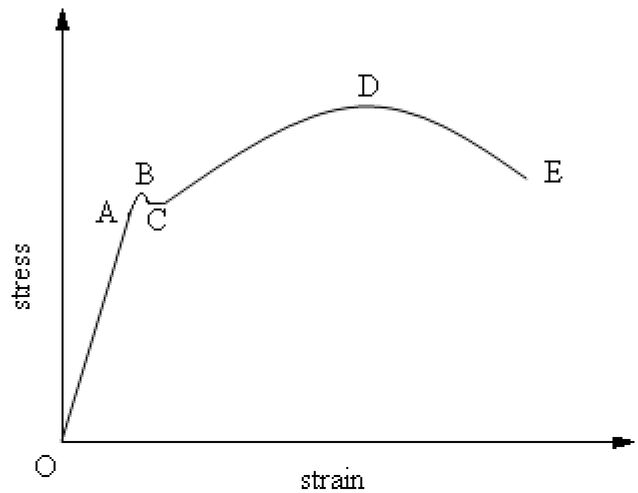
Consider a cube ABCD subjected to equal and opposite forces Q across the top and bottom forces AB and CD. If the bottom face is taken fixed, the cube gets distorted through angle ϕ to the shape ABC'D'. Now strain or deformation per unit length is

Shear strain of cube = CC' / CD = CC' / BC = ϕ radian

4. Relationship between Stress and Strain

Relationship between Stress and Strain are derived on the basis of the elastic behaviour of material bodies.

A standard mild steel specimen is subjected to a gradually increasing pull by Universal Testing Machine. The stress-strain curve obtained is as shown below.



A -Elastic Limit
B - Upper Yield Stress
C - Lower Yield Stress
D -Ultimate Stress
E -Breaking Stress

4.1. Elasticity and Elastic Limit

Elasticity of a body is the property of the body by virtue of which the body regains its original size and shape when the deformation force is removed. Most materials are elastic in nature to a lesser or greater extend, even though perfectly elastic materials are very rare.

The maximum stress upto which a material can exhibit the property of elasticity is called the elastic limit. If the deformation forces applied causes the stress in the material to exceed the elastic limit, there will be a permanent set in it. That is the body will not regain its original shape and size even after the removal of the deforming force completely. There will be some residual strain left in it.

Yield stress

When a specimen is loaded beyond the elastic limit the stress increases and reach a point at which the material starts yielding this stress is called yield stress.

Ultimate stress

Ultimate load is defined as maximum load which can be placed prior to the breaking of the specimen. Stress corresponding to the ultimate load is known as ultimate stress.

Working stress

Working stress= Yield stress/Factor of safety.

4.2. Hooke's Law

Hooke's law states that stress is proportional to strain upto elastic limit. If I is the stress induced in a material and e the corresponding strain, then according to Hooke's law,

$p / e = E$, a constant.

This constant E is called the modulus of elasticity or Young's Modulus, (named after the English scientist Thomas Young).

It has later been established that Hooke's law is valid only upto a stress called the limit of proportionality which is slightly less than the elastic limit.

4.3. Elastic Constants

Elastic constants are used to express the relationship between stresses and strains. Hooke's law , is stress/strain = a constant, within a certain limit. This means that any stress/corresponding strain = a constant, within certain limit. It follows that there can be three different types of such constants. (which we may call the elastic constants or elastic modulae) corresponding to three distinct types of stresses and strains. These are given below.

(i)Modulus of Elasticity or Young's Modulus (E)

Modulus of Elasticity is the ratio of direct stress to corresponding linear strain within elastic limit. If p is any direct stress below the elastic limit and e the corresponding linear strain, then $\mathbf{E = p / e}$.

(ii)Modulus of Rigidity or Shear Modulus (G)

Modulus of Rigidity is the ratio of shear stress to shear strain within elastic limit. It is denoted by N,C or G. if q is the shear stress within elastic limit and ϕ the corresponding shear strain, then $\mathbf{G = q / \phi}$.

(iii) Bulk Modulus (K)

Bulk Modulus is the ratio of volumetric stress to volumetric strain within the elastic limit. If p_v is the volumetric stress within elastic limit and e_v the corresponding volumetric strain, we have $K = p_v / e_v$.

5. Poisson's Ratio

Any direct stress is accompanied by a strain in its own direction and called linear strain and an opposite kind strain in every direction at right angles to it, lateral strain. This lateral strain bears a constant ratio with the linear strain. This ratio is called the Poisson's ratio and is denoted by (1/m) or μ.

Poisson's Ratio = Lateral Strain / Linear Strain.

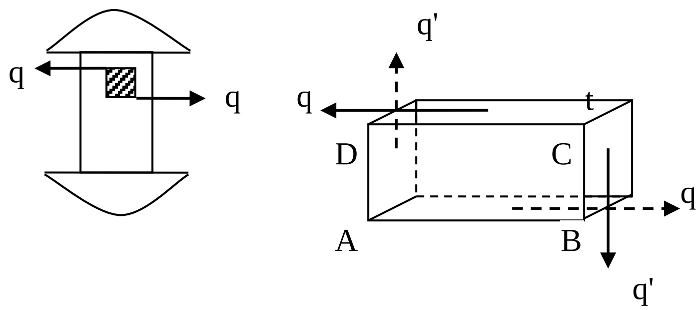
Value of the Poisson's ratio for most materials lies between 0.25 and 0.33.

6. Complementary Strain

Consider a rectangular element ABCD of a body subjected to simple shear of intensity q as shown. Let t be the thickness of the element.

Total force on face AB is , F_{AB} = stress X area = q X AB X t.

Total force on face CD is, F_{CD} = q X CD Xt = q XAB Xt.



FAB and FCD being equal and opposite, constitute a couple whose moment is given by,

$M = F_{AB}$ X BC = q XAB X BC X t

Since the element is in equilibrium within the body, there must be a balancing couple which can be formed only by another shear stress of some intensity q' on the faces BC and DA. This shear stress is called the complementary stress.

F_{BC} = q' X BC X t

F_{DA} = q' X DA X t = q' X BC Xt

The couple formed by these two forces is M' = F_{BC} X AB = q' X BC X t

For equilibrium, M' = M.

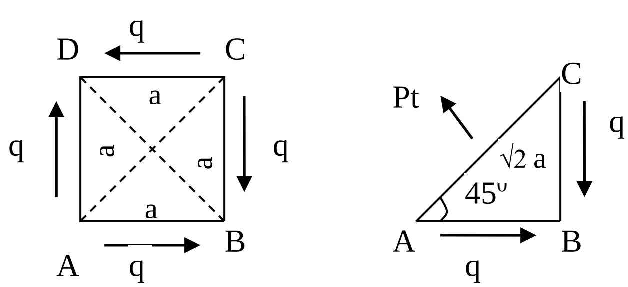
Therefore q' = q

This enables us to make the following statement.

In a state of simple shear a shear stress of any intensity along a plane is always accompanied by a complementary shear stress of same intensity along a plane at right angles to the plane.

7. Direct Stresses Developed Due to Simple Shear.

Consider a square element of side a and thickness t in a state of simple shear as shown in figure. It is clear that the shear stress on the forces of element causes it to elongate in the direction of the diagonal BD. Therefore a tensile stress of same intensity pt is induced in the elements along BD. ie, across the plane of the diagonal AC. The triangular portion ABC of the element is in equilibrium under the action of the following.



(1)F_{AC} = Normal force on face AC = pt X AC X t = pt X √2 aXt

(2)F_{AB} = Tangential force on face AB = q X BC X t = q aXt

(3)FBC = Tangential force on face BC = q X BC X t = q aXt

For equilibrium in the direction normal to AC,

FAC – F_{AB} cos45 – F_{BC} cos45 = 0

Pt X √2 at – q at X 1/√2 - q at X 1/√2 = 0

√2 pt – 2 q /√2 = 0

pt = q

It can also be seen that the shear stress on the faces of the element causes it to foreshorten in the direction of the diagonal BD. Therefore a compressive stress pc is induced in the element in the direction AC, ie across the plane of the diagonal BD. It can also be shown that pc = q.

It can thus be concluded that simple shear of any intensity gives rise to direct stresses of same intensity along the two planes inclined at 45º to the shearing plane. The stress along one of these planes being tensile and that along the other being compressive.

8. Relationship among the elastic constants

8.1. Relationship between modulus of elasticity and modulus of rigidity

Consider a square element ABCD of side 'a' subjected to simple shear of intensity q as shown in figure.

It is deformed to the shape AB'C'D under the shear stress. Drop perpendicular BE to the diagonal DB'. Let Φ be the shear strain induced and let N be the modulus of rigidity.

The diagonal DB gets elongated to DB'. Hence there is tensile strain et in the diagonal.

et = (DB' – DB) / DB = EB' / DB

since this deformation is very small we can take L BB'E = 45º

EB' = BB' / √2 = AB tan Φ / √2 = a tan Φ / √2

DB = √2 a

et = (a tan Φ / √2)/ √2 a = tan Φ /2 = = Φ / 2 since Φ is small

ie et = ½ X q/N ------------------ (1)

We know that stress along the diagonal DB is a pure tensile stress pt = q and that along the diagonal AC is a pure compressive stress pc also equal to q. hence the strain along the diagonal DB is et = q/E + 1/m X q/E

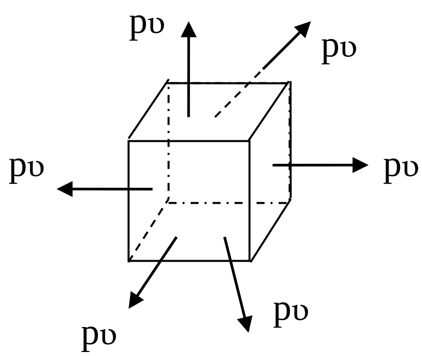
Ie et = q/E (1+1/m) ------------------ (2)

From (1) and (2) we have,

E = 2N(1+1/m)

This is the required relationship between E and N.

8.2. Relationship between Modulus of Elasticity E and Bulk Modulus K



Consider a cube element subjected to volumetric tensile stress pυ in X,Y and Z directions. Stress in each direction is equal to pυ. ie px = py = pz =pυ

Consider strains induced in X-direction by these stresses. px induces tensile strain , while py and pz induces compressive strains. Therefore,

ex = px/E – 1/m[py/E + pz/E] = pυ/E[1-2/m]

due to the perfect symmetry in geometry and stresses

ey = pυ/E[1-2/m]

ez = pυ/E[1-2/m]

K = p υ / e υ =p υ/(ex+ey+ez) = pυ/[3pυ/E(1-2/m)]

ie E = 3K(1-2/m) is the required relationship.

8.3. Relationship among the constants

From above,

E = 2N[1+(1/m)] and E = 3K[1-(2/m)]

E = 3K[1-2(E/2N -1)] = 3K[1-E/N +2]
9K = E[1+(3K/N)] = E[(N+3K)/N]
E = 9NK/(N+3K)

9. Bars of uniform section

Consider a bar of length l and Cross sectional area A. Let P be the axial pull on the bar,p the stresss induced ,e the strain in the bar and δl is the elongation.

Then p= P/A

e= p/E =P/(AE) -------------(1)

e= δl/l -------------(2)

equating (1) and (2)

δl = Pl / (AE)

TYPES OF LOADS

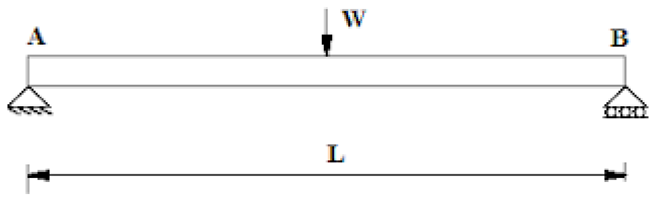
A beam is usually horizontal member and load which will be acting over the beam will be usually vertical loads. There are following types of loads as mentioned here and we will discuss each type of load in detail.

- Point load or concentrated load
- Uniformly distributed load
- Uniformly varying load

- **Point load or concentrated load**

  Point load or concentrated load, as name suggest, acts at a point on the beam. If we will see practically, point load or concentrated load also distributed over a small area but we can consider such type of loading as point loading and hence such type of load could be considered as point load or concentrated load.

  Following figure displayed here indicates the beam AB of length L which will be loaded with point load W at the midpoint of the beam. Load W will be considered here as the point load.

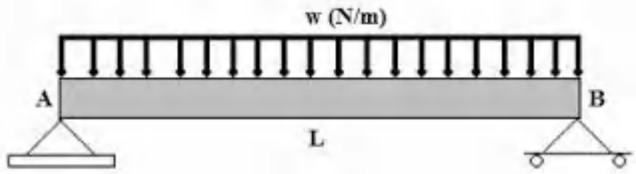


- **Uniformly distributed load**

  Uniformly distributed load is the load which will be distributed over the length of the beam in such a way that rate of loading will be uniform throughout the distribution length of the beam.

  Uniformly distributed load is also expressed as U.D.L and with value as w N/m. During determination of the total load, total uniformly distributed load will be converted in to point load by multiplying the rate of loading i.e. w (N/m) with the span of load distribution i.e. L and will be acting over the midpoint of the length of the uniformly load distribution.

  Let us consider the following figure, a beam AB of length L is loaded with uniformly distributed load and rate of loading is w (N/m).



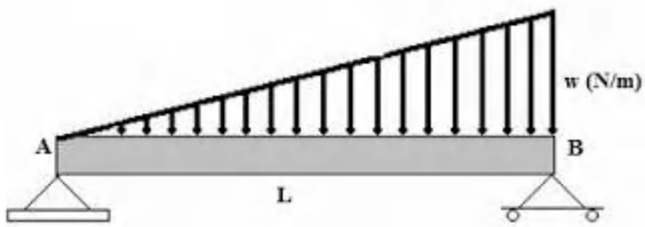
Total uniformly distributed load, P = w*L

- **Uniformly varying load**

  Uniformly varying load is the load which will be distributed over the length of the beam in such a way that rate of loading will not be uniform but also vary from point to point throughout the distribution length of the beam.

  Uniformly varying load is also termed as triangular load. Let us see the following figure, a beam AB of length L is loaded with uniformly varying load.

  We can see from figure that load is zero at one end and increases uniformly to the other end. During determination of the total load, we will determine the area of the triangle and the result i.e. area of the triangle will be total load and this total load will be assumed to act at the C.G of the triangle.



Total load, P = w*L/2

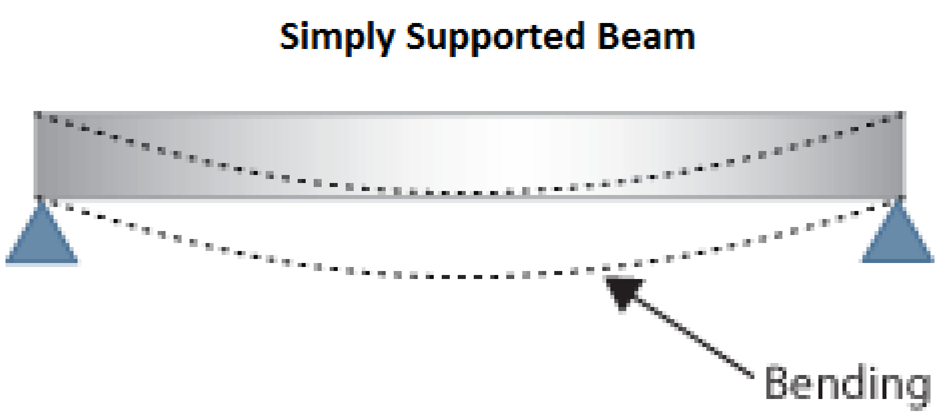
TYPES OF BEAMS

The four different types of beams are:

1. Simply Supported Beam
2. Fixed Beam
3. Cantilever Beam
4. Continuously Supported Beam

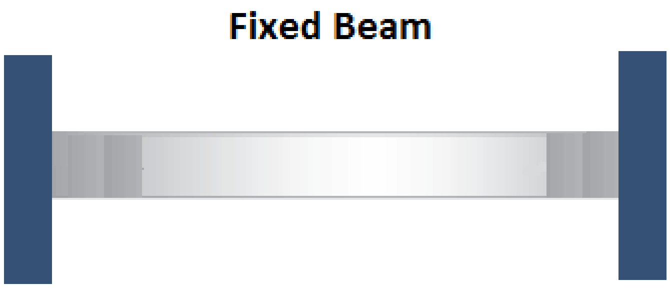
1. Simply Supported Beam

If the ends of a beam are made to rest freely on supports beam, it is called a simple (freely) supported beam.



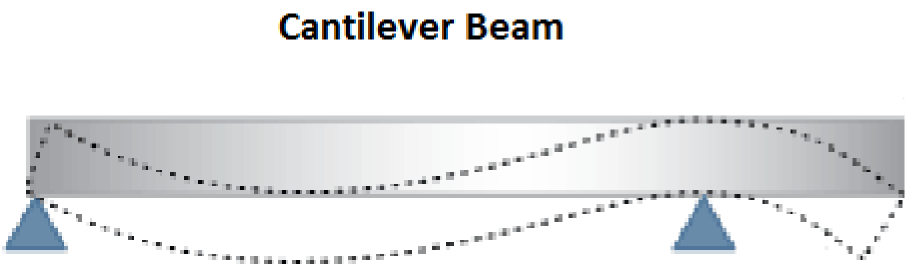
2. Fixed Beam

If a beam is fixed at both ends it is free called fixed beam. Its another name is a built-in beam.



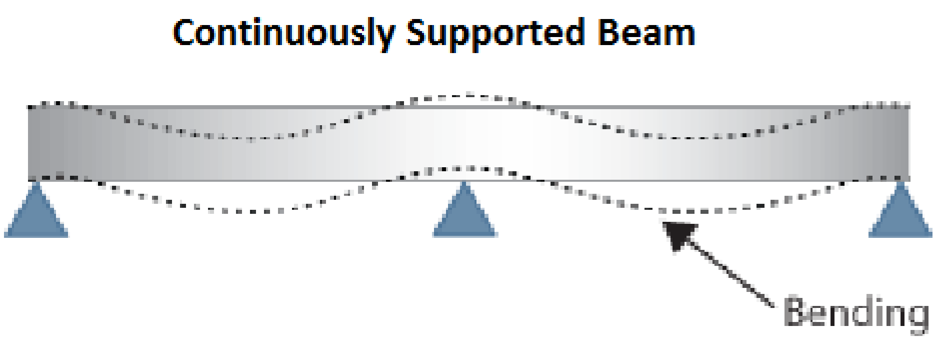
3. Cantilever Beam

If a beam is fixed at one end while the other end is free, it is called cantilever beam.



4. Continuously Supported Beam

If more than two supports are provided to the beam, it is called continuously supported beam.

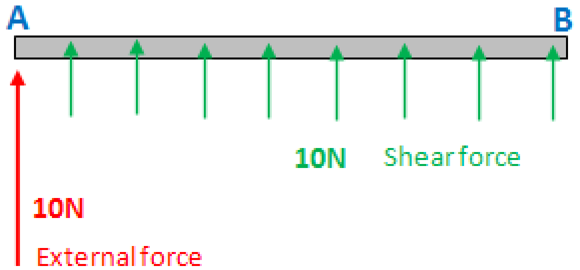


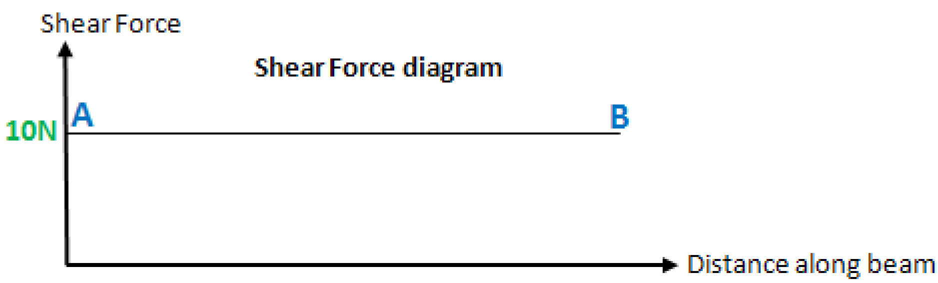
<u>TYPES OF SUPPORT</u>

Different types of external supports are as follows:

- Fixed support
- Pinned support or hinged support
- Roller support
- Link support
- Simple support

Below a force of 10N is exerted at point A on a beam. This is an external force. However because the beam is a rigid structure,the force will be internally transferred all along the beam. This internal force is known as shear force. The shear force between point A and B is usually plotted on a shear force diagram. As the shear force is 10N all along the beam, the plot is just a straight line, in this example.



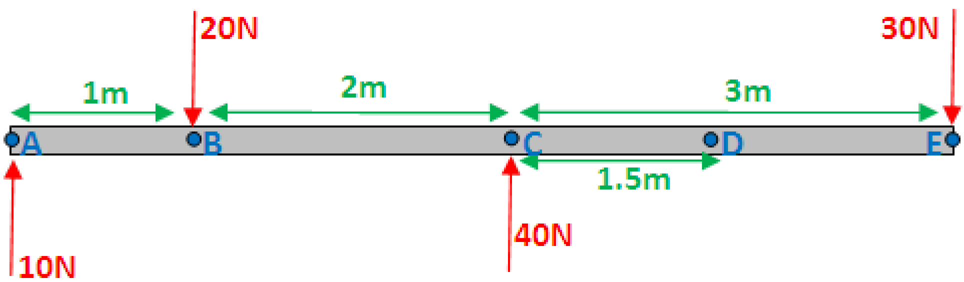


The idea of shear force might seem odd, maybe this example will help clarify. Imagine pushing an object along a kitchen table, with a 10N force. Even though you're applying the force only at one point on the object, it's not just that point of the object that moves forward. The whole object moves forward, which tells you that the force must have transferred all along the object, such that every atom of the object is experiencing this 10N force.

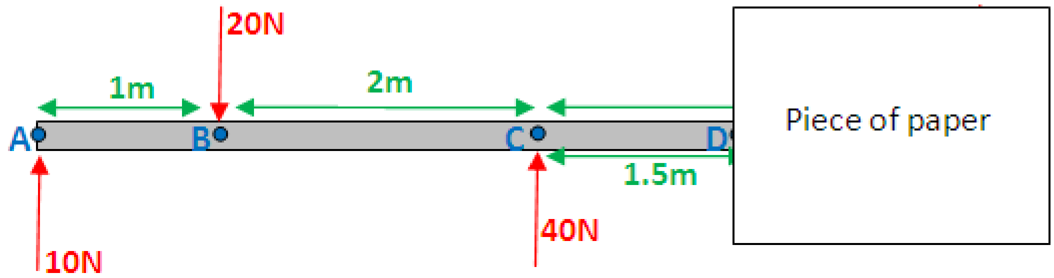
Please note that this is not the full explanation of what shear force actually is.

Basic shear diagram

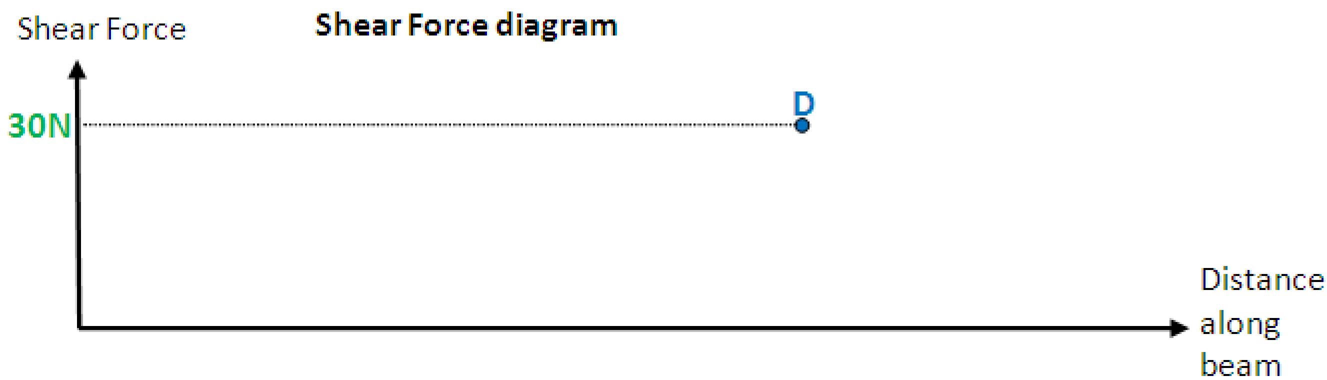
What if there is more than one force, as shown in the diagram below, what would the shear force diagram look like then?



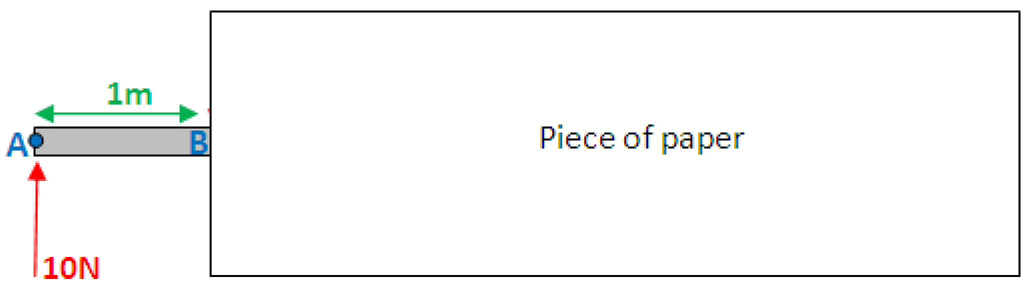
The way you go about this is by figuring out the shear force at points A,B,C,E (as there is an external force acting at these points). The way you work out the shear force at any point, is by covering (either with your hand or a piece of paper), everything to right of that point, and simply adding up the external forces. Then plot the point on the shear force diagram. For illustration purposes, this is done for point D:



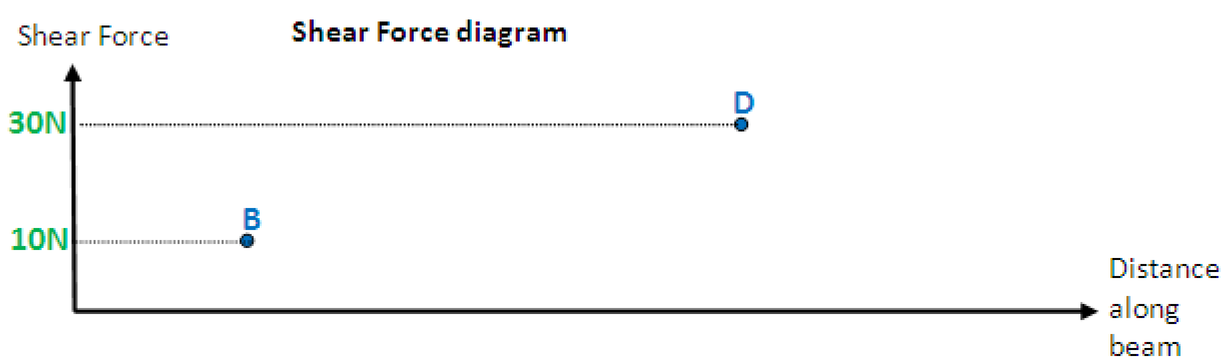
Shear force at D = 10N - 20N + 40N = 30Newtons



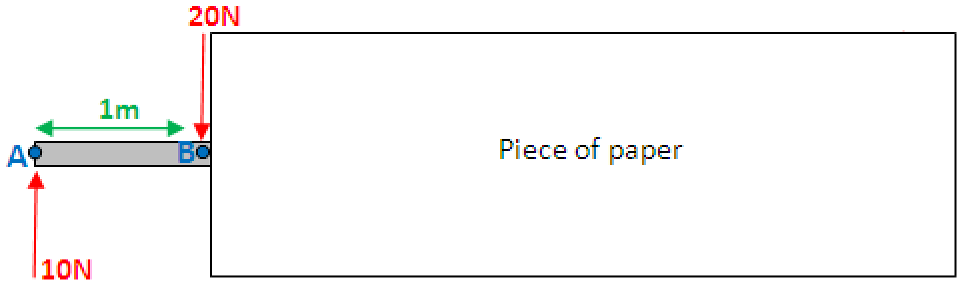
Now, let's do this for point B. BUT - slight complication - there's a force acting at point B, are you going to include it? The answer is both yes and no. You need to take 2 measurements. Firstly put your piece of paper, so it's JUST before point B:



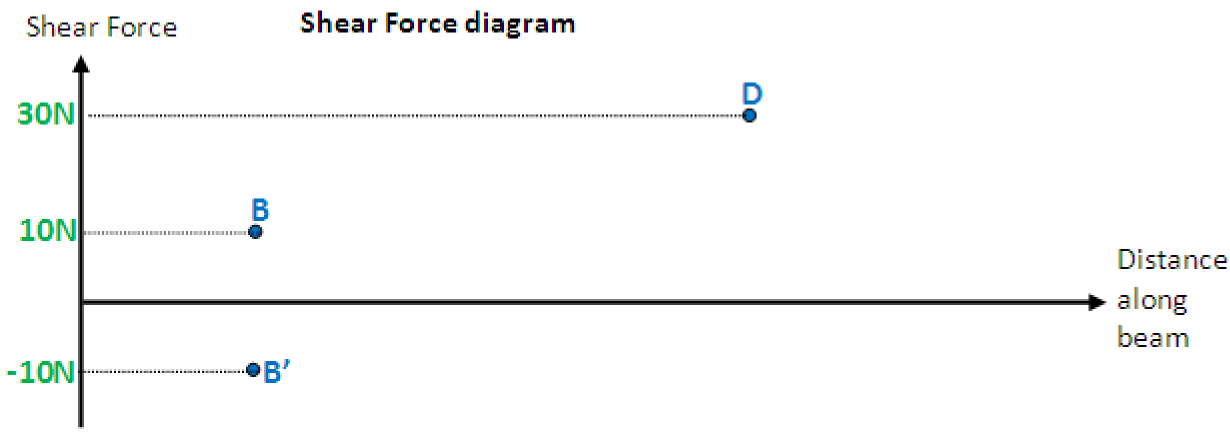
Shear force at B = 10N



Now place your paper JUST after point B:

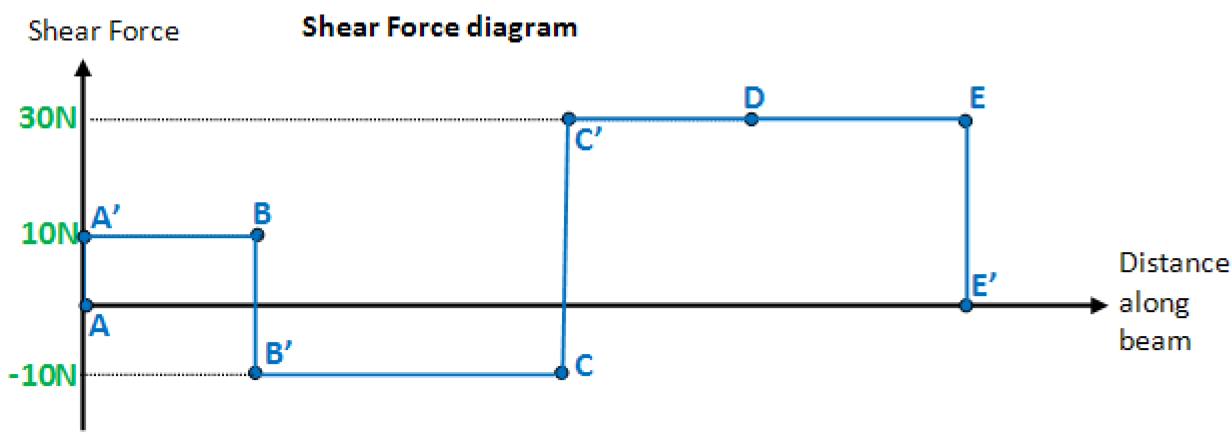


Shear force at B = 10N - 20N = -10N



(B' is vertically below B)

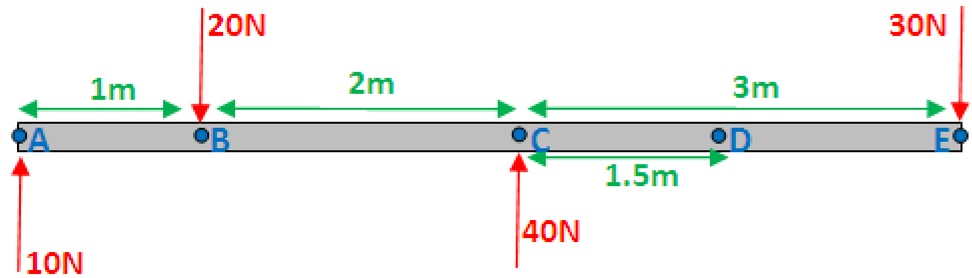
Now, do point A, D and E, and finally join the points. your diagram should look like the one below. If you don't understand why, leave a message on the discussion section of this page (its at the top), I will elaborate on the explanation:



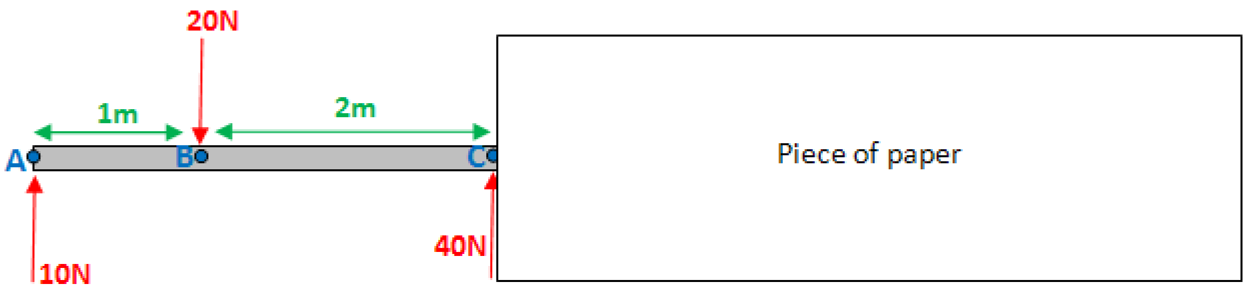
Notice how nothing exciting happens at point D, which is why you wouldn't normally analyse the shear force at that point. For clarity, when doing these diagrams it is recommended you move you paper from left to right, and hence analyse points A,B, C, and E, in that order. You can also do this procedure covering the left side instead of the right, your diagram will be "upside down" though. Both diagrams are correct.

Basic bending moment diagram

Bending moment refers to the internal moment that causes something to bend. When you bend a ruler, even though apply the forces/moments at the ends of the ruler, bending occurs all along the ruler, which indicates that there is a bending moment acting all along the ruler. Hence bending moment is shown on a bending moment diagram. The same case from before will be used here:

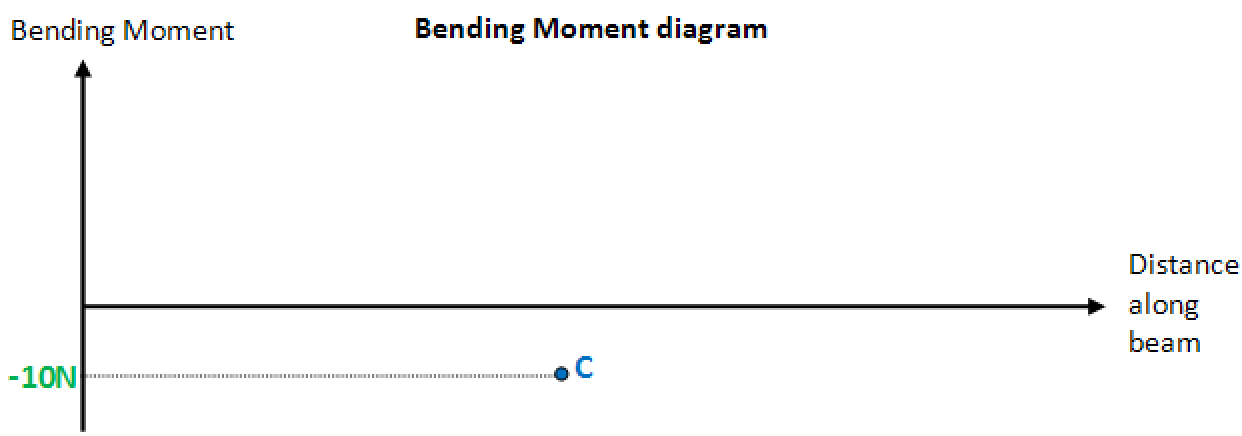


To work out the bending moment at any point, cover (with a piece of paper) everything to the right of that point, and take moments about that point. (I will take clockwise moments to be positive). To illustrate, I shall work out the bending moment at point C:



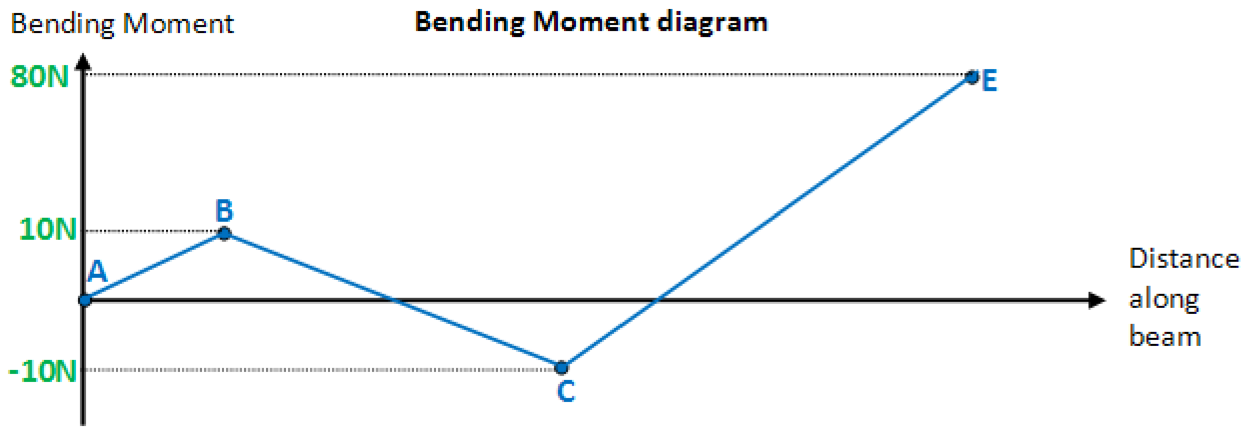
Bending moment at C = 10Nx3m - 20Nx2m = -10Nm

(Please note that the two diagrams below should show units in "Nm", not in "N" as it is currently showing)



Notice that there's no need to work out the bending moment "just before and just after" point C, (as in the case for the shear force diagram). This is because the 40N force at point C exerts no moment about point C, either way.

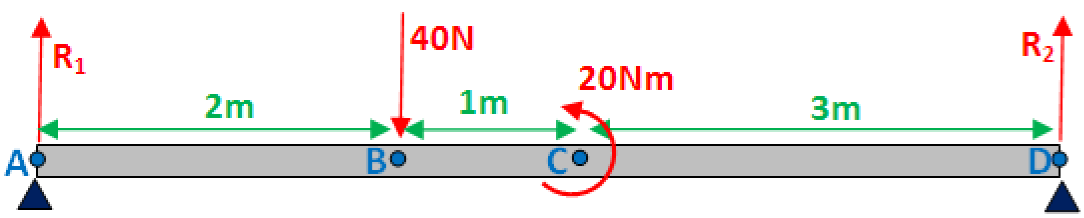
Repeating the procedure for points A,B and E, and joining all the points:



Normally you would expect the diagram to start and end at zero, in this case it doesn't. This is my fault, and it happened because I accidentally chose my forces such that there is a moment disequilibrium. i.e. take moments about any point (without covering the right of the point), and you'll notice that the moments aren't balanced, as they should be. It also means that if you're covering the left side as opposed to the right, you will get a completely different diagram. Sorry about this... Upon inspection, the forces are unbalanced, so it is immediately expected that the diagram will most likely not be balanced.

Point moments

Point moments are something that you may not have come across before. Below, a point moment of 20Nm is exerted at point C. Work out the reaction of A and D:



Force equilibrium: $R_1 + R_2 = 40$

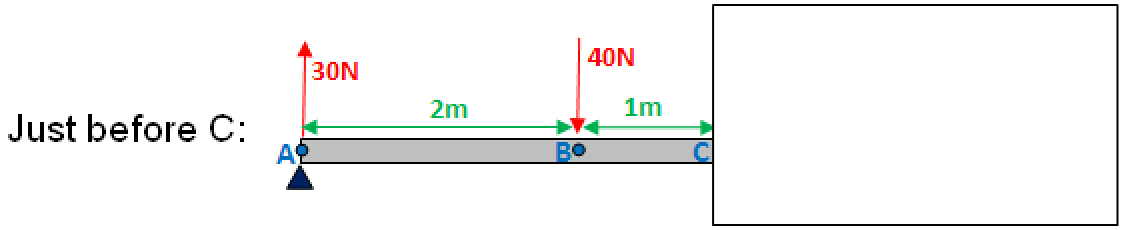
Taking moments about A (clockwise is positive): $40{\cdot}2 - 20 - 6{\cdot}R_2 = 0$

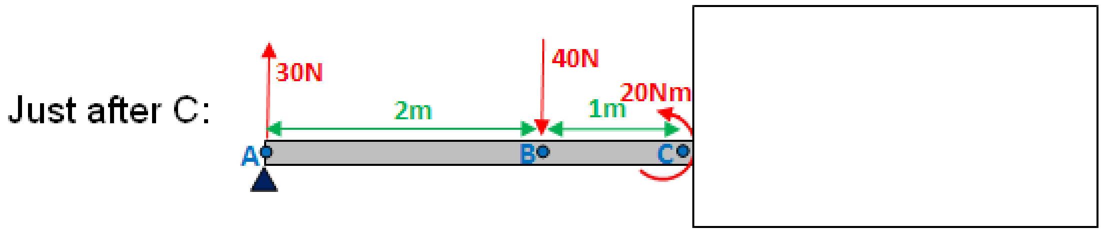
$R_1 = 30N$, $R_2 = 10N$

If instead you were to take moments about D you would get: $- 20 - 40{\cdot}4 + 6{\cdot}R_1 = 0$

I think it's important for you to see that wherever you take moments about, the point moment is always taken as a negative (because it's a counter clockwise moment).

So how does a point moment affect the shear force and bending moment diagrams?
Well. It has absolutely no effect on the shear force diagram. You can just ignore point C when drawing the shear force diagram. When drawing the bending moment diagram you will need to work out the bending moment just before and just after point C:





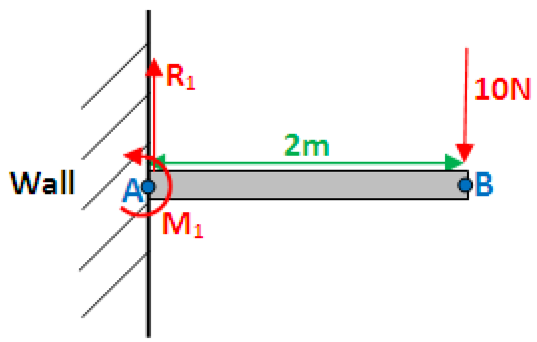
Just before: bending moment at C = 3·30 - 1·40 = 50Nm

Just after: bending moment at C = 3·30 - 1·40 - 20 = 30Nm

Then work out the bending moment at points A, B and D (no need to do before and after for these points). And plot.

Cantilever beam

Until now, you may have only dealt with "simply supported beams" (like in the diagram above), where a beam is supported by 2 pivots at either end. Below is a cantilever beam, which means - a beam that rigidly attached to a wall. Just like a pivot, the wall is capable of exerting an upwards reaction force R_1 on the beam. However it is also capable of exerting a point moment M_1 on the beam.

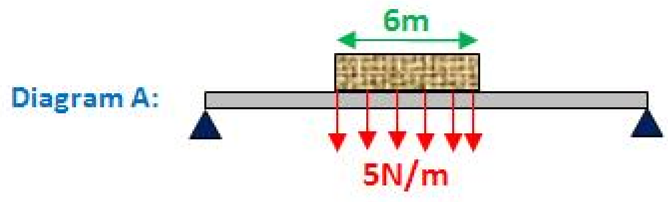


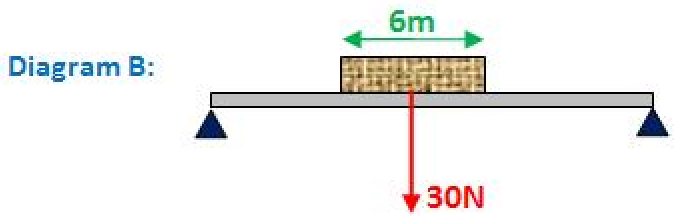
Force equilibrium: $R_1 = 10N$

Taking moments about A: $-M_1 + 10 \cdot 2 = 0 \rightarrow M_1 = 20Nm$

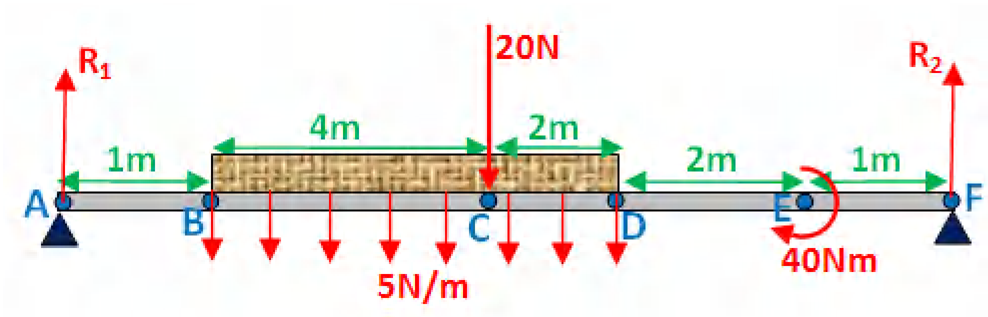
Uniformly Distributed Load (UDL)

Below is a brick lying on a beam. The weight of the brick is uniformly distributed on the beam (shown in diagram A). The brick has a weight of 5N per meter of brick (5N/m). Since the brick is 6 meters long the total weight of the brick is 30N. This is shown in diagram B. Diagram B is a simplification of diagram A. As you will see, you will need to be able to convert a type A diagram to a type B.





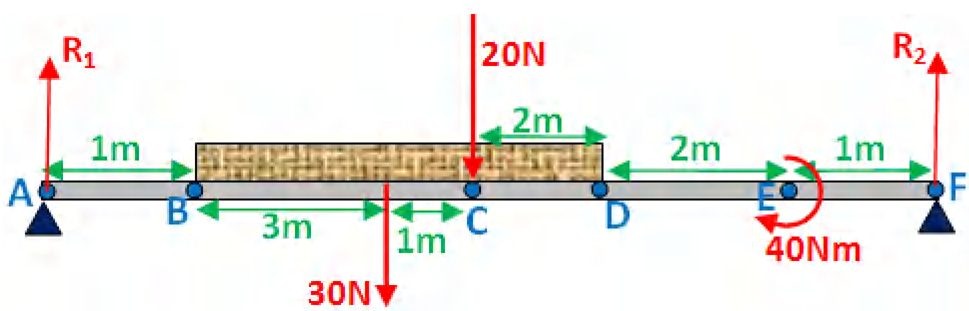
To make your life more difficult I have added an external force at point C, and a point moment to the diagram below. This is the most difficult type of question I can think of, and I will do the shear force and bending moment diagram for it, step by step.



Firstly identify the key points at which you will work out the shear force and bending moment at. These will be points: A,B,C,D,E and F.

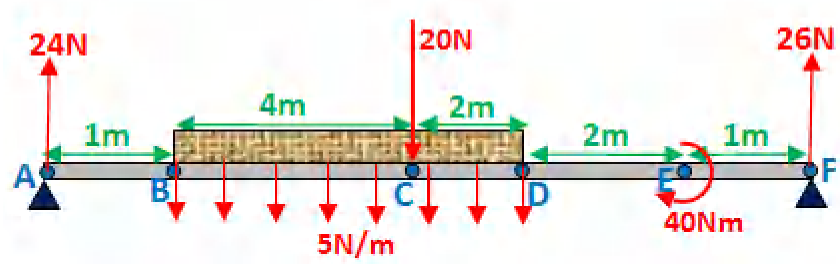
As you would have noticed when working out the bending moment and shear force at any given point, sometimes you just work it out at the point, and sometimes you work it out just before and after. Here is a summary: When drawing a shear force diagram, if you are dealing with a point force (points A,C and F in the above diagram), work out the shear force before and after the point. Otherwise (for points B and D), just work it out right at that point. When drawing a bending moment diagram, if you are dealing with a point moment (point E), work out the bending moment before and after the point. Otherwise (for points A,B,C,D, and F), work out the bending moment at the point.

After identifying the key points, you want to work out the values of R_1 and R_2. You now need to convert to a type B diagram, as shown below. Notice the 30N force acts right in the middle between points B and D.



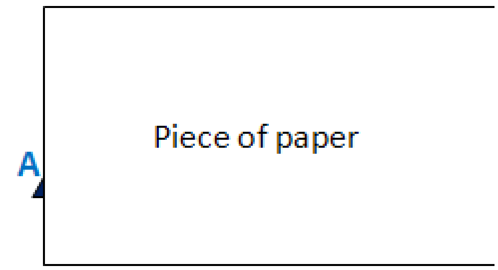
Force equilibrium: $R_1 + R_2 = 50$
Take moments about A: $4 \cdot 30 + 5 \cdot 20 + 40 - 10 \cdot R_2 = 0$
$R_1 = 24N$, $R_2 = 26N$

Update original diagram:

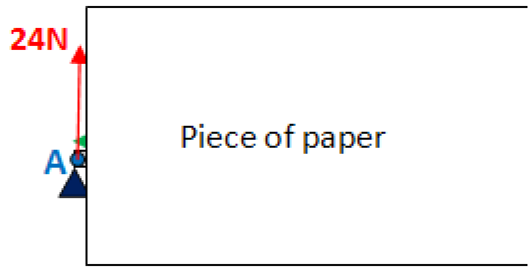


Shear force diagram

point A:

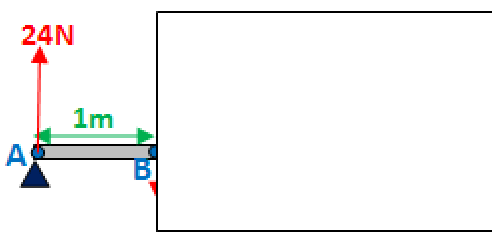


Before A – shear force = 0N



After A – shear force = 24N

point B:

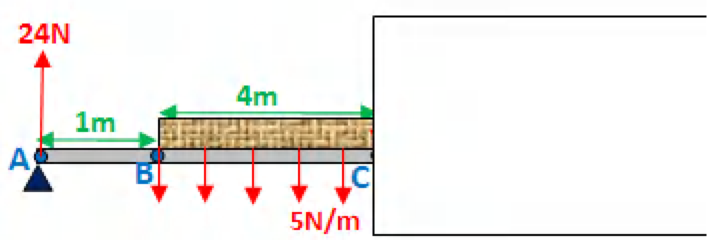


Shear force at B = 24N

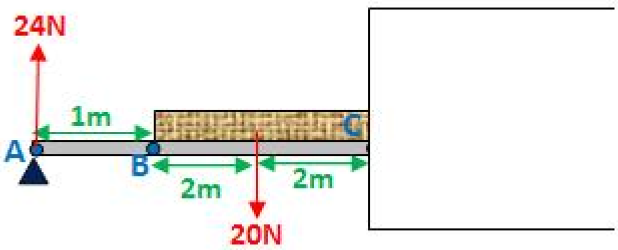
Notice that the uniformly distributed load has no effect on point B.

point C:

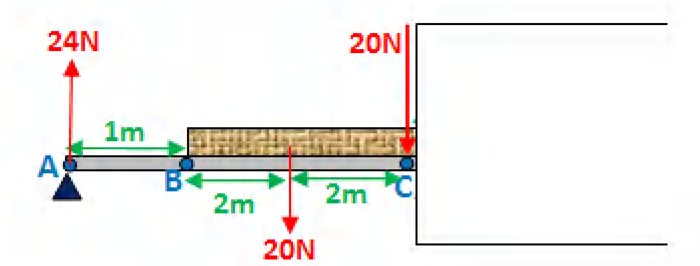
Just before C:



Now convert to a type B diagram. Total weight of brick from point B to C = 5x4 = 20N

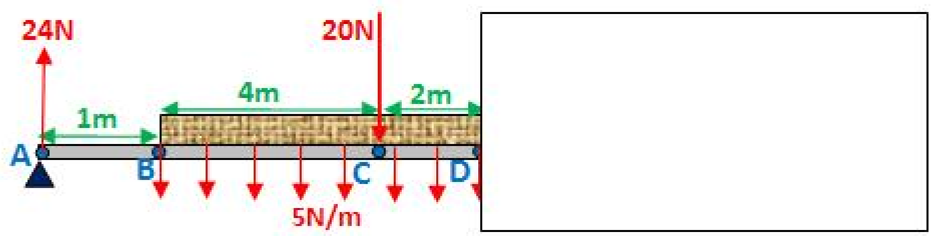


Shear force before C: 24 - 20 = 4N

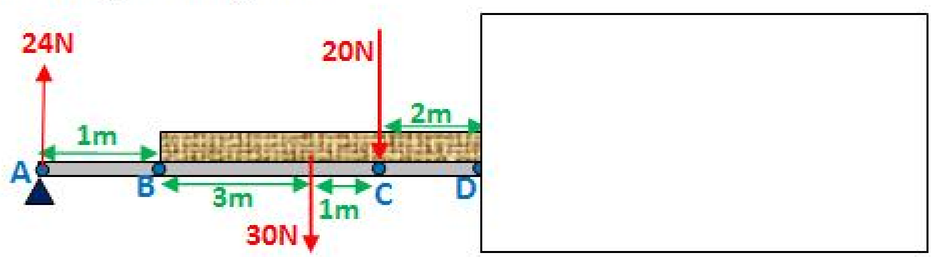


Shear force after C: 24 - 20 - 20 = -16N

point D:



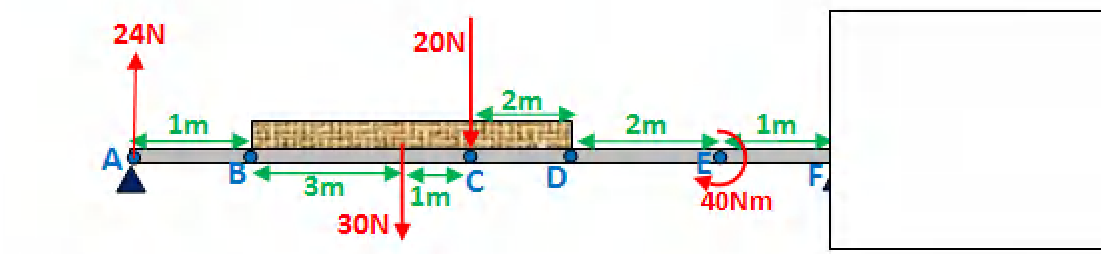
Convert to type B diagram:



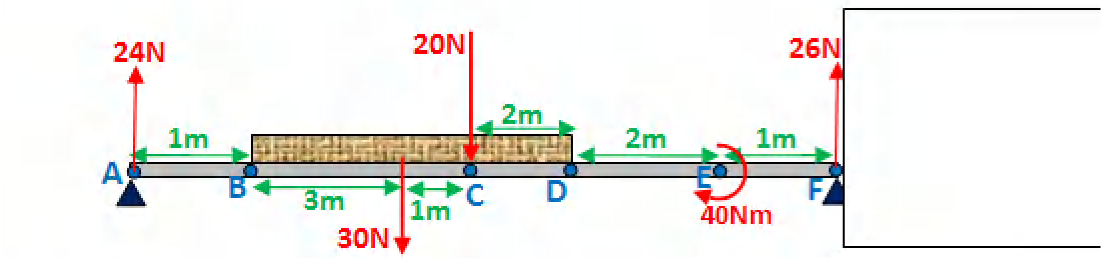
Shear force at D: 24 - 30 - 20 = -26N

point F:

(I have already converted to a type B diagram, below)

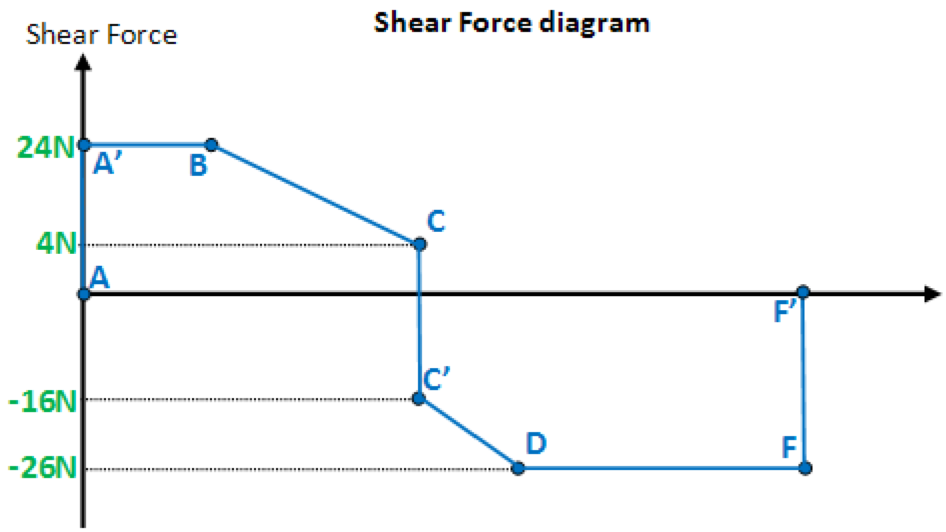


Shear force before F: 24 -30 -20 = -26N



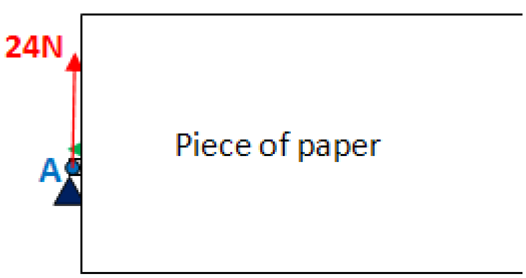
Shear force after F: 24 -30 -20 +26 = 0N

Finally plot all the points on the shear force diagram and join them up:



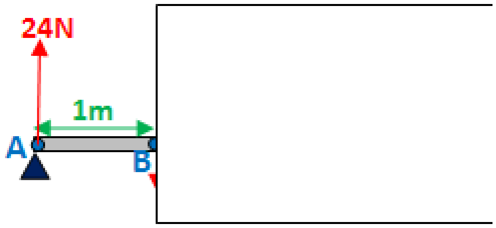
Bending moment diagram

Point A



Bending moment at A: 0Nm

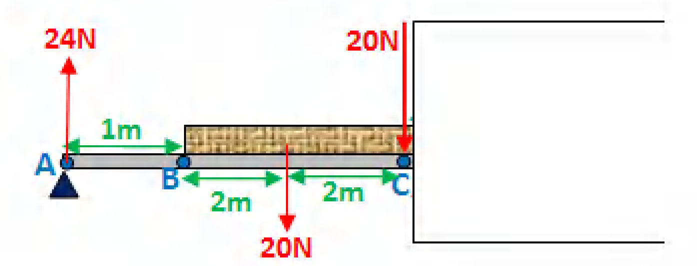
Point B



Bending moment at B: 24·1 = 24Nm

point C:

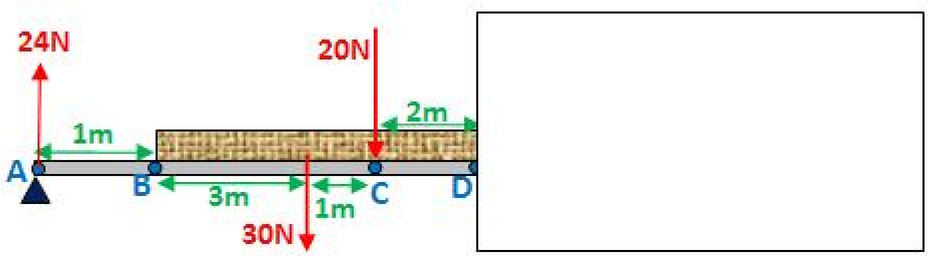
(I have already converted to a type B diagram, below)



Bending moment at C: 24·5 - 20·2 = 80Nm

point D:

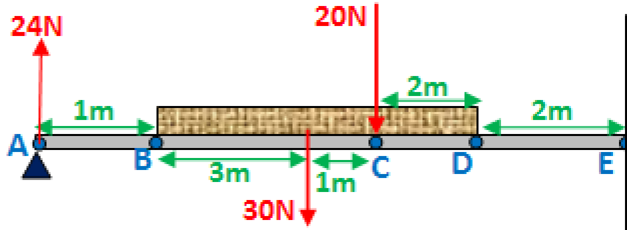
(I have already converted to a type B diagram, below)



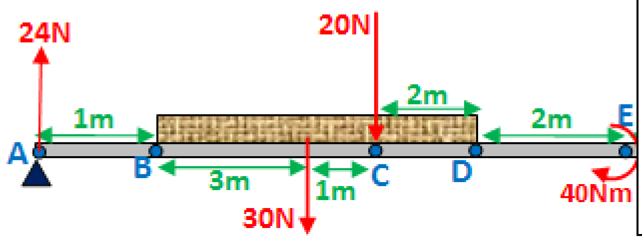
Bending moment at D: 24·7 - 30·3 - 20·2 = 38Nm

point E:

(I have already converted to a type B diagram, below)



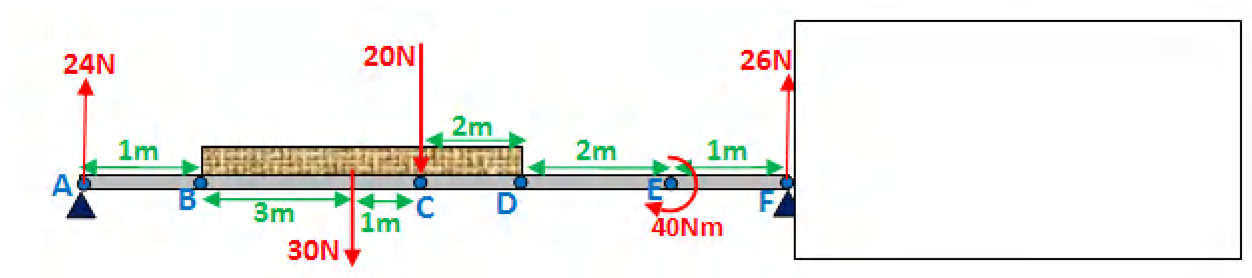
Bending moment just before E: 24·9 - 30·5 -20·4 = -14Nm



Bending moment just after E: 24·9 - 30·5 -20·4 + 40 = 26Nm

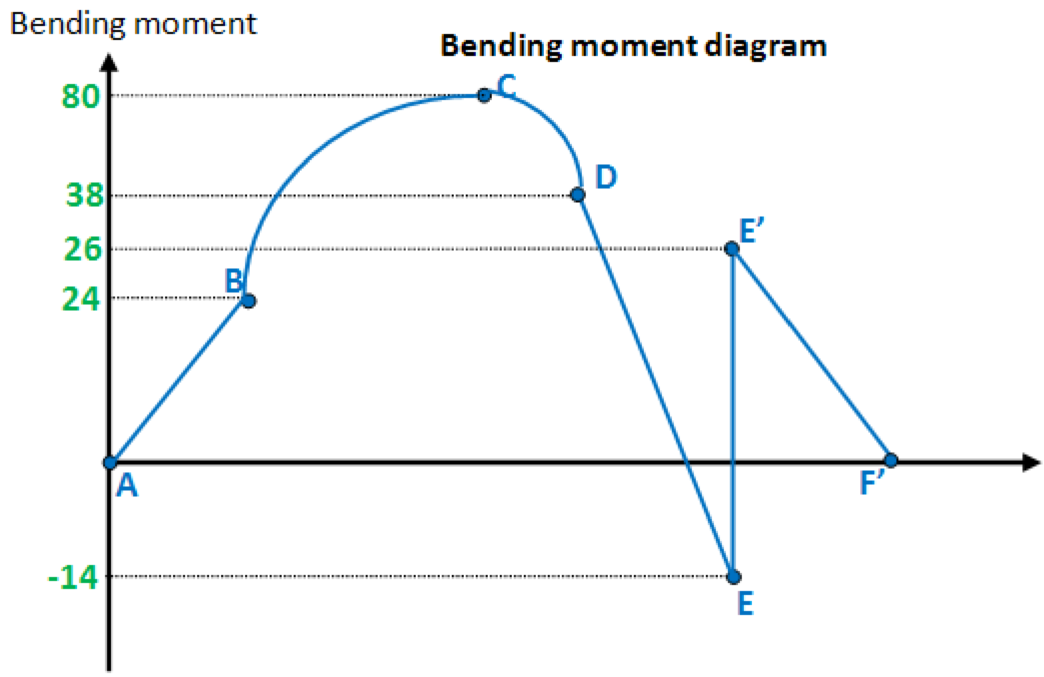
point F:

(I have already converted to a type B diagram, below)



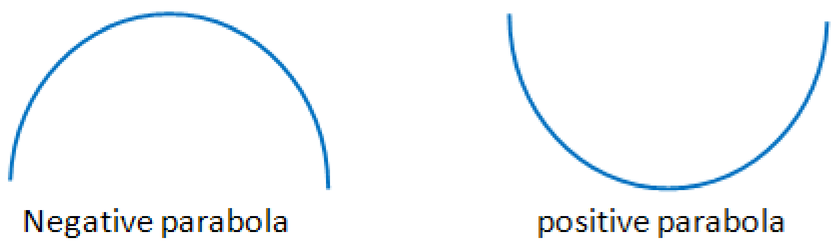
Bending moment at F: 24·10 - 30·6 - 20·5 + 40 = 0Nm

Finally, plot the points on the bending moment diagram. Join all the points up, EXCEPT those that are under the uniformly distributed load (UDL), which are points B,C and D. As seen below, you need to draw a curve between these points. Unless requested, I will not explain why this happens.



Note: The diagram is not at all drawn to scale.

I have drawn 2 curves. One from B to C, one from C to D. Notice that each of these curves resembles some part of a negative parabola.



Rule: When drawing a bending moment diagram, under a UDL, you must connect the points with a curve. This curve must resemble some part of a negative parabola.

Note: The convention used throughout this page is "clockwise moments are taken as positive". If the convention was "counter-clockwise moments are taken as positive", you would need to draw a positive parabola.

Hypothetical scenario

For a hypothetical question, what if points B, C and D, were plotted as shown below. Notice how I have drawn the curves for this case.