Contreol System

The control System is Very important fore all Engineering. In contreol system the behavious of the system is described by the differential equations.

The differential equation may be ordinary differential equation ore the difference equations.

A control system is a group of physical component arranged in a such a way that it gives the desired output fore the given input by means of control ore regulations either direct ore indirect.

→ Based on controller action control system are classified into two ways.

① Open loop Control System.

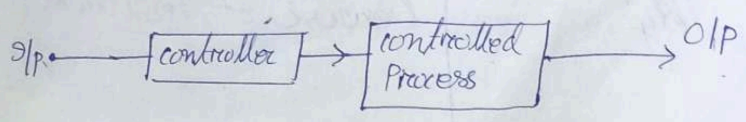
② Close loop Control System.

Open loop Control System :-

The open loop Control System is also known as control system without feed back.

A system in which the controller action complete indepandent of output then it is open loop Control System.

In this system the output is not compared with the reference input.



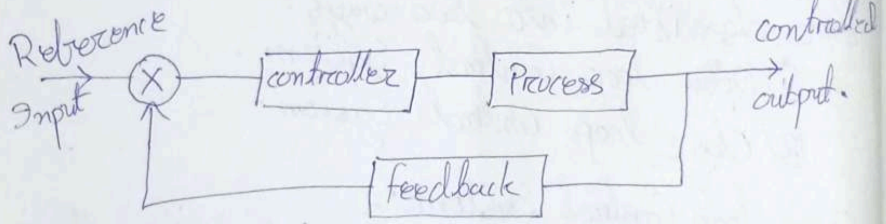
Ex → Automatic washing machine
Immersion rod.

Close loop Control System

Close loop control system are also known as feedback control system. In the closed loop control system the control action is dependent on the desired output. If any system having one or more feedback paths forming a closed loop system.

A system in which the controller action depends on the o/p it is called closed loop control system.

In close loop system the output is compared with the reference input and error signal is produced.



Ex- Air Conditioner.

comparision between Open loop & Close loop

Open loop	Close loop
→ These are not reliable	→ These are reliable.
→ It is easier to build	→ It is difficult to build.
→ If Calibration is good, they perform accurately	→ They are accurate because of feedback.

Open loop	Close loop
-> Open loop System are generally more stable.	-> These are less stable.
-> Optimization is not possible	-> Optimization is possible.

Feedback Network

If the property of the close loop control system which bring the output to the Input and compared with Reference Input, so that that the control action formed to make the error equal to zero.

-> feedback Network consists of passive elements like Resistor, Inductor and Capacitor.

-> The maximum value of feedback Network is one

-> Ideal value of feedback Network is one.

-> feedback Network is noting but a transducer which converts the energy ~~from~~ from one ~~form~~ form to other.

Transfer function

The Transfer function gives the mathmatical equvalent model for physical model.

-> The transfer function of a linear time invarant system is defined as the ratio of Laplace transfer of o/p to ~~Input~~ Laplace transber of i/p with all initial condition are zero.

$$T/F = \frac{L[O/P]}{L[I/P]}\Bigg|_{\text{all initial condition zero}}$$

Linear Time Invarient System:-

→ The transfor characteristics of the system must be linear. To satisfy the linearity between output to input the initial conditions is zero.

→ The Lineare time Invarient system is R L C network are passive network.

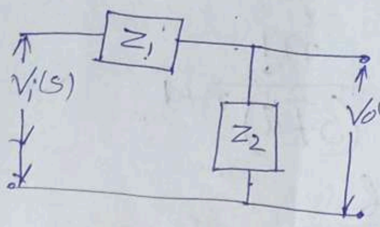
→ The Linear time Invarient system Transber function analysic is valid fore R L C network.

2nd defination of Transber Function:-

The Transfer function of a Linear Time Invarient system is defined as the Laplace Transformation of Implulse response with all intial conditions are zero.

$$T/F = L[\text{Impulse Response}]\Big|_{\text{with all initial condition zero}}$$

Transferfunction to the Electrical N/w :-



$$T/F = \frac{V_0(s)}{V_i(s)} = \frac{\text{Impedance across output}}{\text{Impedance of the total circuit.}}$$

$$\rightarrow \frac{V_0(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

~~De~~

→ Denominator of the T/F = 0 will give poles location.

→ Neumerator of the T/F = 0 will give zeros location.

→ By changing network element position the no. of poles and its locations are not changes. But the number of zeros and its location is changes.

Laplaces Transformation of the following:-

$L(\text{unit Impulse}) = 1$

$L[\text{unit step}] = \frac{1}{s}$

$L[\text{unit Ramp}] = \frac{1}{s^2}$

$L[\text{unit parabolic}] = \frac{1}{s^3}$

$L[t^n] = \frac{n!}{s^{n+1}}$

$$L\left[e^{\pm at}\right] = \frac{1}{s \mp a}$$

$$L\left[t^{n} e^{\pm at}\right] = \frac{n!}{(s \mp a)^{n+1}}$$

$$L\left[\sin bt\right] = \frac{b}{s^2+b^2}$$

$$L(\cos bt) = \frac{s}{s^2+b^2}$$

$$L\left[e^{\pm at} \sin bt\right] = \frac{b}{(s \mp a)^2 + b^2}$$

$$L\left[e^{\pm at} \cos bt\right] = \frac{(s \mp a)}{(s \mp a)^2 + b^2}$$

$$L\left[f(t-T)\right] = e^{-sT} F(s)$$

delay in the system

$$L\left[R\right] = R$$

$$L\left[L\right] = sL$$

$$L\left[C\right] = \frac{1}{sC}$$

Time Constant

The system time constant is

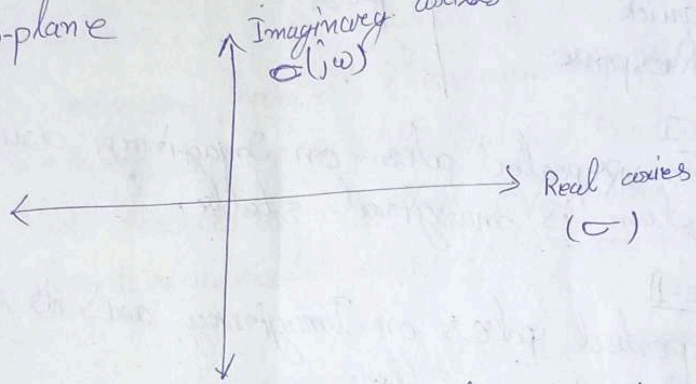
$$T = -\frac{1}{\text{Real part of the Dominate poles location}}$$

<u>Poles</u> Poles is nothing but a -ve of inverse of system time constant at which the magnetude of T/F is infinite (∞)

<u>Zeros</u>

zero is nothing but a -ve of inverse of system time constant at which the magnetude of T/F is zero.

S-plane



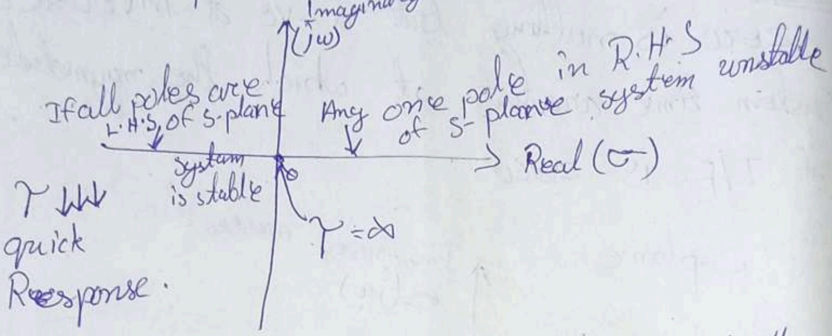
If poles on the -ve real axis gives exponential decay.

-> If many poles are locatated in the -ve real axis at different location irrespective of zero's location the system response is exponential decay.

-> The system time constant is nothing but dominatent pole time constant.

-> We are not consider zero's because it never effect the system Response.

→ we are not ~~cond~~ considering zero's for stability of the system, ~~not cond~~ for system time constant, not for system Response.



case-I
non-Repeated poles on Imaginary axi's the system is marginal stable.

case-II
Repeated poles on Imaginary axi's the system is unstable.

note
on imaginary axi's the system time constant is infinity (∞) as poles moves towards the L.H.S the system time constant decreases and its stability impove system response is fast (Quick).

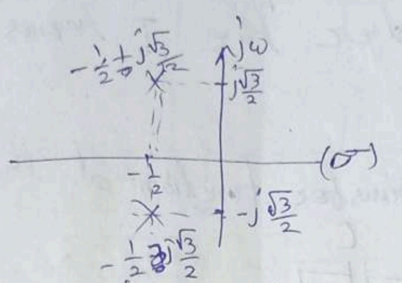
$$\frac{V_0}{V_i} = \frac{\frac{1}{CS}}{R+SL+\frac{1}{CS}} = \frac{1}{RCS+S^2LC+1}$$

$$= \frac{1}{S^2+S+1}$$

$$s^2+s+1 = 0$$

$$S = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

complex conjugate poles



→ The Real term of the complex pole is gives the system time constant and exponential decay response.

→ The imaginary term of the complex pole is gives the frequency of oscillation and hence the total response is called exponential decay ~~signi~~ sinusoidal ossilation or damped oscillation.

note when the poles (non-repeted) the system response is constant amplitude frequency of oscillation hence called undamped oscillation.

→ Damped means decreasing amplitude osillation

note If any one pole in the R.H.S and remain all the pole and zero in L.H.S then it is exponential rise,

→ Time constant is defind for only stable system

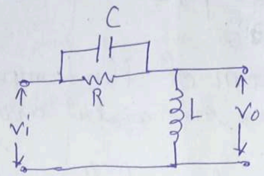
→ If the complex conjugate poles locatated on the Right side of S-plane the system response is exponentia rise sinusoidal

Oscillation and the system is unstable.

-> If the poles are ~~repeated~~ repeated, the ~~response~~ response must be consider the T terms.

Problem!

① Find the Transfer function of the electrical network?

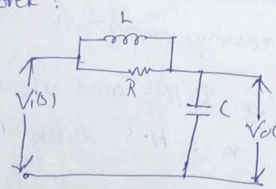


Sol^n

$$T/F = \frac{V_o}{V_i} = \frac{LS}{\frac{R \times \frac{1}{SC}}{R + \frac{1}{SC}} + LS} = \frac{LS}{\frac{\frac{R}{SC}}{\frac{RSC+1}{SC}} + LS}$$

$$= \frac{LS(1+RSC)}{R+LS(1+RSC)} = \frac{SL(1+RSC)}{R+LRS^2C+LS}$$

② Find the Transfer function of the electrical network?

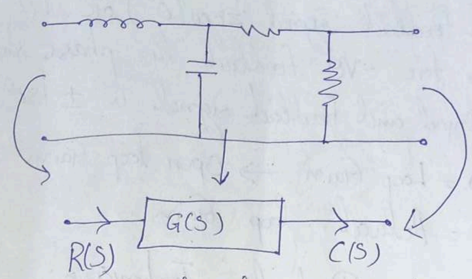


Sol^n

$$T/F = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}}{\frac{SLR}{R+SL} + \frac{1}{SC}}$$

$$= \frac{R+SL}{S^2LRC+SL+R}$$

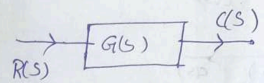
BLOCK DIAGRAM



Short hand pictorial Representation of the system in I/p and Output is called Block diagram.

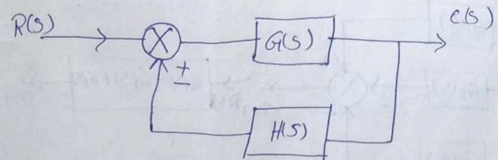
The system can be represented by the forms based on the controller action.

Open loop Transfer function



~~Op~~ open loop Transfer function = $\frac{C(S)}{R(S)} = G(S)$

Close loop Transfer function :-



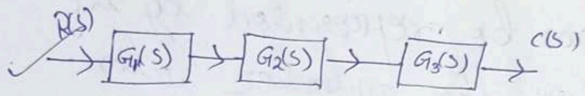
close loop Transfer function = $\frac{C(S)}{R(S)} = \frac{G(S)}{1 \mp G(S)H(S)}$

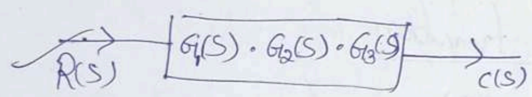
for positive feedback the phase shift between Input and feedback signal zero(0°) or 360°. where as for -ve feedback the phase shift between Input and feedback signal is $\pm 180^0$.

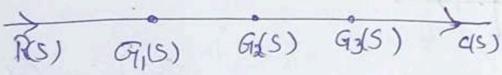
$G(S)\,H(S)$ = Loop Gain → Open loop Gain
= Actual loop Gain.

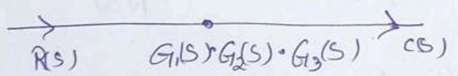
Block diagram Reduction Technique :-

① Block in Series or cascade.

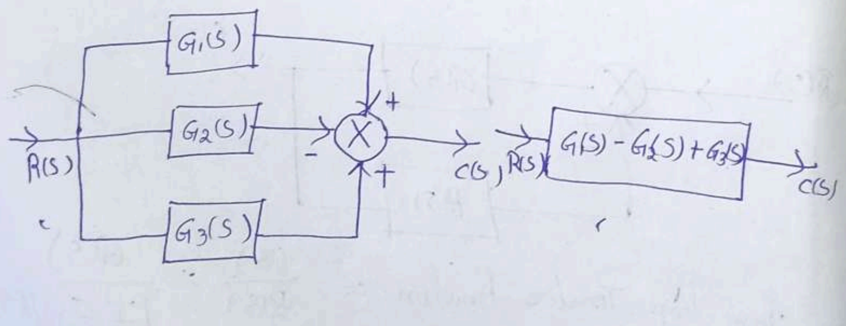


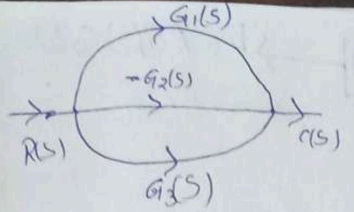


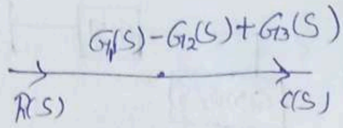




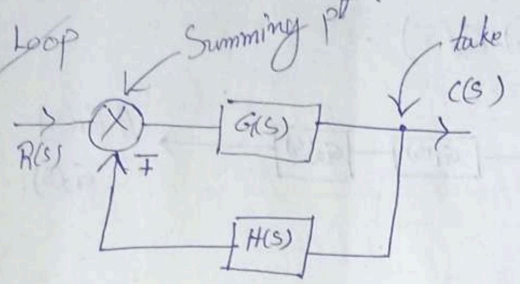
(2) Block in Parallel

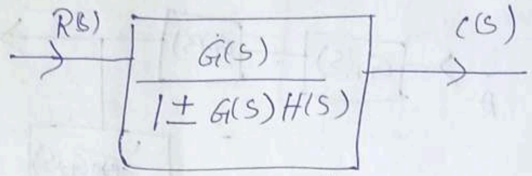




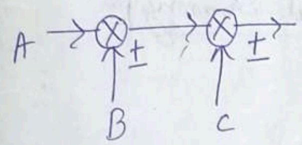


③ Loop

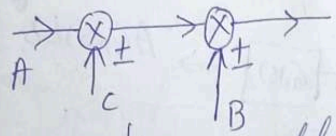




④ Adjusting the Summing pt :-



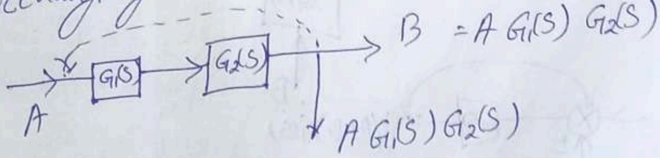
$A \pm B \pm C$

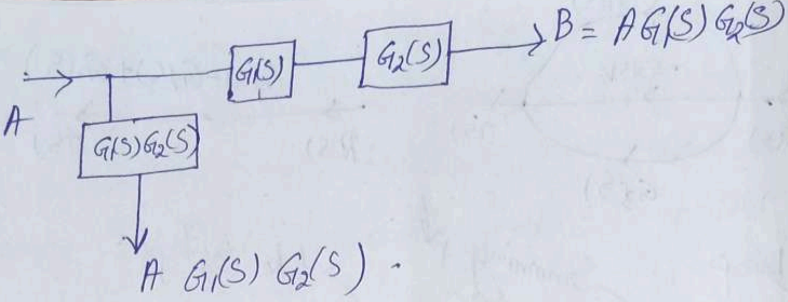


$A \pm C \pm B = A \pm B \pm C$

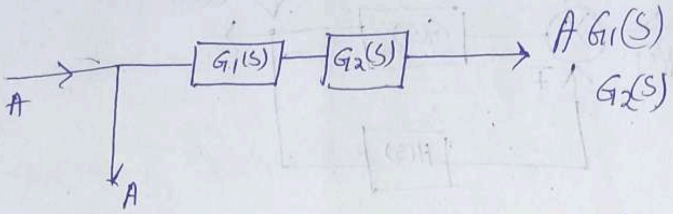
⑤ Interchanging block and take off pt :-

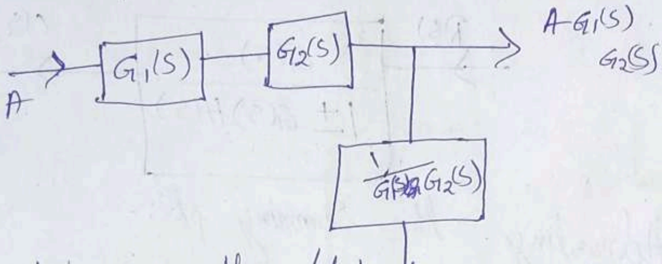
a-



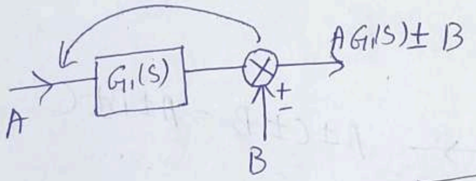


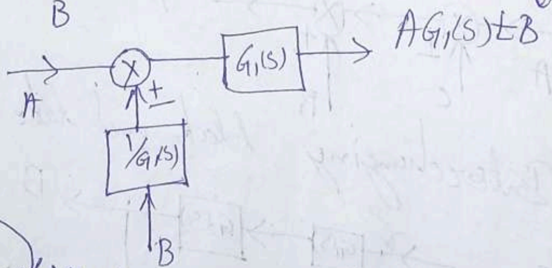
b.

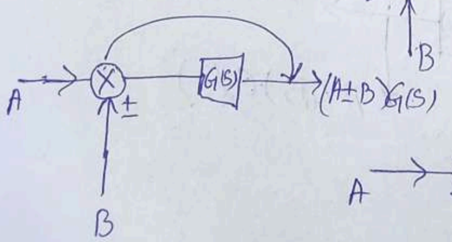


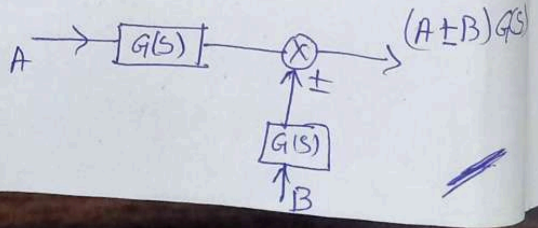


⑥ Interchanging the block and Summing pt.



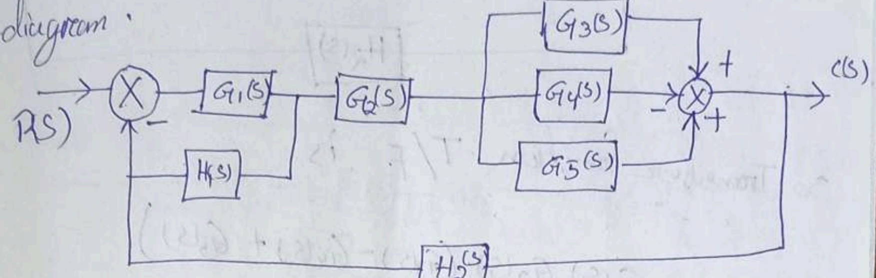




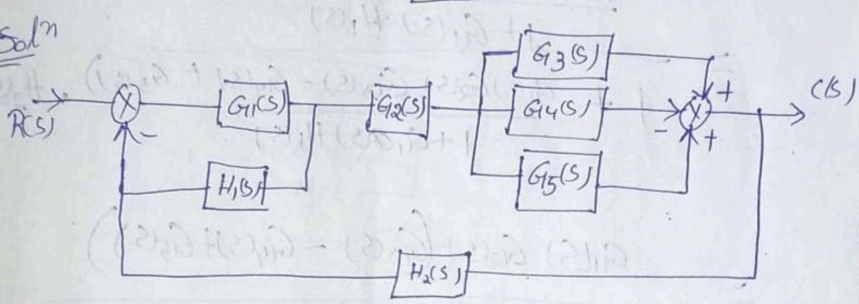


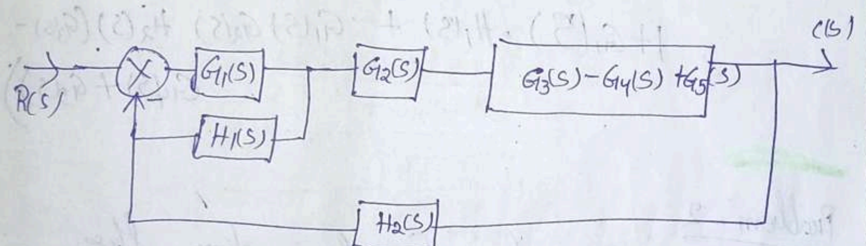
Problem

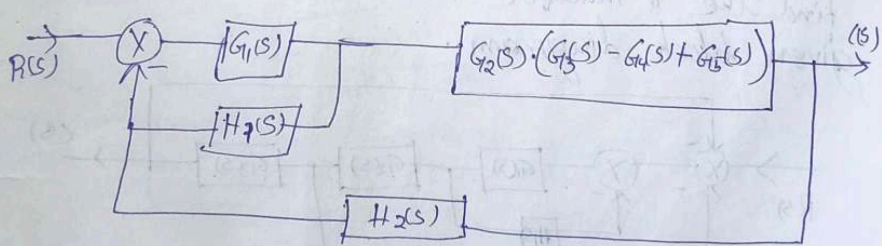
① Find the Transfer function for following block diagram.

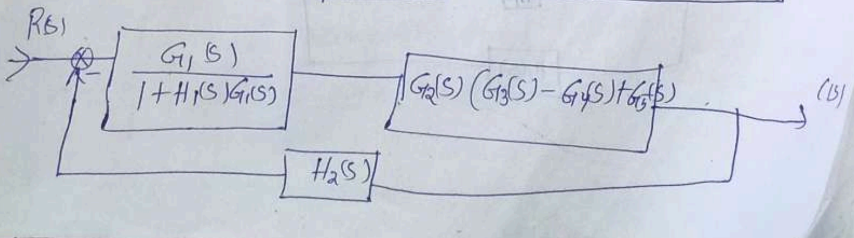


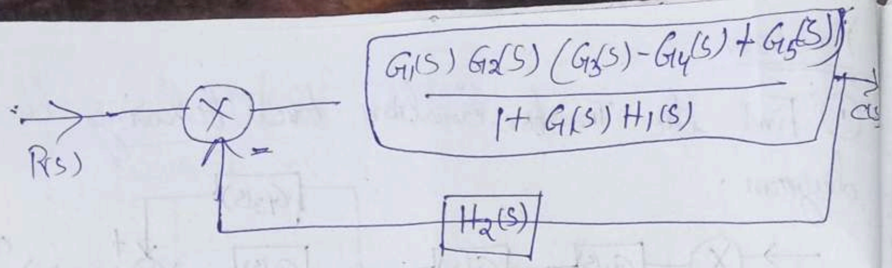
Soln











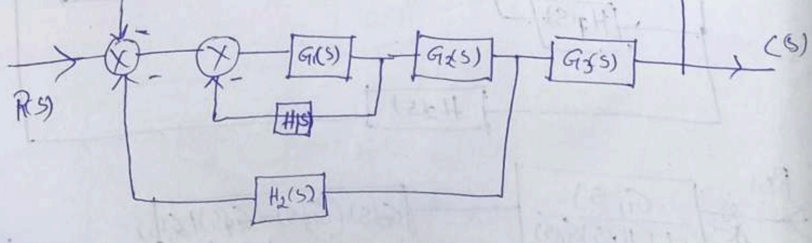
So Transfer function T/F is

$$= \frac{\dfrac{G_1(S)\,G_2(S)\,(G_3(S)-G_4(S)+G_5(S))}{1+G_1(S)\cdot H_1(S)}}{1+\dfrac{G_1(S)G_2(S)\,(G_3(S)-G_4(S)+G_5(S))}{1+G_1(S)\,H_1(S)}\cdot H_2(S)}$$

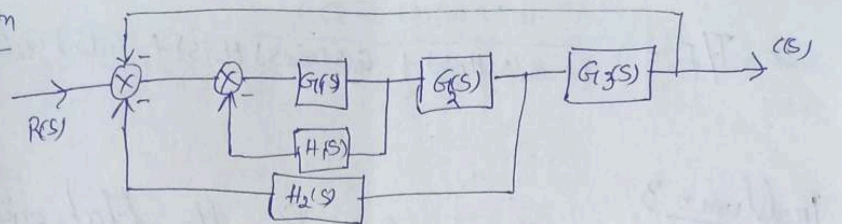
$$= \frac{G_1(S)\,G_2(S)\,(G_3(S)-G_4(S)+G_5(S))}{1+G_1(S)\cdot H_1(S)+G_1(S)\,G_2(S)\,H_2(S)\,(G_3(S)-G_4(S)+G_5(S))}$$

Problem - 2

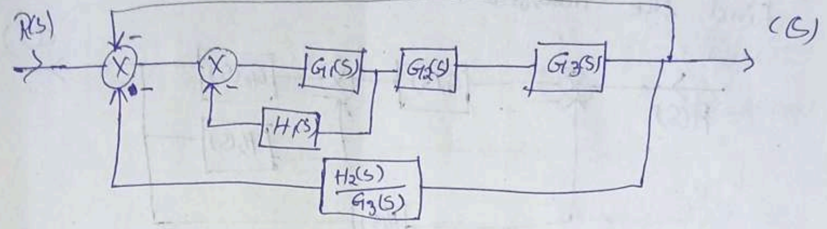
Find the Transfer function for the given block diagram.



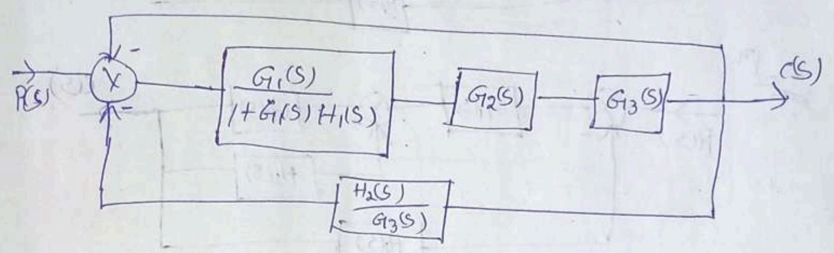
Sol^n



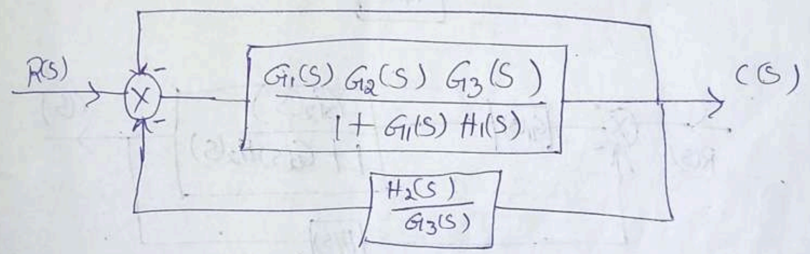
R(S)
G1(S)
G2(S)
G3(S)
H1(S)
H2(S)
C(S)



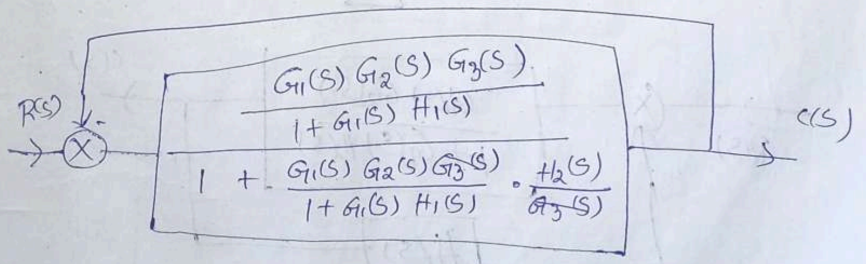
R(S)
G1(S)
G2(S)
G3(S)
H1(S)
$\frac{H_2(S)}{G_3(S)}$
C(S)



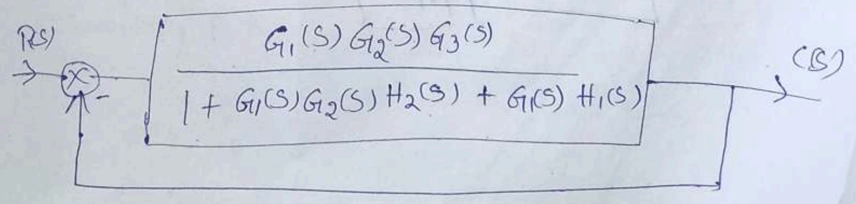
R(S)
$\frac{G_1(S)}{1+G_1(S)H_1(S)}$
G2(S)
G3(S)
$\frac{H_2(S)}{G_3(S)}$
C(S)



R(S)
$\frac{G_1(S)\,G_2(S)\,G_3(S)}{1+G_1(S)\,H_1(S)}$
$\frac{H_2(S)}{G_3(S)}$
C(S)



R(S)
$\dfrac{\dfrac{G_1(S)\,G_2(S)\,G_3(S)}{1+G_1(S)\,H_1(S)}}{1+\dfrac{G_1(S)\,G_2(S)\,G_3(S)}{1+G_1(S)\,H_1(S)}\cdot\dfrac{H_2(S)}{G_3(S)}}$
C(S)

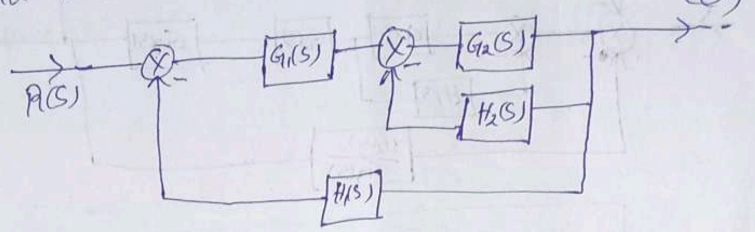


R(S)
$\frac{G_1(S)\,G_2(S)\,G_3(S)}{1+G_1(S)G_2(S)H_2(S)+G_1(S)\,H_1(S)}$
C(S)

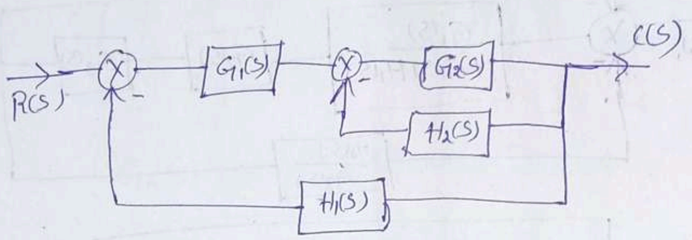
$$\text{So } T/F = \frac{G_1(S)\,G_2(S)\,G_3(S)}{1 + G_1(S)H_1(S) + G_1(S)G_2(S)H_2(S) + G_1(S)G_2(S)G_3(S)}$$

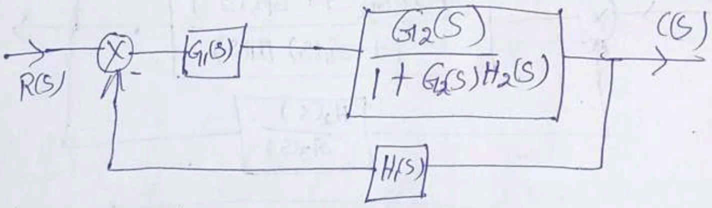
Problem - 3

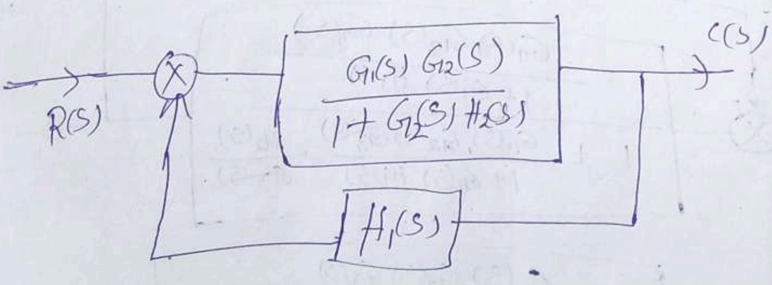
Find the Transfer function for the block diagram



Soln



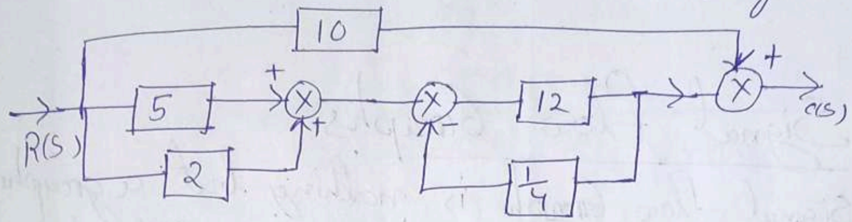




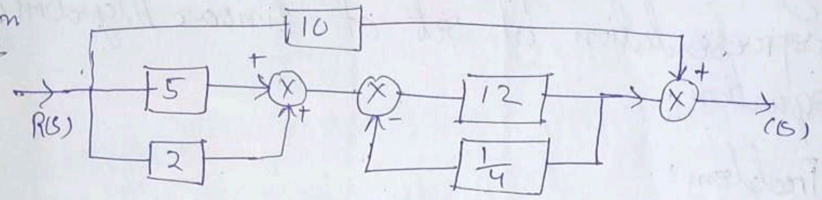
So $T/F = \frac{C(S)}{R(S)} = \frac{G_1(S)\,G_2(S)}{1 + G_2(S)\,H_2(S) + G_1(S)G_2(S)H(S)}$

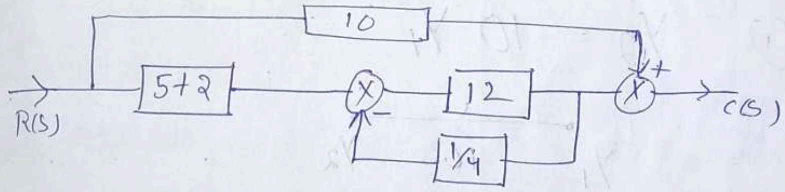
Problem - 4

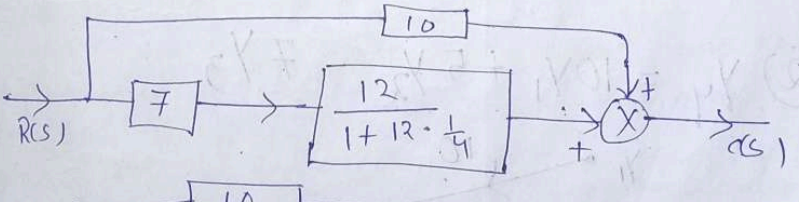
Find the Transfer function for the block diagram.

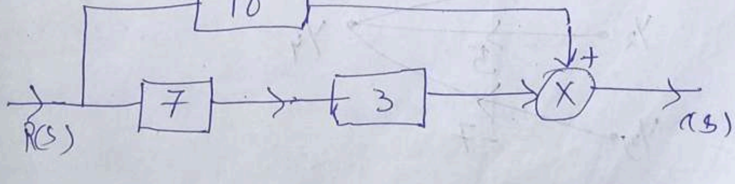


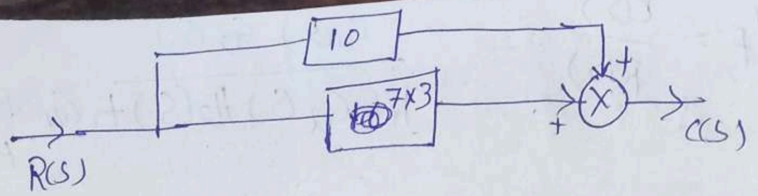
Solⁿ











So Transfer function is

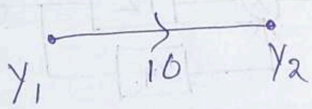
$$T/F = \frac{C(S)}{R(S)} = 21 + 10 = 31$$

Signal flow Graphs

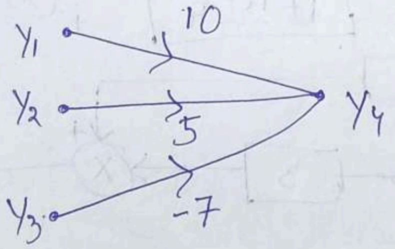
Signal flow Graph is nothing but a graphical representation of set of linear Algebric equation.

Problem :-

① $Y_2 = 10\, Y_1$



② $Y_4 = 10Y_1 + 5\, Y_2 - 7\, Y_3$

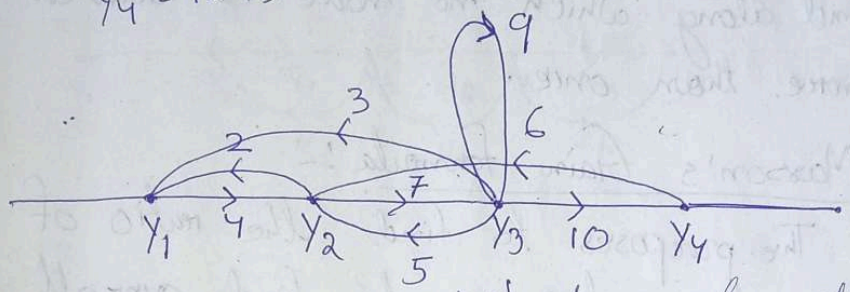


③ $y_1 = 2y_2 + 3y_3$

$y_2 = 4y_1 + 5y_3 + 6y_4$

$y_3 = 7y_2 + 9y_3$

$y_4 = 10y_3$



Loop :- It is a path which terminates with Same node.

Non touching loop :→ If their is no common node between two or more loop, then it is called non-touching loop.

Input node or Source node :- An input node is a node which has only outgoing branches.

output node or sink node :- An output node is a node that has only one or more incomming branches.

Mixed node :- A node having incoming and outgoing branches is known as mixed nodes.

Transmittance :- Transmittance also known as Transfer function, which is noremally written on the branch near the arrow.

<u>forward path</u> :- forward path is a path which originates from the input node and terminates at the output node and along which no node is traveled more than once.

<u>Masson's Gain formula</u> :-

The purposes to find the ratio of any two nodes or to find overall Transfer function.

Overall Transfer function

$$T/F = \sum_{K=1}^{\ell} \frac{P_K \, \Delta_K}{\Delta}$$

where P_K = kth forward path Gain

$$\Delta = 1 - [(\sum L_1 + L_2 + L_3 + \cdots) + \sum L_1 L_2 + L_2 L_3 + \cdots - \sum L_1 L_2 L_3 + L_3 L_4 L_5 + \cdots) + \cdots$$

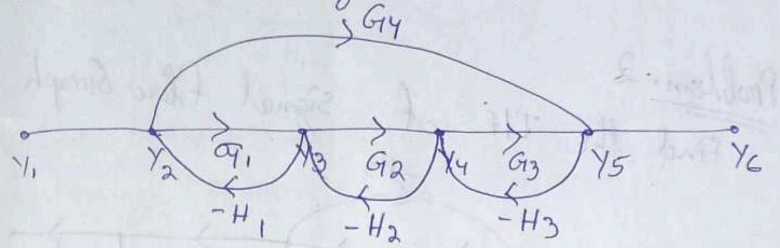
-ve sign is for odd no of Loops.

In Δ takes the -ve sign for odd no of non-touching loops and +ve for even no of non-touching loop.

Δ_K = It is obtain from Δ by removing the loops touching the kth forward path.

Problem-1

Find the Transfer function of the signal flow Graph with the help of Masson's Gain formula for the fig.



Solⁿ

forward paths

$P_1 = G_1 G_2 G_3$

$P_2 = G_4$

Loops

$L_1 = -G_1 H_1$

$L_2 = -G_2 H_2$

$L_3 = -G_3 H_3$

$L_4 = -G_4 H_1 H_2 H_3$

2 non touching loops.

$L_1 L_3 = G_1 G_3 H_1 H_3$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_3$

$= 1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_4 H_1 H_2 H_3 + G_1 G_3 H_1 H_3$

$\Delta_1 = 1$

$\Delta_2 = 1 - L_2$

$= 1 + G_2 H_2$

According the masson's Gain Formula is

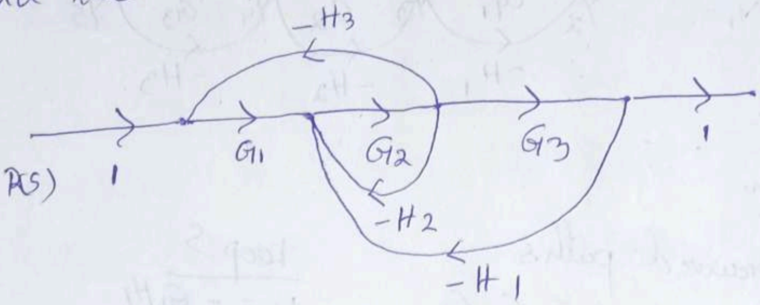
T/F is $= \sum_{K=1}^{l} \frac{P_K \Delta_K}{\Delta}$

$= \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

$$= \frac{G_1G_2G_3 + G_4(1+G_2H_2)}{1+G_1H_1+G_2H_2+G_3H_3+G_4H_1H_2H_3+G_1G_3H_1H_3}$$

Problem-2

Find the T/F of signal flow Graph ?



Sol^n

The foreward paths are

$P_1 = G_1 G_2 G_3$

Loops

$L_1 = -G_2H_2$

$L_2 = -G_1G_2H_3$

$L_3 = -G_2G_3H_1$

Δ= There is no non-touching loops

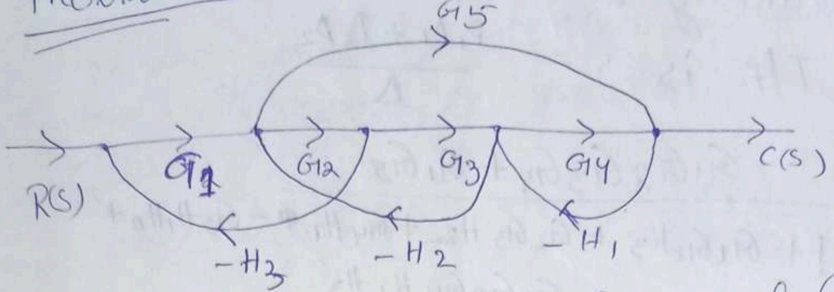
So $\Delta = 1-(L_1+L_2+L_3)$

$= 1+G_2H_2+G_1G_2H_3+G_2G_3H_1$

$\Delta_1 = 1$

So $T/F = \frac{P_1\Delta_1}{\Delta} = \frac{G_1G_2G_3}{1+G_2H_2+G_1G_2H_3+G_2G_3H_1}$

Problem-3



Find the Transfer function of Signal flow Graph using Massion's Gain formula ?

Sol^n

The foreward path are

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_5$

Loop S are

$L_1 = - G_1 G_2 H_3$

$L_2 = - G_2 G_3 H_2$

$L_3 = - G_4 H_1$

$L_4 = G_5 H_1 H_2$

The Two non touching loops are

$L_1 L_3 = G_1 G_2 H_3 G_4 H_1$

So $\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1 L_3$

$= 1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_5 H_1 H_2$

$+ G_1 G_2 H_3 G_4 H_1$

$\Delta_1 = 1$

$\Delta_2 = 1$

So According to Mason's Gain formula the T/F is $= \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

$$= \frac{G_1 G_2 G_3 G_4 + G_1 G_5}{1 + G_1 G_2 H_3 + G_2 G_3 H_2 + G_4 H_1 - G_5 H_1 H_2 + G_1 G_2 G_4 H_1 H_3}$$

Sensitivity

It is used to measure the effectiveness of feedback. The system sensitivity is used to describe the relative variations in the parameter.

Sensitivity with respect to variation in $G(S)$

$$= \frac{\%\text{ of change in } T/F}{\%\text{ of change in } G(S)}$$

$$= \frac{\partial T / T}{\partial G / G} = \frac{\partial T}{\partial G} \times \frac{G}{T}$$

$$S_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T}$$

Sensitivity with respect to variation in $H(S)$

$$S_H^T = \frac{\partial T}{T} \Big/ \frac{\partial H}{H} = \frac{\partial T}{\partial H} \times \frac{H}{T}$$

Problem-1

Find the Sensitivity of close loop Control system with respect to variation in parameter

Solution

close loop $T/F = T(S) = \frac{G(S)}{1+G(S)H(S)}$

$= \frac{G(S)}{1+G(S)H(S)}$

Parameter difference ~~w~~ with respect to G

$= \frac{\partial T}{\partial G} = \frac{1(1+GH) - GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$

$S_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T}$

$= \frac{1}{(1+GH)^2} \times \frac{G}{\frac{G}{1+GH}} = \frac{1}{1+GH}$

S_G^T for Close loop Control system

$= \frac{1}{1+GH}$

Sensitivity with respect to $H(S)$

$S_H^T = \frac{\partial T}{\partial H} \cdot \frac{H}{T}$

$\frac{\partial T}{\partial H} = \frac{0 - G \cdot G}{(1+GH)^2} = \frac{-G^2}{(1+GH)^2}$

$S_H^T = \frac{-G^2}{(1+GH)^2} \times \frac{H}{\frac{G}{1+GH}} = \frac{-GH}{1+GH}$

So $S_H^T = \frac{-GH}{1+GH}$

note
The sensitivity with respect to feedback H(S) is more then sensitivity with respect to forward path G(S)
Hence while designing the feedback network the components should be selected so that it gives the stable values.

Problem:- 2
Find the Sensitivity for Open loop control system?

$T/F = T(S) = G(S)$

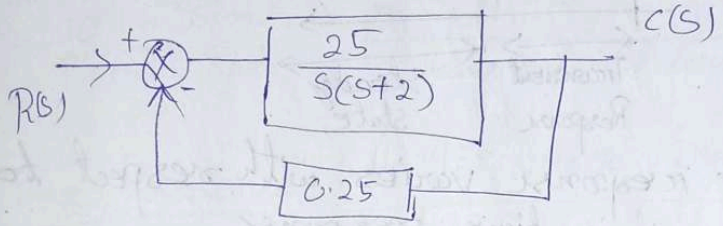
$$S_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T}$$

$$= 1 \times \frac{G}{G} = 1$$

The sensitivity of the open loop control system is '1'. i.e any variation in G(S) is directly effect the o/p where as in close loop control system the sensitivity decreased by $1+G(S)H(S)$.

Problem

Determine the sensitivity of overall T/F fore the system shown in fig at the frequency $\omega = 1$ rad/sec with respect to
(a) forward path T/F (b) feed back path.



$$S_G^T = \frac{1}{1+GH} =$$

$$GH = \frac{25}{S(S+2)} \cdot (0.25)$$

$$= \frac{6.25}{(S+2)\,S}$$

$$= \frac{1}{1+\frac{6.25}{(S+2)S}} = \frac{S^2+2S}{S^2+2S+6.25}$$

$$S = j\omega \Rightarrow S = j$$

$$S_G^T = \frac{-1+2j}{-1+2j+6.25} = \left|\frac{-1+2j}{5.25+2j}\right| = \frac{\sqrt{5}}{\sqrt{25+4}}$$

$$= \frac{1}{\sqrt{6}} = 0.399$$

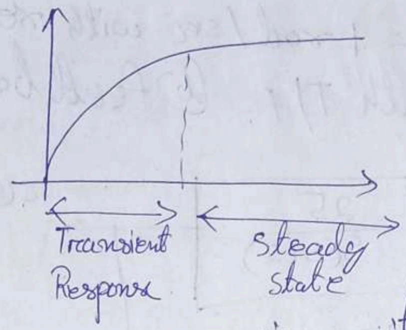
$$S_H^T = \frac{-GH}{1+GH} = \frac{-\frac{6.25}{S(S+2)}}{1+\frac{6.25}{(S+2)S}} = \frac{-6.25}{S^2+2S+6.25}$$

$$S = j\omega = j$$

$$= \frac{-6.25}{5.25+2j} = -1.11$$

Time domain Analysis

Time Response (t)



If the response varies with respect to time then it is time response

The time response is nothing but a transient Response + Steady state response.

Transisent Response:-
It is a part of a time response that goes to zero as time becomes very large i.e $\lim_{t \to \infty} C_{tt}(t) = 0$

steady state resssponse:- It is a part of a time response that after the transient died out are nullified.

Problem
Find the treansient toom in the given complete time response

$$c(t) = 10+5\sin t + e^{-10t}t + e^{-2t}\sin t$$

Sol^n $c(t) = 10 + 5\sin t + e^{-10t} t + e^{-2t}\sin t$.

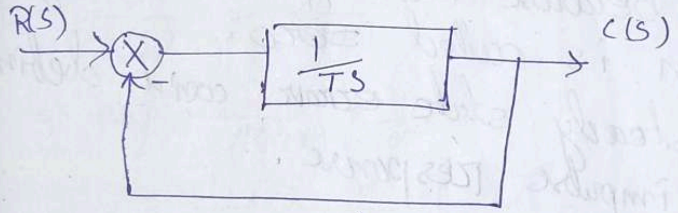
$\lim_{t \to \infty} C_{tr}(t) = 0$

i.e $e^{-10t} t = 0$ & $e^{-2t}\sin t = 0$

so $\underbrace{10 + 5\sin t}_{\text{steady state}} + \underbrace{e^{-10t} t + e^{-2t}\sin t}_{\text{Transient terms.}}$

Time Response of 1st order System

The standard from of the block diagram is



Loop Gain or Open loop T/F $= G(S) = \dfrac{1}{TS}$

close loop T/F $= \dfrac{\frac{1}{TS}}{1 + \frac{1}{TS}} = \dfrac{1}{1+TS}$

for Impulse Response:-

$r(t) = \delta(t)$

$R(S) = 1$

T/F for 1st order is $= \dfrac{1}{1+TS}$

$\dfrac{C(S)}{R(S)} = \dfrac{1}{1+TS}$

$$\Rightarrow C(S) = \frac{1}{TS+1} R(S)$$

$$\Rightarrow C(S) = \frac{1}{TS+1} = \frac{1}{T\left(S+\frac{1}{T}\right)}$$

Taking Inverse Laplace transformation

$$c(t) = \frac{1}{T}\left(e^{-t/T}\right) u(t)$$

The impulse Response is consists of only system time constants hence it is called system response. The impulse response is consists only transient from because the I/p exist only at origin is called zero.

The steady state error can't defined for impulse response.

<u>Unit step Response</u>

$$r(t) = 1$$

$$R(S) = \frac{1}{S}$$

$$T/F = \frac{C(S)}{R(S)} = \frac{1}{TS+1}$$

$$\Rightarrow C(S) = \frac{1}{TS+1} \times R(S) = \frac{1}{TS+1} \times \frac{1}{S}$$

$$= \frac{1}{S} - \frac{T}{(TS+1)}$$

Taking Inverse Laplace transformation

$$c(t) = \cancel{1 \pm 0}\left(1 - e^{-t/T}\right) u(t)$$

Step Response consists the steady state and transient completly.
steady state response of the system is depends on I/p but its value depends on system gain and magnitude of I/p.
Transient Response is depends only on system time constant.

Ramp Response :-

$r(t) = t$

$R(S) = \frac{1}{S^2}$

$T/F = \frac{C(S)}{R(S)} = \frac{1}{TS+1}$

$\Rightarrow C(S) = \frac{1}{T(S)+1} * \frac{1}{S^2}$

$= \frac{1}{S^2} - \frac{T}{S} + \frac{T^2}{TS+1}$

Taking Inverse Laplace transfer we have

$c(t) = \underbrace{t - T}_{\text{Steady state}} + \underbrace{Te^{-t/T}}_{\text{Transient state}}$

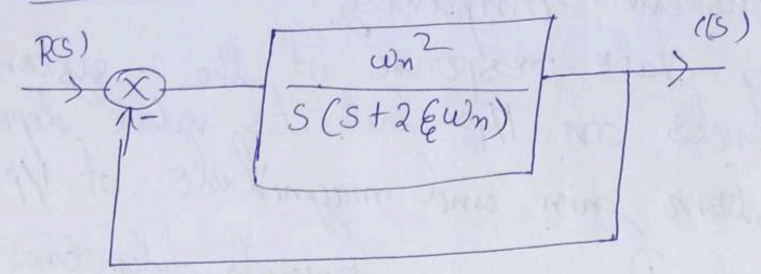
Unit parabolic Response

$r(t) = \frac{t^2}{2}$

$R(S) = \frac{1}{S^3}$

$C(S) = \frac{1}{S^3(TS+1)}$

2nd Order System response :-

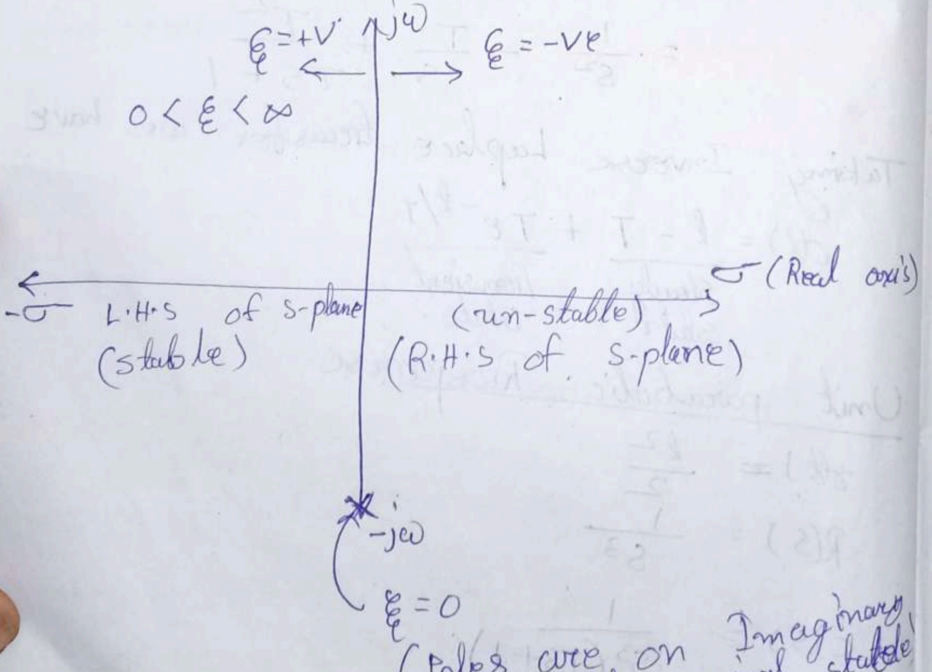


$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$$

The 2nd order system response is depends on damping ration ξ

$$0 < \xi \leq \infty$$

→ The 2nd order system are stable because the close loop poles are in the left of S-plane



Impulse response to the 2nd order

$r(t) = \delta(t)$

$R(S) = 1$

$$T/F = \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$\Rightarrow C(S) = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} \cdot R(S)$$

$$= \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

Case-I $\xi = 0$

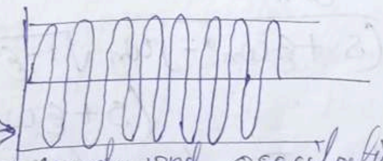
$$C(S) = \frac{\omega_n^2}{S^2 + \omega_n^2}$$

Taking Inverse laplace transformation

$$c(t) = \omega_n \sin \omega_n t$$

(Time const. $T = \infty$)

→ when $\xi = 0$, the system poles located on imaginary axis hence the system is ~~marg~~ marginal stable.



c(t)

undamped osscilation

frequency of osscilation $= \omega_n$ rad/sec^2

(constant amplitude & frequency osscilation.)

→ when system poles are located on imaginary side $(0 \pm j\omega_n)$ the system response is called undamped system.

→ The 2nd order system nature is completely depends on ξ even if the I/p changes the nature of the system not change.

Case-2

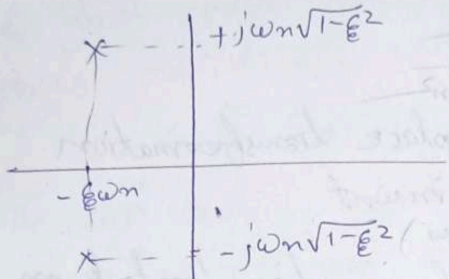
$0 < \xi < 1$

$$-\frac{2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$C(S) = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2} = \frac{-2\xi\omega_n \pm 2\omega_n\sqrt{\xi^2-1}}{2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$= -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



complex conjugate poles (stable)

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{(S + \xi\omega_n - j\omega_n\sqrt{1-\xi^2})(S + \xi\omega_n + j\omega_n\sqrt{1-\xi^2})}$$

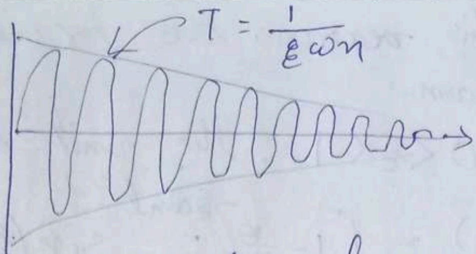
Inverse Laplace Transformation

$$c(t) = k\, e^{-\xi\omega_n t} \sin(\omega_n\sqrt{1-\xi^2})\, t$$

The frequency of osscilation are called Damped osscillation.

$$\boxed{\omega_d = \omega_n\sqrt{1-\xi^2}} \text{ rad/sec.}$$

$T = \frac{1}{\xi \omega_n}$



<u>case-3</u>

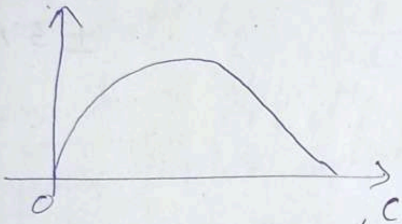
$\xi = 1$, critical dumped system.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

Inverse Laplace transformation $= \frac{\omega_n^2}{(s+\omega_n)^2}$

$c(t) = k\, t\, e^{-\omega_n t}$ $\quad (s+\omega_n)^2 = 0$

System is stable $\quad s = -\omega_n, -\omega_n$



critical dumped system

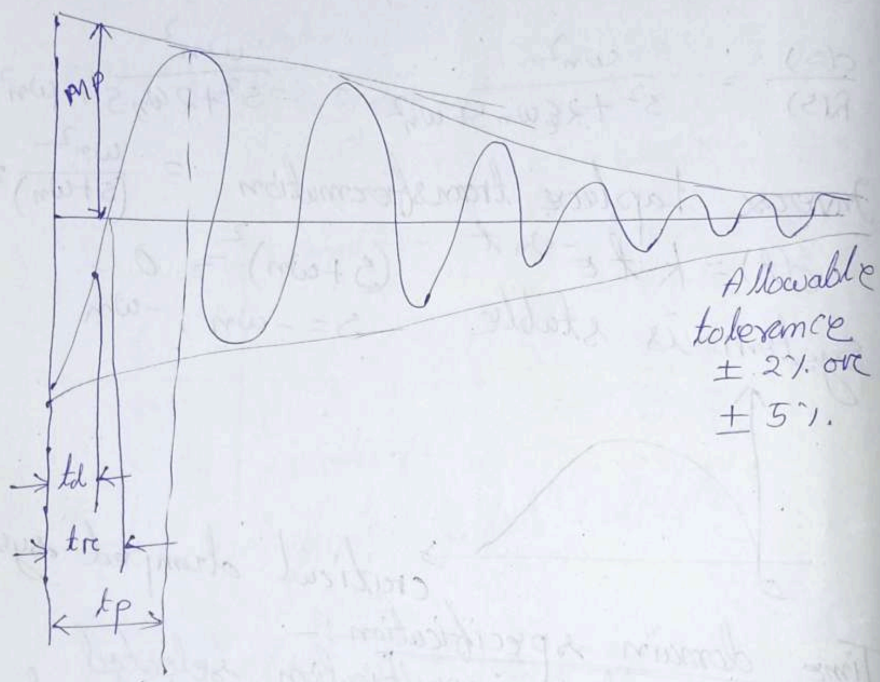
<u>Time domain specification:-</u>

For time domain specification selected underdamped system because the critical and over damped system have large rise time and less setting time (t_s) where as if consider the undamped system the t_r decreces but $t_s \to \infty$. So in above two case either one parameter is less but other is very ~~lage~~ large. Practically we required t_s & t_r is small it is possible in a under damped system. Parctically to design any system ξ value is 0.4 to 0.6

in this reason. the t_r and t_s are medium.

→ $0 < \xi < 1$, the unit step response is

$$c(t) = \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n\sqrt{1-\xi^2}\,t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)\right]$$



delay time

It is the time required for the response to rise 0 to 50% of the final value.

$$\boxed{t_d = \frac{1 + 0.7\xi}{\omega_n}} \text{ sec.}$$

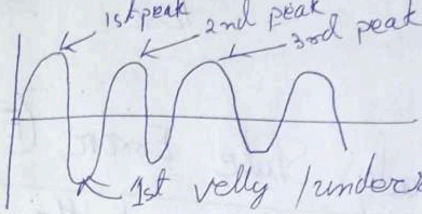
rise time (t_r) :- It is time required for the response to rise 0 to 100% for under damp
05 - 95% for critical damp
10 - 90% for over damped.

$$\boxed{t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_d}} \text{ sec}$$

or $\boxed{t_r = \frac{3}{2} t_d}$ sec

Peak time (t_p) :- It is the time required for the response to rise the peak of the response.

$\boxed{t_p = \frac{n\pi}{\omega_d}}$ (where $n=1$ default for 1st peak)



$\boxed{t_p = \frac{\pi}{\omega_d}}$

2nd peak $t_p = \frac{3\pi}{\omega_d}$

1st undershoot $t_p = \frac{2\pi}{\omega_d}$

Settling time :- It is the time required for the response to reise and reach specified tollerance band $\pm 2\%$ or $\pm 5\%$.

$\boxed{T_s = 4T = \frac{4}{\xi\omega_n} \text{ for } \pm 2\%}$

$\boxed{T_s = 3T = \frac{3}{\xi\omega_n} \text{ for } \pm 5\%}$

% of Peak overshoot :- It gives the nutrallization difference between time response peak to steady state O/P.

$\boxed{\% M_P = \left(e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}\right) \times 100}$

Time Period of oscillation

$$\boxed{T_{osc} = \frac{2\pi}{\omega_d}}$$

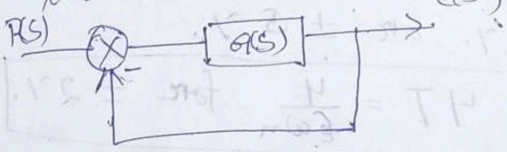
The no of oscillation before reaching the steady state.

$$N = \frac{t_s}{T_{osc}} \quad (\pm 2\% \text{ or } \pm 5\%)$$

Steady State Error (ESS)

The deviation of the output from the Refarance Input at $t \to \infty$ gives the steady state Erarorc.

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\,E(s)$$



$$E(s) = R(s) - C(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

If $H(s) = 1$, $G(s)$ is called open loop T/F system.

$$e_{ss} = \lim_{S \to 0} S\, E(S)$$

$$= \lim_{S \to 0} \frac{S\, R(S)}{1 + G(S)}$$

The steady state Errore are depends on two factore ① Type of Input
② Type of system.

→ Steady State Errore are ~~depends on~~ valid fore close loop stable system.

→ steady state Errore are calculated fore unit feedback system by using open loop Transfer function.

Type of Input

(0) step i/p :- ~~condi~~ considerring the zero '0'

$$r(t) = A\, t^0\, u(t) = A\, u(t)$$

$$R(S) = \frac{A}{S}$$

$$e_{ss} = \lim_{S \to 0} S \frac{R(S)}{1 + G(S)} = \lim_{S \to 0} \frac{S \frac{A}{S}}{1 + G(S)}$$

$$= \lim_{S \to 0} \frac{A}{1 + G(S)} = \frac{A}{1 + \lim_{S \to 0} G(S)}$$

Let $K_p = \lim_{S \to 0} G(S)$ → Position errore constant

$$\Rightarrow e_{ss} = \frac{A}{1 + K_p}$$

① Ramp Input :-

$$r(t) = A\,t\,u(t)$$

$$R(S) = A/S^2$$

$$e_{ss} = \lim_{S\to 0} \frac{S\,A/S^2}{1+G(S)} = \lim_{S\to 0} \frac{A}{S(1+G(S))}$$

$$= \frac{A}{\lim_{S\to 0} SG(S)}$$

Let $K_V = \lim_{S\to 0} S\,G(S)$ Velocity errore const.

$$e_{ss} = \frac{A}{K_V}$$

② Paraboblic Input :-

$$r(t) = A\frac{t^2}{2}u(t)$$

$$R(S) = \frac{A}{S^3}$$

$$e_{ss} = \lim_{S\to 0} \frac{S\frac{A}{S^3}}{1+G(S)} = \frac{A}{\lim_{S\to 0} S^2(1+G(S))}$$

$$= \frac{A}{\lim_{S\to 0} S^2 G(S)}$$

Let $K_a = \lim_{S\to 0} S^2 G(S)$ accelaration errore const.

$$e_{ss} = \frac{A}{K_a}$$

Type of system

$$\text{OL T/F } G(s) = \frac{K(1+ST_1)(1+ST_2)\cdots}{s^{n}(1+ST_a)(1+ST_b)\cdots}$$

when $H(s) = 1$

n = Type n system.

→ Type is nothing ~~too~~ but no of poles at origin

→ Order is nothing but total no of poles in the s-plane..

consider step i/p

$$e_{ss} = \frac{A}{1+K_p}$$

consider the type 0

when $\lim_{s\to 0} \frac{K(1+ST_1)(1+ST_2)\cdots}{s^0(1+ST_a)(1+ST_b)\cdots}$

$$= \lim_{s\to 0} K = K$$

$$e_{ss} = \frac{A}{1+K}$$ is a constant.

Type-1 system

$$K_p = \lim_{s\to 0} \frac{K(1+ST_1)(1+ST_2)\cdots}{s^1(1+ST_a)(1+ST_b)\cdots}$$

$$= \frac{K}{0} = \infty$$

$$e_{ss} = \frac{A}{1+\infty} = \frac{A}{\infty} = 0$$

when ever type = i/p ess = constant

type > i/p ess = 0

type < i/p ess = ∞

—x—

STABILITY

The linear time Inveriable system is said to be stable:

① If the system i/p is excited by Bounded i/p then o/p must be Bounded.

② If the i/p of the system is zero o/p must be equal to zero irrespective of all the initial condition.

The linear time Inveriant system is said to be marginal stable if for the bounded i/p the o/p maintains the constant amplitude and frequency of osscilation is called maginal stable.

The stability are classified into 2ways

① absolutily stable system :- Here the system is stable for all the value of system parameter from (0 to ∞)

(ii) Conditional stable :- Here the system is stable for a certain time like $0 < k < 11$

(iii) relative stability :- It is applicable only for close loop stable system.

→ As poles moves to the left hand side ($-\infty$) the relative stability improve.

→ By using relative stability we can find low pass transient are died out system time const (T) and (t_s) settling time.

Routh - Hurwitz Criterion.

Purpose - (1) To find close loop system stability

(2) No of close loop poles lies either left or Right or on imaginary axis.

(3) Range of k for system stable.

(4) value of k for marginal stable.

(5) frequency of oscilation for marginal stable

(6) Relative stability.

→ In a close loop system.

$1 + G(s) H(s) = 0$ is the characterstic eqn.

In RH Creteria to find the close loop system stability we required characterisc eqn.

The general characteristic equation for nth order is

$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots\ldots + a_{n-1} s^1 + a_n s^0 = 0$

$$\begin{array}{c|ccc}
s^n & a_0 & a_2 & a_4 \ldots\ldots \\
s^{n-1} & a_1 & a_3 & a_5 \ldots\ldots \\
s^{n-2} & \frac{a_1 a_2 - a_0 a_3}{a_1} & \frac{a_1 a_4 - a_0 a_5}{a_1} & \\
\vdots & & & \\
s^0 & a_n & &
\end{array}$$

The condition to make the system stability is

-> All the co-efficient in the first colum should be same sign and no ~~coffi~~ coefficient is zero.

-> If any sign change in the 1st colum then the system is unstable.

-> The no of sign change = no of poles in the right side of s-plane

Problem

① Find the stability of the characteristic eqn ?

$s^3 + 2s^2 + 10s + 50 = 0$

s^3	1	10
$+s^2$	2	50
$-s^1$	$\frac{20-50}{2} = -15$	
$+s^0$	50	

There is sign change in the 1st coloum So system is unstable. Two sign change means 2 poles are in the right half of the s-plane.

② $s^3 + 8s^2 + 5s + 40 = 0$

s^3	1	5
s^2	8	40
s^1	$\frac{40-40}{8} = 0$	
s^0	40	

If one zero in the 1st coloum then the system is marginal stable.

(3) Check for Stability.

$$s^3+9s^2+10s+10=0$$

s^3	1	10
s^2	9	10
s^1	$\frac{90-10}{9}=\frac{80}{9}$	
s^0	10	

All the co-efficient in the 1st colourm are same sign. So the system is stable.

(4) Find the no. of close loop poles in the Left & Right half of S-plane for the characteristics equation.

$$s^4+2s^3+3s^2+4s+5=0$$

s^4	1	3	5
s^3	2	4	
$+s^2$	$\frac{6-4}{2}=1$	5	
$-s^1$	-6		
$+s^0$	5		

Two sign change in the 1st colomn. So the system is unstable.

-> Two Roots in the Right half of S-plane

-> Two Roots in the left half of S-plane.

(5) find the stability

$$s^4 + 2s^3 + 2s^2 + 4s + 8 = 0$$

s^4	1	2	8
s^3	2	4	
s^2	0	8	
s^1			
s^0			

If any one element is zero in the 1st colomn replace zero by smallest positive constant (ϵ) and continue the R-H table. finally subsititute $\epsilon = 0$ and check the no of sign change.

$$\begin{array}{c|ccc} s^4 & 1 & 2 & 8 \\ s^3 & 2 & 4 & \\ +s^2 & \epsilon & 8 & \\ -s^1 & \frac{4\epsilon-16}{\epsilon} & & \\ +s^0 & 8 & & \end{array}$$

$$\frac{4\epsilon-16}{\epsilon} = 4 - \frac{16}{\epsilon}$$
$$= 4 - \infty$$
$$= -\infty$$

So the system is unstable. Two poles are in the Right Half of the S-plane

note:- The rows of zero occurs in the Routh-array only when all the poles are located symatical above the origin.

-> The auxilarly eqn consists of the only even power of the s-terms. because it gives the sysmatical poles above the origin.

-> In Routh array tabular from the only one row of zero occures and all the co-efficient of the first coloum are +ve then the poles are in the imaginary axies with out nonrepeated poles. Hence the system is marginal stable.

Problem $s^5 + s^4 + 3s^3 + 3s^2 + 2s + 2 = 0$

s^5	1	3	2
s^4	1	3	2
s^3	$\cancel{0}$ 4	$\cancel{0}$ 6	
s^2	$\frac{3}{2}$	2	
s^1	$\frac{2}{3}$		
s^0	2		

$A.E = s^4 + 3s^2 + 2 = 0$

$\frac{d(A.E)}{ds} = 4s^3 + 6s$

As all the sign in 1st coloum is same. So the system is marginal stable.

note :- If many time rows of zeros occurs and all the coefficients in the 1st colum are +ve then the roots must be on imaginary axis with ~~respect~~ ~~repir~~ repeated poles. Hence it is unstable.

find the range of k for system stable.

-> To find the value of k minimum. we have ~~equal~~ to equate s^0 coefficient equal to zero.

-> To find the maximum value of k we have to equate s^1 co-efficient equal to zero.

k_{min} Value < k < k_{max} Value.

for marginal stable $k_{max} = 0$

Problem

(1) $2s^3 + 10s^2 + 4s + k + 4 = 0$

(2) $G(s)\,H(s) = \frac{k}{s(s+2)(s+4)(s+6)}$. find the range of k & the value for marginal stable.

—x—

Root locus

The Root locus is nothing but the close loop poles path for the value of k from zero to ∞.

-> In R-H Criteria we can find only either poles located in L.H.S or R.H.S of S-plane where as in Root locus we can identify the nature of the system.

-> open loop T/F $G(s)H(s) = k\frac{N(s)}{D(s)}$

open loop T/F poles => $D(s) = 0$

open loop T/F zeros => $N(s) = 0$

Angle and Magnitude Condition :-

→ The close loop poles are given by the characteristic eqn i.e $1+G(S)H(S) = 0$

$$\Rightarrow G(S) \cdot H(S) = -1$$

→ Angle of $G(S) \cdot H(S) \Rightarrow \angle G(S) H(S) = \angle -1$

$$= \text{odd multipul of } \pm 180^\circ$$

$$= \pm(2q+1)\, 180^\circ$$

where $q = 0, 1, 2, \ldots$

Proposes

To check any point exist in the root locus or not i.e All the point in the Root locus must be satisfy the angle condition.

Problem

check whether the following pt exists on the root locus or not for $G(S) \cdot H(S) = \frac{k}{S(S+2)(S+4)}$ for $S = -1$ & $S = -3$.

Soln

for $S = -1$

$$\angle G(S) H(S) = \frac{\angle K}{\angle -1 \angle 1 \angle 3} = \frac{0^\circ}{\pm 180^\circ \cdot 0^\circ \cdot 0^\circ}$$

$$= \mp 180^\circ$$

Satified Angle condition.

for $S = -3$.

$$\angle G(S) H(S) = \frac{\angle K}{\angle -3 \angle -1 \angle 1} = \frac{0^\circ}{\pm 180^\circ \pm 180^\circ \cdot 0^\circ}$$

$= \mp 360^\circ$ not satified Angle condition.

So the given $s=-3$ is not inroot locus.

Magnitude Condition

$$G(s) \cdot H(s) = -1$$

$$\left| G(s)\, H(s) \right|_{\substack{\text{at a pt} \\ \text{which is on RL}}} = 1$$

→ The magnitude condition is valid only when angle condition satified.

Problem

find the System Gain at a pt $s = -2+j2$

fore $G(s)\, H(s) = \dfrac{k}{s(s+4)}$

Solⁿ

$$\angle G(s)\, H(s) = \frac{\angle k}{\angle -2+2j \;\; \angle 2+2j} = \frac{0}{+135^\circ \cdot 135^\circ}$$

$$= -180^\circ$$

Satisfy the angle condition.

$$\left| G(s)\, H(s) \right| = \left| \frac{k}{s(s+4)} \right|_{\text{at } s=-2+j2} = 1$$

$$= \left| \frac{k}{(-2+j2)(2+j2)} \right| = 1$$

$$\Rightarrow \left| \frac{k}{\sqrt{4+4}\,\sqrt{4+4}} \right| = 1 \Rightarrow \boxed{k = 8}$$

Rules for Constructing Root locus

Rule-1 Symmetrical :- Root locus must be symmetrical about real axis, because the location of zeros and poles are symmetrical about real axis.

Rule-2

No of root locus branch (LOCI)

① P > Z No of root ~~locus~~ locus branches = No of poles.

② P < Z No of root locus branches = No of zeros.

Actually the no. of poles = no. of zero because the Root locus branch starts at poles and ends at zero.

Rule-3

Real axis Root locus Branch

Real axis Root locus branch

-> A pt to exist real axis Root locus branch the sum of poles and zeros to the R.H.S of the pt should be odd.

-> A Root locus branches exist means there exist the path to the close loop poles.

-> Number of Root locus means number of path exist to the close loop poles.

Rule-4

Assymptotes :- It is the Root locus branch which are aprossess to ∞.

No of assymptotes is $N = P - Z$

angle of Assymptotes $\theta = \frac{(2q+1)180^\circ}{P-Z}$

where $q = 0, 1, 2 \dots [(P-Z)-1]$

The assymptotes are symmetrical about real axis.

The assymptotes shows the direction of zero towards the ∞.

Rule-5 Centroid

It is nothing but a intersection pt of assymptotes on the real axis.

$$\sigma = \frac{\sum \text{Real Part of Poles} - \sum \text{Real Part of zeros}}{P-Z}$$

The Centroid may be located any where on the real axis it may or mayn't be on the root locus branch.

Rule-6 Break pts

-> It is the pt where the poles meet.

classification of Break pts.

Break ~~ay~~ away pt :- The point at which the root locus Branches lives the real axis is called Break away point.

Break in point:- The point at which the Root locus Branches enter into the Real axis is called Break in point.

-> The Root locus Branches enter or ~~leve~~ leaves the real axis with a angle $\pm 180°/n$ where n is the no of poles at Break point.

Finding the existance ~~a~~ of Break pts.

case-1 If any two poles are adjusently placed in between exist the root locus branches their should be a minimum one Break away point in between adjusently place pole.

case-2

when ever any two zeros are adjusently placed in between their exist a root locus branches. their should be a one Break in point in between this two zeros.

note

At any location their exist a 2 or more poles ore zeros their should be a Break away ore Break in pt at that location respectively.

Cass-3

when ever left most side zero is their to the left most side zero their exist a Root locus branch then their should be the minimum one Break point to the left most side of zero when the $P > Z$ only.

Find the co-ordinate of the Break pt.

step-I → Replace $G(S)H(S) = -1$

step-II → Rewrite the above eqn in the from of $k = f(s)$.

step-III → ~~Differentiat~~ differentiate k with respect to S and find $\frac{dk}{ds} = 0$

step-IV → The roots of $\frac{dk}{ds} = 0$ gives the valid and invalid Break pt.

Rule-7

Intersection point with imaginary axis

step-1 :- Intersection point with imaginary axis can obtained by R-H Criteria. form the characteristic eqn $1 + G(S)H(S) = 0$

step-2 :- form the Routh array.

step-3 :- find the k ~~margy~~ marginal value

step-4 :- form the auxilary eqn of above line.

→ The roots of the auxilary eqn gives the ~~vaild~~ valid or ~~invaild~~ invalid intersection pt with imaginary axis.

→ The valid intersection pt is the one for which k-marginal is +ve.

Rule-8

Angle of departure | Angle of Arrival

→ Angle of departure is at complex conjugate pole.

→ Angle of arrival is at complex conjugate zero

Angle of departure :- It gives at which angle the pole depart from the initial position will give Angle of departure.

$$\phi_d = 180 - \phi \quad (\phi = \Sigma\phi_p - \Sigma\phi_z)$$

Angle of arrival :- It gives at which angle the zero arrive inner the complex zero

$$\phi_d = 180 + \phi \quad [\phi = \Sigma\phi_p - \Sigma\phi_z]$$

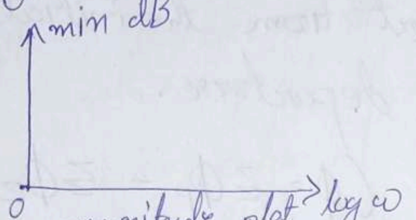
Bode Plots

Purpose :-

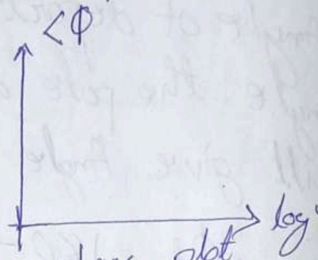
① To draw the open loop transfer function.

② To find close loop System stability, by using, gain margine, phase margine.

③ Bode Plots consists 2 plots.
magnitude plots & Phase plots.



magnitude plot



phase plot

note:- The Root locus & Nyquest plots are symmetrical about Real axis.

④ Bode plots are valides fore min phase system i.e finite pole and zeros must be left half of S-place.

⑤ The control system are low pass filter because at low frequency gives high magnitude as frequency increase magnitude decrease.

⑥ The general form of the system representation by

$$G(s)\,H(s) = \frac{k(1+sT_1)(1+sT_2)\cdots}{s^n(1+sT_a)(1+sT_b)\cdots}$$

The k gives the shift in the magnitude plot.

-> The phase plot independance of k value.

$$G(s)\,H(s) = k$$

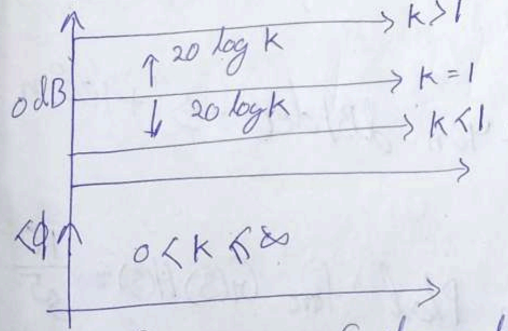
$$m\ \text{in dB} = 20 \log k$$

when $k=1 \Rightarrow m = 20 \log k = 20 \log 1$
$= 0\,dB$

no shift in the plot.

when $k>1 \Rightarrow m = 20 \log k = 20\,dB$
plot shifted up words.

when $k<1 \Rightarrow m = 20 \log k = -20\,dB$
plot shifted down words.



Corner frequency :- finite pole and zero location in the form at magintude.

defn :- The frequency at which the slope change from one frequency to other frequency.

G(S) H(S)	slope	phase angle
K	$\rightarrow$ 0 dB/dec	$\rightarrow$ 0°
$\frac{1}{S^n}$ (n-poles at origin)	$\rightarrow$ $-20n$ dB/dec	$\rightarrow$ $-90^\circ n$
S^n (n-zeros at origin)	$\rightarrow$ $+20n$ dB/dec	$\rightarrow$ $+90^\circ n$
$\frac{1}{(1+ST)^n}$	$\rightarrow$ $<$ CF $\rightarrow$ 0 dB/dec; $>$ CF $\rightarrow$ $-20n$ dB/dec	$\rightarrow$ $-90^\circ n$
$(1+ST)^n$	$\rightarrow$ $+20n$ dB/dec	$\rightarrow$ $+90^\circ n$
$\frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$	$\rightarrow$ $-40n$ dB/dec	$\rightarrow$ $-180^\circ n$
$\frac{S^2+2\xi\omega_n S+\omega_n^2}{\omega_n^2}$	$\rightarrow$ $+40n$ dB/dec	$\rightarrow$ $+180^\circ n$

Problem-1

Draw the Bode Plot for $G(S)H(S) = \frac{10}{S^5}$

Soln

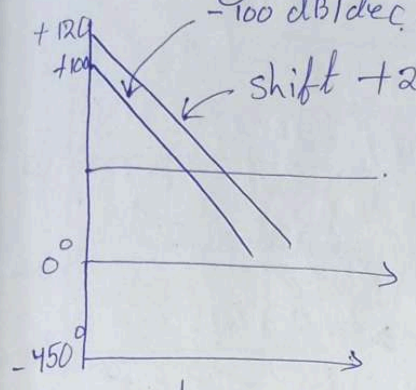
5 poles are at origin so 100 dB/dec

shift in magnitude plot = 20 log k

= 20 log 10

= 20 dB/dec

phase angle is $-90° \times 5 = -450°$



note :- when T/F consisting the poles and zero at origin the plot start with magnitude of opposite sine of the slope and it should be passes through 0 dB line intersecting at $\omega = 1$ and extended upto 1st corner frequency.

Problem-2

Draw the Bode plots for

$$G(S)\ H(S) = \frac{S^6}{100}$$

Soln

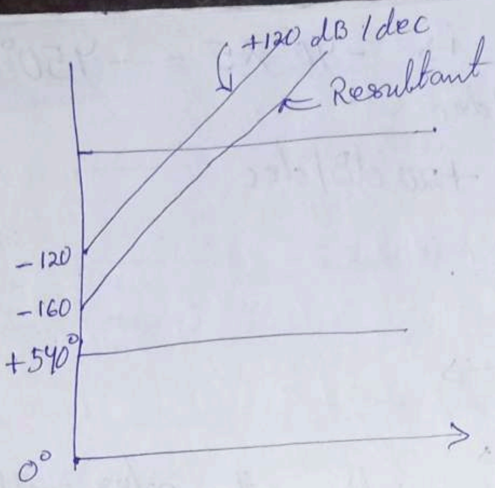
magnitude slope is $+20 \times 6$ dB/dec $= 120$ dB/dec

Shift in magnitude slope $= 20 \log k$

$k = 0.01$ $\qquad = 20 \log 0.01$

$\qquad = -40$ dB

Phase angle is $+90 \times 6 = 540°$



Problem - 3

Draw the Bode plot $G(s)H(s) = \frac{100S}{(S+2)(S+5)}$

Solⁿ

$$G(s)H(s) = \frac{100S}{(S+2)(S+5)} = \frac{100S}{2\times5\left(1+\frac{S}{2}\right)\left(1+\frac{S}{5}\right)}$$

$$= \frac{10S}{\left(1+\frac{S}{2}\right)\left(1+\frac{S}{5}\right)}$$

shift $= 20 \log k$ when $k = 10$

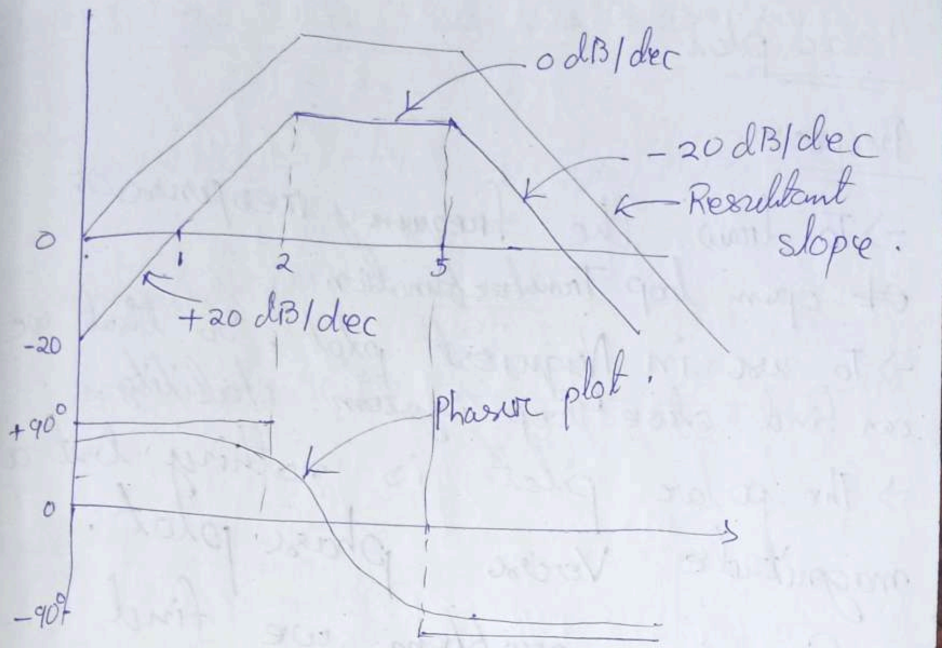
$= 20$ dB/dec

phase plot :- $90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{5}$

$= 90^\circ - 45$

$= 45^\circ$

note :- change in slope = (new slope - earlier slope)



0 dB/dec
-20 dB/dec
Resultant slope.
0
1
2
5
+20 dB/dec
-20
+90°
Phasor plot.
0
-90°

Polar plot

Purpose

-> To draw the frequency response of ~~of~~ open loop Transfer function.

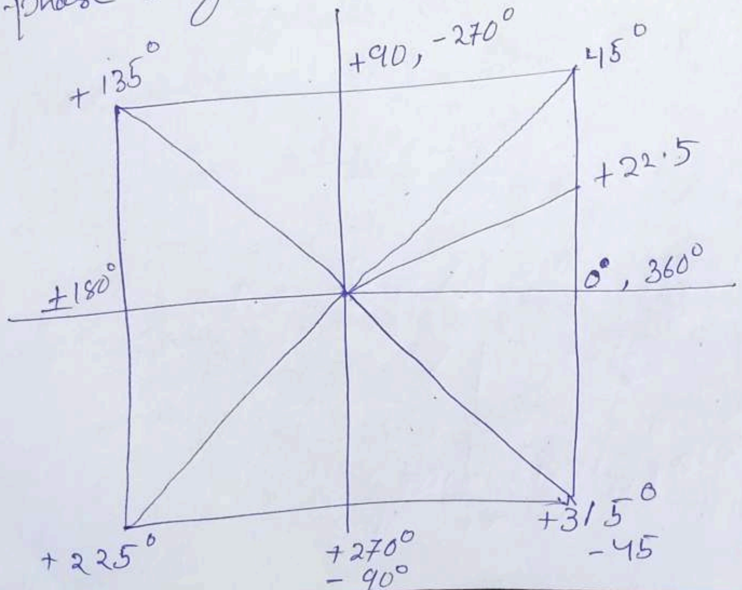
-> To use in Nyquiest plot. So that we can find close loop system stability.

-> The polar plot is nothing but a magnitude Verse phase plot.

-> for any problem we find magnitude (M) & phase angle ($\angle\phi$)

-> It will start from $\omega=0$ to end at $\omega=\infty$

-> Here we find magnitude (M) and the phase angle $\theta(\phi)$



Problem: Draw the Polar Plot for $G(s)H(s) = \frac{1}{s}$

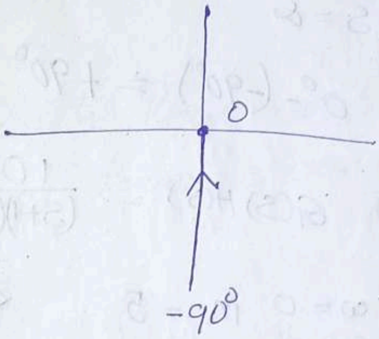
Soln

$$G(s)\,H(s) = \frac{1}{s} \Rightarrow G(j\omega)\,H(j\omega) = \frac{1}{j\omega}$$

$$\text{Mangitude } (M) = \frac{1}{\sqrt{\omega^2}} = \frac{1}{\omega}$$

$$\text{phase angle } \angle\phi = \frac{\angle 1}{\angle j\omega} = \frac{0^0}{90^0} = -90^0$$

ω	M	$\angle\phi$
0	∞	-90^0
1	1	-90^0
2	0.5	-90^0
5	0.2	-90^0
10	0.1	-90^0
⋮	⋮	
∞	0	-90^0



(2) Draw the polar plot for $G(s)\,H(s) = \frac{1}{s+1}$

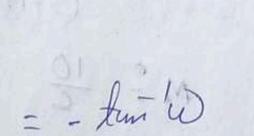
Soln

$$G(j\omega)\,H(j\omega) = \frac{1}{j\omega+1}$$

$$M = \frac{1}{\sqrt{\omega^2+1}}$$

$$\angle\phi = \frac{\angle 1}{\angle j\omega+1} = \frac{0^0}{\tan^{-1}\omega} = -\tan^{-1}\omega$$

ω	M	$\angle\phi$
0	1	0
1	0.707	-45^0
10	0.1	-86^0
⋮	⋮	⋮
∞	0	-90^0



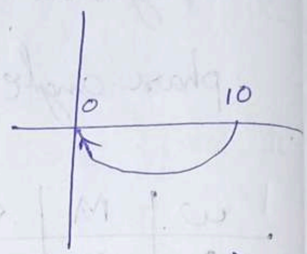
Angle direction = Starting Angle - Ending Angle.

= +ve clock wise

= -ve Anti clockwise.

<u>Problem</u>

① $G(s)\,H(s) = \dfrac{10}{s+1}$

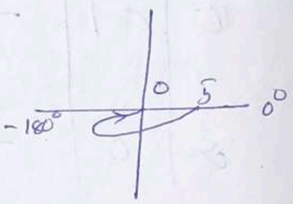


$\omega = 0$ $\quad M = 10 \quad \angle\phi = 0^\circ$
$s = 0$

$\omega = \infty$ $\quad M = 0 \quad \angle\phi = -90^\circ$
$s = \infty$

$0^\circ - (-90^\circ) = +90^\circ$ (+ve → clk wise)

② $G(s)\,H(s) = \dfrac{10}{(s+1)(s+2)}$

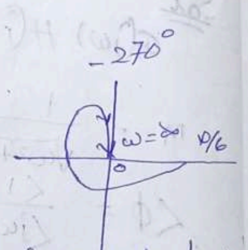


$\omega = 0 \quad M = 5 \quad \angle\phi = 0^\circ$
$s = 0$

$\omega = \infty \quad M = 0 \quad \angle\phi = -180^\circ$
$s = \infty$

$0^\circ - (-180^\circ) = 180^\circ$ (direction → clkwise)

③ $G(s)\,H(s) = \dfrac{10}{(s+1)(s+2)(s+3)}$



$\omega = 0 \quad M = \dfrac{10}{6} \quad \angle\phi = 0^\circ$
$s = 0$

$\omega = \infty \quad M = 0 \quad \angle\phi = -270^\circ$
$s = \infty$

$0 - (-270^\circ) = 270^\circ$ (direction clockwise

→ The addition of each finite pole shift the $\angle\phi$ by -90° in clk wise direction

Find the intersection pt with Real and imaginary axis ?

* In the previous problem, we have

$$G(s)\,H(s) = \frac{10}{(s+1)(s+2)(s+3)}$$

Expanding the term

$(s+1)(s+2)(s+3) = 0$

$(s^2+3s+2)(s+3) = 0$

$\Rightarrow s^3+3s^2+2s+3s^2+9s+6 = 0$

$\Rightarrow s^3+6s^2+11s+6 = 0$

The Intersection pt with Real axis

By equating imaginary term $=0$ or odd power of s terms $=0$

$s^3+11s = 0$

Put $s=j\omega$

$(j\omega)^3+11j\omega = 0$

$\Rightarrow -\omega^2+11 = 0$

$\Rightarrow \omega = \sqrt{11}$ rad/sec

$$\text{magnitude} = \left.\frac{10}{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}}\right|_{\omega=\sqrt{11}}$$

$$= \frac{10}{\sqrt{12\times15\times10}} = \frac{10}{\sqrt{4\times3\times5\times3\times4\times5}}$$

$$= \frac{10}{4\times3\times5} = \frac{1}{6}$$

The intersection pt with Imaginary axis By equating Real term = 0 or even power of s term = 0

$$6s^2 + 6 = 0$$

$$s^2 + 1 = 0 \Rightarrow s = \pm 1$$

$$\Rightarrow j\omega = \pm j$$

$$\omega = 1 \text{ rad/sec}$$

$$M = \frac{10}{\sqrt{(1+\omega^2)(4+\omega^2)(9+\omega^2)}} = \frac{10}{\sqrt{2 \times 5 \times 10}} = \frac{10}{10} = 1$$

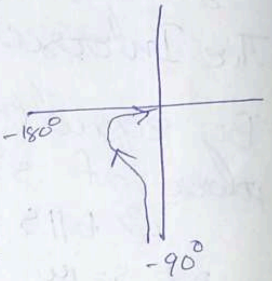
Problem

① $\frac{1}{s(s+1)}$

$\omega = 0$, $s = 0$ $M = \infty$ $\angle\theta = -90^0$

$\omega = \infty$, $s = \infty$ $M = 0$ $\angle\theta = -180^0$

clkwise

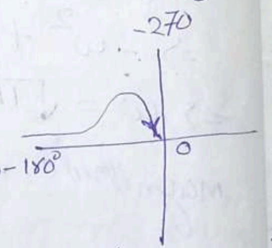


② $\frac{1}{s^2(s+1)}$

$\omega = 0$, $s = 0$ $M = \infty$ $\angle\theta = -180^0$

$\omega = \infty$, $s = \infty$ $M = 0$ $\angle\theta = -270^0$

clkwise

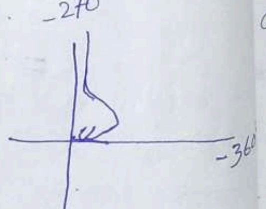


③ $\frac{1}{s^3(s+1)}$

$\omega = 0$, $s = 0$ $M = \infty$ $\angle\theta = -270^0$

$\omega = \infty$, $s = \infty$ $M = 0$ $\angle\theta = -360^0$

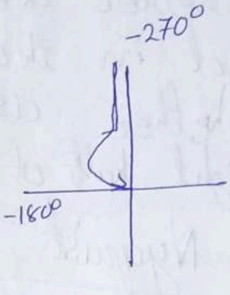
clkwise



So the addition of each pole at oregin shift the total plot by -90^0 in clkwise direction.

Problem

$$\frac{s+1}{s^3}$$



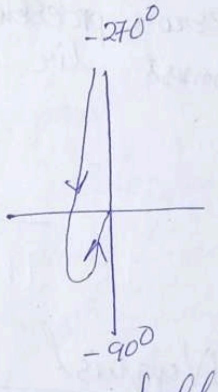
$\omega = 0$ $\quad$ $M = \infty$ $\quad$ $\angle\theta = -270^0$
$s = 0$

$\omega = \infty$ $\quad$ $M = 0$ $\quad$ $\angle\theta = -180^0$
$s = \infty$

Anti clockwise

Problem

$$\frac{(s+1)(s+2)}{s^3}$$



$\omega = 0$ $\quad$ $M = \infty$ $\quad$ $\angle\theta = -270^0$
$s = 0$

$\omega = \infty$ $\quad$ $M = 0$ $\quad$ $\angle\theta = -90^0$
$s = \infty$

Anticlockwise

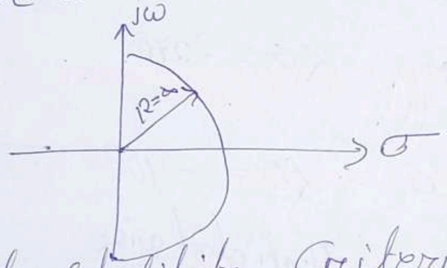
So the addition of each finite zero shift the total plot by -90^0 in Anticlockwise direction.

—: Nyquist Stability Critoria :-

The main purpose in study the stability of the closed loop system is to determine whether the characteristic equation has any root in the righ half of S-plane i.e whether $C(S)/R(S)$ has any pole in ~~H~~ right half of S-plane.

Nyquist Path or Nyquist Countour :-

This Countour encloses the right half of S-plane completely with an infinite radious of a semicircle and It assumes any poles or zeros present in right half of S-plane must lie within the infinite contour.



Nyquist Stability Criterion :-

→ If the path $G(S)H(S)$ of the open loop transfer function $G(S)\cdot H(S)$ corresponding to the Nyquist Contour in the S-plane encircle the point $(-1+j0)$ in the contour clockwise direction as many times as the no of right half S-plance poles of $G(S)H(S)$, then closed loop System is Stable.

-> The polar plot of Nyquist contour is called Nyquist plot.

-> The Number of poles in right half of S-plane is equal to no. of anticlock wise encirclement of $(-1+j0)$ point in Nyquist plot, the system is stable.

-> No. of zeros in the right half of S-plane is equal to the no of clock wise, encirclement of $(-1+j0)$ point in Nyquist plot, then system is stable.

-> $N = P - Z$

N = no. of anticlockwise encirclement of $(-1+j0)$ point, system is stable

-> It based on principle of argument in which states that If there are P Poles and Z zeros are enclosed by the S-plane then the corresponding $G(s)H(s)$ plane must encircle the origin $P - Z = N$ time.

-> To become a Open loop system stable then their should not be any open loop poles in the Right side of S-plane.

-> Open loop pole = pole of characteristic eqn which must be zeros in the Right half of S-plane

$P = 0$ or $N = P - Z = 0 - Z = -Z$

$\Rightarrow \boxed{N = -Z}$

→ To become a close loop system stable then theire should not be any close loop zeros in the Right half of S-plane.

→ close loop zero = Zeros of characteristic equation which must be zero in the Right half of S-plane.

$$Z = 0 \Rightarrow \boxed{N = P}$$

Nyquist Stability Criteria :-

It states that the no of encirclement about the critical point $(-1+j0)$ should be equal to poles of characteristic eqⁿ (open loop T/F poles which are Right half of S-plane)

Rules for Mapping the Nyquist Plot :-

step 1 :- Draw the polar plot.

step 2 :- fold with respect to real axis and take the image of the polar plot.

step 3 :- Observe the no of poles and zeros in the origin and draw the Half circle and the Half circle starts where the mirror image starts.

Problem

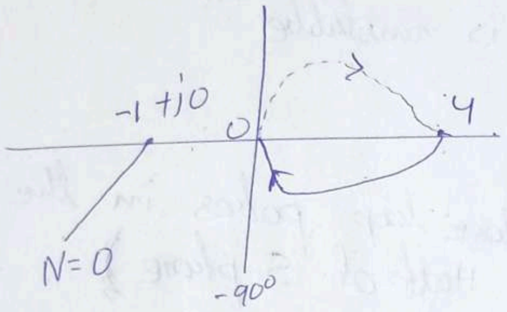
$G(S)\ H(S) = \dfrac{4}{S+1}$ $\quad M = \dfrac{4}{\sqrt{\omega^2+1}}$ $\quad \angle\theta = \dfrac{\angle 0^\circ}{\angle \tan^{-1}\frac{\omega}{1}}$

$= -\angle \tan^{-1}\omega$

Here ~~P = 0~~ Z = 0

$\omega = 0$	$M = 4$	$\angle\theta = 0^\circ$
$\omega = \infty$	$M = 0$	$\angle\theta = -90^0$

So N = P and the system is stable

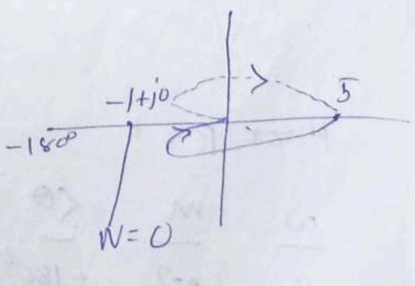


Problem -2

$G(S)\ H(S) = \dfrac{10}{(S+1)(S+2)}$

Here $M = \dfrac{10}{\sqrt{\omega^2+1}\ \sqrt{\omega^2+4}}$ $\quad \angle\theta = \dfrac{\angle 0^\circ}{\angle \tan^{-1}\omega\ \angle \tan^{-1}\frac{\omega}{2}}$

ω	M	$\angle\theta$
0	5	0°
∞	0	-180^0



So N = P
and the system is stable

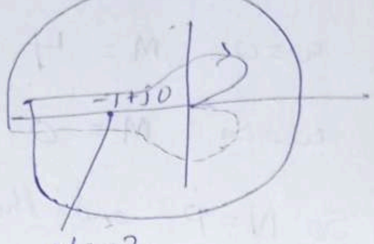
Problem-3

$$G(s)\,H(s) = \frac{10}{s^2(s+1)(s+2)}$$

$$\text{Here } M = \frac{10}{\omega^2\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}} \qquad \angle\theta = \frac{\angle 0^\circ}{\angle 180^\circ \angle \tan^{-1}\omega \angle \tan^{-1}\frac{\omega}{2}}$$

Here $Z = 0$

ω	M	$\angle\theta$
0	∞	$\angle -180^\circ$
∞	0	$\angle -360^\circ$



$N = -2$

$N \neq P$ so it is unstable

$N = P - Z$

$-2 = 0 - Z$

$\boxed{Z = 2}$ close loop poles in the Right Half of S-plane

Problem-4

$$G(s)\,H(s) = \frac{(s+2)}{(s+1)(s-1)}$$

$$M = \frac{\sqrt{\omega^2+4}}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+1}} \qquad \angle\theta = \frac{\angle \tan^{-1}\frac{\omega}{2}}{\angle \tan^{-1}\omega \;\angle \tan^{-1}(-\omega)}$$

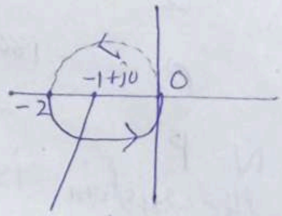
$$= -\angle\tan^{-1}\omega - (180 - \angle\tan^{-1}\omega) + \angle\tan^{-1}\frac{\omega}{2}$$

$$= -180^\circ + \tan^{-1}\frac{\omega}{2}$$

Here $P = 1$

ω	M	$\angle\theta$
0	2	-180°
∞	0	-90°

Anticlockwis



$N = 1$

So $N = P$ So close loop system is stable and open loop system is unstable